An Algorithm to Compute the Likelihood Ratio Test Statistic of the Sharp Null Hypothesis for Compliers

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Abstract

In a randomized experiment with noncompliance, scientific interest is often in testing whether the treatment exposure X has an effect on the final outcome Y [2, 1]. We have proposed a finite-population significance test of the sharp null hypothesis that X has no effect on Y, within the principal stratum of compliers, using a generalized likelihood ratio test [4].

As both the null and alternative hypotheses are composite hypotheses (each comprising a different set of distributions), computing the value of the generalized likelihood ratio test statistic [6] requires two maximizations: one where we assume that the sharp null hypothesis holds, and another without making such an assumption.

In our work [4], we have assumed that there are no Always Takers, such that the nuisance parameter is a bivariate parameter describing the total number of Never Takers with observed outcomes $y=0$ and $y=1$. Extending the approach to the more general case in which there are also Always Takers would require a nuisance parameter of higher dimension that describes the total number of Always Takers with observed outcomes $y=0$ and $y=1$ as well. This increases the size of the nuisance parameter space and the computational effort needed to find the likelihood ratio test statistic.

We present a new algorithm that extends [5] to solve the corresponding integer programs in the general case where there are Always Takers. The procedure for the finite-population significance test may be illustrated using a toy example from [3].

References

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