Inference of Cause and Effect with Unsupervised Inverse Regression

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Abstract

We address the problem of causal discovery in the two-variable case, given a sample from their joint distribution. Since $X \to Y$ and $Y \to X$ are Markov equivalent, conditional-independence-based methods [Spirtes et al., 2000, Pearl, 2009] can not recover the causal graph. Alternative methods, introduce asymmetries between cause and effect by restricting the function class (e.g., [Hoyer et al., 2009]).

The proposed causal discovery method, CURE (Causal discovery with Unsupervised inverse REgression), is based on the principle of independence of causal mechanisms [Janzing and Schölkopf, 2010]. For the case of only two variables, it states that the marginal distribution of the cause, say $P(X)$, and the conditional of the effect given the cause $P(Y|X)$ are "independent", in the sense that they do not contain information about each other. This independence can be violated in the backward direction: the distribution of the effect $P(Y)$ and the conditional $P(X|Y)$ may contain information about each other because each of them inherits properties from both $P(X)$ and $P(Y|X)$, hence introducing an asymmetry between cause and effect. For deterministic causal relations ($Y = f(X)$), all the information about the conditional $P(Y|X)$ is contained in the function f. In this case, previous work [Janzing et al., 2012] formalizes "independence" as uncorrelatedness between $\log f'$ and the density of $P(X)$, both viewed as random variables. For non-deterministic relations, we propose an implicit notion of independence, namely that $p_{Y|X}$ cannot be estimated based on p_X (lower case denotes density). However, it may be possible to estimate $p_{X|Y}$ based on the density of the effect, p_Y.

In practice, we are given empirical data $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{y} \in \mathbb{R}^N$ from $P(X, Y)$ and estimate $p_{X|Y}$ based on $\mathbf{y}$ (intentionally hiding $\mathbf{x}$). The relationship between the observed $\mathbf{y}$ and the *latent* $\mathbf{x}$ is modeled by a Gaussian Process (GP). Then, the required conditional $p_{X|Y}$ is estimated as $\hat{p}^{\mathbf{y}}_{X|Y} : (x, y) \mapsto p(x|y, \mathbf{y})$, with $p(x|y, \mathbf{y})$ estimated by marginalizing out the latent $\mathbf{x}$ and the GP hyperparameters.

CURE infers the causal direction using the procedure above two times: one to estimate $p_{X|Y}$ based only on $\mathbf{y}$ and another to estimate $p_{Y|X}$ based only on $\mathbf{x}$. If the first estimation is better, $X \to Y$ is inferred. Otherwise, $Y \to X$. To evaluate the conditional's estimation, we compare it to the one using both $\mathbf{x}$ and $\mathbf{y}$. CURE was evaluated on synthetic and real data and often outperformed existing methods. On the downside, its computational cost is comparably high. This work was recently published at AISTATS 2015 [Sgouritsa et al., 2015].

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