Modeling and Decision Support for the Firms' Pricing Policy under a Chaotic Dynamic of Market Prices

Ekaterina V. Orlova

Ufa State Aviation Technical University, Ufa, Russia
ekorl@mail.ru

Abstract. The article presents the results of the study of nonlinear market prices dynamic, simulated using game theory model, maps and theory of bifurcation. Market pricing is presented as a two-dimensional map. Qualitative analyses of the firms' pricing system properties using the fixed points, analysis of the trajectories near these fixed points were fulfilled Simulation of the market prices dynamic showed that the fixed point of the map coincides with the local Nash equilibrium, so the analysis of the stability of Nash equilibrium was done based on the maps' fixed points sustainability analysis. Numerical simulation results were visualized, bifurcations of the fixed point were identified and transition from periodic to chaotic mode was demonstrated. Sustainability analysis was carried out with using Jacobian. The mechanism for the pricing decision support, allowing under a chaotic market dynamics to ensure maximum efficiency of the firms was proposed.

Keywords: nonlinear economic dynamics; visualization of chaos; pricing decision making; fixed point stability criteria

1 Introduction

The last two decades the scientific literature has been widely discussing the concept of deterministic chaos, chaotic dynamics occurring in different systems. Different approaches and methods for chaos control in theoretical and applied problem solution are suggested. At the same time a lot of attention is given to managing the impact of low power, meaning that the system has a number of characteristics, inherent properties and laws which allow to achieve expected results using weak management actions (without spending of significant resources).

In a number of studies of nonlinear systems dynamic (physical, chemical, biological, economic), it was found that the dynamic chaos mode is a typical phenomenon. Chaotic properties are manifested in a variety of systems, and if the chaos is not found, the reasons for this may be either the existence of chaos in a small area of the parameter space, or it is out of range of parameters.

Research associated with the problems of predictability of chaotic systems, control of system dynamics and stabilization of chaos is rapidly developing in different fields. Theoretical and applied studies in these fields have revealed an unexpected property of chaotic dynamical systems: they are controlled by external actions [3,13,18,21]. That is, by smaller impacts it is possible to significantly affect the dynamics of chaotic systems, to stabilize their dynamics, transferring it from the chaotic mode to the required periodic mode.

In relation to economic systems the following studies in the field of chaos control were conducted: [1,2,5–8,11,12,14,15,19,20]. Chaos control is proposed on the basis of production cost reduction [2], an increase in investment activity [11,19], or control impacts are confirmed on the basis of revealed connection between the previous and current variables values, i. e. by effective use of feedback [2,19,22].

2 Phenomenon of Chaotic Market Dynamic: the Model

Chaotic systems are a class of uncertain models that differs from deterministic and stochastic systems in their properties [4]. In deterministic systems it is possible to build the future system trajectory from the initial state to infinite time interval. In stochastic systems it is possible to estimate the future system state into short time interval,determined by the prediction accuracy.

We consider a duopoly as a market model, when two firms-producers cooperate on the market, and the pricing process is controlled by price (which firms offer there differential products to consumers) change. Further it will be illustrated that the pricing system is chaotic and then we will offer the decision support model for stabilization market prices dynamic.

The main precondition of the model:

- The model of duopoly (model of price competition) with a differentiated product in which the consumer demand function is given a utility function with constant elasticity of substitution is used;
- Pricing dynamics is modeled as a map (recurrence relations);
- Modeling of pricing decisions is realized with using the game theory and methods of nonlinear dynamic.

The model designations: $i \in I$ – producer's number; p_i – price of the i-th producer (firm); q_i – the volume of sales of i-th producer; $q_i(\bar{p})$ – demand for the product of i-th producer; c_i – unit costs for the product of i-th producer; $\Pi_i(\bar{p}) = (p_i - c_i)\, q_i(\bar{p})$ – profit of i-th producer.

Prices vector $\bar{p}^* = (p_i^*, i \in I)$ is a local Nash equilibrium when it satisfies the following conditions of local maxima conditional of the profit function:

$$\frac{\partial \Pi_i}{\partial p_i} = 0, \quad \frac{\partial^2 \Pi_i}{\partial (p_i)^2} < 0.$$

The strategy of i-th producer is a price changing, which is proportional to the change of its profits with some constant $k_i > 0$ in order to maximize their effectiveness, i. e. to achieve the global sustainable Nash equilibrium. The model of competitive interaction of firms described by two-dimensional system of differential equations was offered in [16, 17] and represented as:

$$\begin{cases} p_1(t+1) = p_1(t) + k_1 \frac{\left(-p_1^2(t)p_2(t)+2c_1p_1(t)p_2(t)+c_1p_2^2(t)\right)}{\left(p_1^2(t)+p_1(t)p_2(t)\right)^2}, \\ p_2(t+1) = p_2(t) + k_2 \frac{\left(-p_1(t)p_2^2(t)+2c_2p_1(t)p_2(t)+c_2p_1^2(t)\right)}{\left(p_2^2(t)+p_1(t)p_2(t)\right)^2}, \end{cases} \tag{1}$$

where $p_1(t)$, $p_2(t)$ are product prices of first and second producers at discrete time t; second terms in both equations show the prices changing in period t, and how this changing will affect the price in the next period $(t+1)$. Parameters k_1 and k_2 characterizes the prices increasing due to changes in the firms' pricing policy; c_1 and c_2 represents a production cost of the first and second producer respectively.

Nash equilibrium, characterized by a pair of prices p_1^* and p_2^*, is the solution of the following equations:

$$\begin{cases} p_1^* = c_1 + c_1\sqrt{1 + \frac{p_2^*}{A_1}}, \\ p_2^* = c_2 + c_2\sqrt{1 + \frac{p_1^*}{A_2}}. \end{cases} \tag{2}$$

Analyze the evolution of the system (1) according to the parameter values k_1, k_2, A_1, A_2 identify the area of stability, bifurcations and chaos. Such analysis has been made based on the known analytical and graphical criteria of dynamic chaos [10, 13]: the Lyapunov exponents, bifurcation diagrams, attractors of the system by varying the system parameters. Then, in order to control the system dynamic on the basis of the revealed laws, it is necessary to find the method for parameters changing in order to provide an expected mode of the system dynamic. For each firm, this means the monitoring and controlling of their costs A_i and changing in profit for the period and the selection of appropriate control k_i. In practice this means the variation of price, which causes both firms to balance interests, i. e. the Nash equilibrium.

From the economic point of view, this means that firms choose the mode (and parameters), which will lead to a change in the market, resulting in price levels will evolve predictable dynamic. If such control is permissible, then a transition to the market equilibrium will proceed, that ensure for each firm the maximum effectiveness.

The fixed point (p_{10}, p_{20}) of the system (1) is a point which goes into itself under a single iteration of the map and is determined on the basis of equations:

$$\begin{cases} p_{10} = f(p_{10}, p_{20}), \\ p_{20} = g(p_{10}, p_{20}), \end{cases}$$

where f – the function $p_1(t+1)$ of $p_1(t)$ and $p_2(t)$, g – the function $p_2(t+1)$ of $p_1(t)$ and $p_2(t)$ of the map (1).

The decision of the latter system of equations will obviously be the same as the solution of the system (2). Therefore, the fixed point of map (1) coincides with the Nash equilibrium, for a certain competitive interaction between firms. The nature of the stability of a fixed point is defined by its multipliers, which are the eigenvalues of the perturbation matrix (Jacobian) and their number is equal to the dimension of the display. The bifurcation analysis and stability analysis of two-dimensional maps is carried out on the basis of parameters – invariants of Jacobi matrix. For the two-dimensional map there are track and Jacobian of Jacobi matrix. Jacobi matrix of the dynamic system (1) at a fixed point is as follows:

$$\widehat{M} = \begin{pmatrix} f'_{p_1} & f'_{p_2} \\ g'_{p_1} & g'_{p_2} \end{pmatrix}_{(p_{10}, p_{20})}. \tag{3}$$

Eigenvalues of this matrix are multiples of the map (1) μ_1 and μ_2, for which the relation is performed: $\mu^2 - S\mu + J = 0$, where S and J – two invariants of Jacobi matrix – track and Jacobian, also $S = \mu_1 + \mu_2$, $J = \mu_1 \cdot \mu_2$. In accordance with a triangle of stability [9], the conditions of stability of a fixed point are presented as:

$$\begin{cases} 1 - S + J > 0, \\ 1 + S + J > 0, \\ J < 1. \end{cases} \tag{4}$$

3 Pricing Decision Making: Visualization and Justification

We form the prices Nash equilibrium p_1^* and p_2^*, and find the conditions to achieve and maintain this balance for given costs A_i. If we fix a cost A_1,vA_2 at the level 0.07 and 0.12 respectively, the Nash equilibrium prices (2) are: $p_1^* = 0.15$; $p_2^* = 0.23$. Jacobi matrix (3) at this point would be:

$$\widehat{M} = \begin{pmatrix} 1 - 30.5k_1 & 4k_1 \\ 2.9k_2 & 1 - 9.9k_2 \end{pmatrix}_{(p_1^*, p_2^*)}$$

for which the characteristic polynomial $\mu^2 - S\mu + J = 0$ has a trace and the Jacobian:

$$S = 2 - 9.9k_2 - 30.5k_1,$$
$$J = (9.9k_2 - 1)(30.5k_1 - 1) - 11.7k_1k_2 = 290.25k_1k_2 - 30.5k_1 - 9.9k_2 + 1.$$

The equilibrium stability conditions formulated by the system (4). Solving this system with respect to k_1 and k_2 we obtain the range of the speed of adjustment k_1 and k_2 for Nash equilibrium (0.15; 0.23):

$$\begin{cases} 290.25k_1k_2 > 0, \\ 290.25k_1k_2 - 61k_1 - 19.8k_2 + 4 > 0, \\ 290.25k_1k_2 - 30.5k_1 - 9.9k_2 + 1 < 1. \end{cases}$$

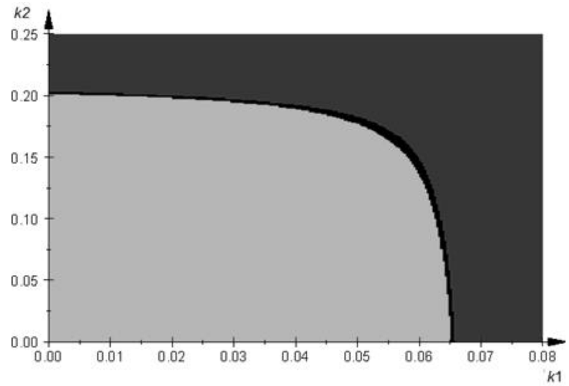

Fig. 1. Decision making area (gray) for the parameters k_1 and k_2

The solution of this system of inequalities that define the triangle of stability for the fixed point of the two-dimensional map prices (1), is shown in Fig. 1.

Thus, the identified area of admissible values of adaptive price parameters k_1 and k_2 is highlighted in gray. To ensure the stability of the Nash price equilibrium (0.15; 0.23) in the conditions of the given cost values of both firms at 0.07 and 0.12 will allow the use of such firms adaptation strategy, which is based on a set of adaptation options combinations that are within the acceptable area.

As soon as the control parameters deviate from the permissible values, there occurs the equilibrium stability loosing and the system goes to another unstable mode – chaos. That is, in any initial price of firms if they use a pricing strategy in accordance with the values determined above, the firm definitely reaches the Nash equilibrium. These pricing values are not able to change the Nash price equilibrium, so to increase the efficiency the firm should use the proposed decisions.

Figure 2 shows the chaotic attractors of system (1) in terms of unit costs $A_1 = 0.07$; $A_2 = 0.12$ and variations in the parameters k_1, k_2. For instance, when the values of the chaotic attractor of price adaptation parameters are $k_1 = 1$, $k_2 = 1.15$ it demonstrates the chaotic pricing system dynamic. This mode is determined by nonlinear system properties and manifests itself in an exponentially rapid divergence of initially close trajectories in a bounded phase space.

The chaotic nature of the prices system dynamics is due to the instability of the phase trajectories, the growth of small initial perturbations in time, mixing elements of the phase space and, as a consequence, leads to unpredictable system dynamic over long term.

In making pricing decisions in addition to the criterion of economic efficiency the firms must take into account the objective nature of the market dynamics as a whole and take into account the possibility of a chaotic regime of market dynamics.

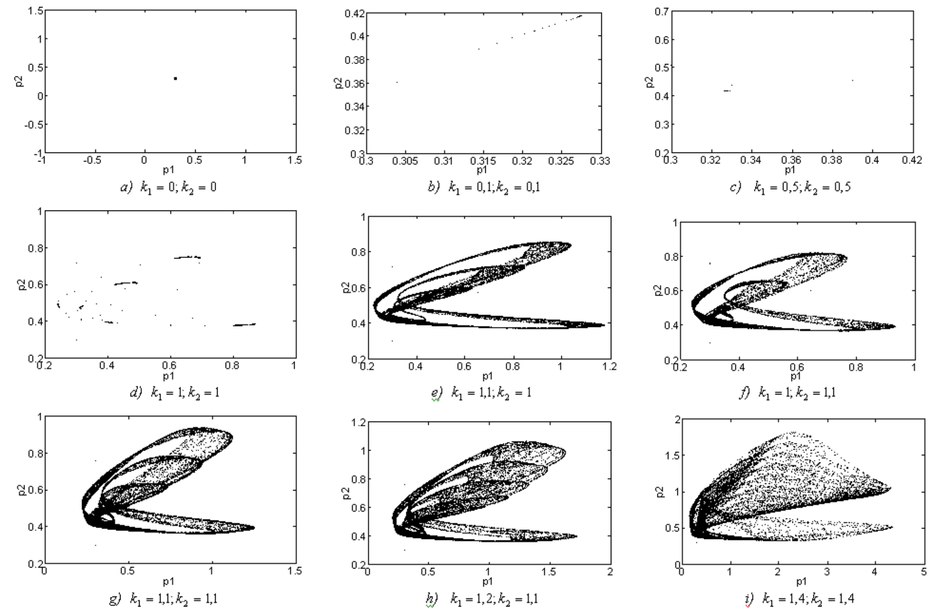

Fig. 2. Chaotic attractors of market price system (1) under variations of parameters k_1 and k_2 (a-i)

4 Results and conclusions

The described approach for firms pricing using the methods of game theory, nonlinear dynamics and bifurcation theory provides a new perspective on the dynamics of the process of competitive interaction of the firms and makes it possible to conduct a qualitative and visual analysis of the system properties with help of the singular points of the phase space (fixed points) and to analyze the systems trajectories near these fixed points.

It is shown that the market pricing system has complex and diverse types of dynamics, so that the structure of the phase space and its dependence on the parameters of this structure are very complex. The phase space of the system is heterogeneous and has two basic types of system dynamic – stability and chaos.

Prices dynamics is modeled using a two-dimensional map; coordination of firms' pricing decisions is based on monitoring the stability of the Nash equilibrium. The analysis of the developed model shows that the Nash equilibrium coincides with the map fixed point prices. Therefore, the analysis of Nash equilibrium stability is carried out on the basis of the analysis of the map fixed points sustainability. Numerical simulations are demonstrated the existence of fixed point bifurcations. Chaos in market pricing model means that when one firm change its price even slightly this can lead to unpredictable market prices changing of another producers and total market in long term. Therefore, all producers must have the tools of chaos control.

In order to avoid unexpected chaotic dynamics in market prices the mechanism for decision making support is offered and is based on the price changing proportional to marginal profit changing of each firm. These mechanism would ensure the stability of the Nash equilibrium, and therefore would balance the firms' economic interests, would coordinate price decisions and maintain maximum firms efficiency.

For the complex research and analysis of firms competitive interaction we use the author program for modeling and visualization of nonlinear pricing dynamics and decision support in firms price strategies. The program is designed to simulate the strategic cooperation in the firms pricing process, use a four-parameter map and form the optimal pricing policy in oligopoly, provided the effective control and decision making under prices chaotic dynamics. The program has the following functions:

- Assessment of local Nash equilibrium of prices;
- Identification of bifurcations of fixed price points in the map;
- Identification of modes of stability and dynamical chaos in pricing;
- Identification of transition scenarios to dynamical chaos;
- Forming the pricing decisions, ensuring stable mode of market prices dynamic.

References

1. Agliari, A., Gardini, L., Puu, T.: Global bifurcations in duopoly when the cournot point is destabilized through a subcritical neimark bifuraction, URL: http://www.cerum.umu.se/digitalAssets/18/18883_cwp_66_03.pdf
2. Ahmed, E., Elsadany, A.A., Puu, T.: On bertrand duopoly game with differentiated goods. Applied Mathematics and Computation 251, 169–179 (2015)
3. Ahmed, E., Hassan, S.Z.: On controlling chaos in cournot games with two and three competitors. Nonlinear Dynamics, Psychology, and Life Sciences 4, 189–194 (2000)
4. Andrievskij, B.R., Fradkov, A.L.: Chaos control: Methods and applications. Automation and Remote Control 4, 3–34 (2004)
5. Farmer, J.D., Ott, E., Yorke, J.A.: The dimension of chaotic attractors. Physica D: Nonlinear Phenomena 7(1), 153–180 (1983)
6. Feichtinger, G.: Nonlinear Threshold Dynamics: Further Examples for Chaos in Social Sciences, pp. 141–154. Springer Berlin Heidelberg, Berlin, Heidelberg (1992)
7. Hołyst, J.A., Urbanowicz, K.: Chaos control in economical model by time-delayed feedback method. Physica A: Statistical Mechanics and its Applications 287(3–4), 587–598 (2000)
8. Kopel, M.: Improving the performance of an economic system: Controlling chaos. Journal of Evolutionary Economics 7(3), 269–289 (1997)
9. Kuznetsov, A.P., Savin, A.V., Sedova, J.V., Turukina, L.V.: Bifurcation Maps. "Science" Publishing Center Ltd., Saratov, Russia (2012)
10. Kuznetsov, S.P.: Dynamical Chaos, vol. 2006. FIZMATLIT, Moscow, Russia
11. Lorenz, H.W.: Nonlinear dynamical economics and chaotic motion. Springer-Verlag (1989)

12. Loskutov, A.Y.: Non-linear optimization of chaotic dynamics of the market. Economics and Mathematical Methods 46(3), 58–70 (2010)
13. Loskutov, A.Y., Mikhailov, A.S.: Fundamentals of the Theory of Complex Systems. Institute of Computer Science, Moscow – Izhevsk, Russia (2007)
14. Moon, F.: Chaotic Oscillations: Introductory Course for Scientists and Engineers. Mir, Moscow, Russia (1990)
15. Neimark, Y.I., Ostrovsky, A.V.: On some models of pricing in the market economy. Izvestiya VUZ. Applied Nonlinear Dynamics 6, 35–41 (1999)
16. Orlova, E.V.: Model for economic interests agreement in duopoly's making pricing decision. Computer Research and Modeling 7(6), 1309–1329 (2015)
17. Orlova, E.V.: Concept for industrial and economic systems management based on criteria coordination of interested agents. Program Engineering 2, 86–96 (2016)
18. Ostrovsky, A.A.: About one class of models of competitive pricing in the market economy. Differential Equations and Control Processes 2, 58–77 (2000)
19. Puu, T.: Nonlinear Economic Dynamics. Springer Berlin Heidelberg, Berlin, Heidelberg (1991)
20. Puu, T.: Attractors, Bifurcations, and Chaos: Nonlinear Phenomena in Economics. Springer Berlin Heidelberg, Berlin, Heidelberg (2000)
21. Schuster, H.G., Just, W.: Deterministic chaos : an introduction. Wiley-VCH, Weinheim (2005)
22. Stachurski, J.: Economic Dynamics: Theory and Computation. MIT Press (2009)