Forming of the Competitive Investment Programs for Enterprises

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Abstract. The paper presents three economic-mathematical models for the formation of the company's investment program based on: (1) principle of guaranteed net present value; (2) principle of maximizing of the average expected net present value under predetermined upper estimation of its dispersion; (3) principle of maximizing the average expected net present value under predetermined upper estimation of the probability of its inaccessibility. Proposed solutions of the problems allow us to give a system estimation of the enterprise investment attractiveness which can be used in selecting an effective investment portfolio based on risk appetite of decision makers.

Keywords: investment program, net present value, risk dispersion, probability, stochastic programming

1 Introduction

Currently, there are many models of the choice of investment policy. The best known model of financial markets is the model of Markowitz-Tobin for portfolio management [6, 10]. This model allows maximization of the expected result with acceptable risk. The main element of control is a periodic portfolio diversification.

To implement the strategic objectives of the enterprise also requires effective management of its investment activities. The goal of this management is the most efficient implementation of possible projects bringing the maximum financial result with minimal risk under limited investment resources and the uncertainty of their volumes [2, 3, 9, 12, 13, 16, 17]. In this case the investments are long term, and the investment program is essentially a strategic plan for development of the company [3, 13, 16, 17]. This plan is calculated for certain time, and includes a list of various projects which are listed with the detail volumes of financial investments.

The aim of the paper is to show mathematical models determining the optimal investment program for the enterprise, i. e. finding the order of implementation of many independent projects. There are some known heuristic algorithms [3, 14] for building an investment program which provides the system of author's preferences. Analyses of the set of effective investment programs, each of which: (i) maximizes the expected net present value (NPV) under a certain level of risk of impossibility to execute the program, (ii) ensures minimal risk of impossibility of the program for a certain value of the expected NPV, – are more reasonable way for selection of the optimal enterprise investment program. As a risk measure, either variance of NPV by analogy with the approach of Markowitz-Tobin [6, 10] at formation of a portfolio of securities [4, 7], or the probability of inaccessibility of the desired mean NPV [5, 8] can be used.

Construction of the efficient investment programs set (i. e. Pareto set by the criteria space of "risk – NPV") allows to select an effective investment program taking into account the risk appetite of decision makers. The paper presents mathematical models of building a set of efficient investment programs.

The article consists of four sections, conclusion and bibliography (17 references). Designations and the basic relations are introduced in section 2. A mathematical model implementing the principle of guaranteed payoff is presented in section 3. Search efficient portfolios under uncertainty and risk [10] are discussed in the section 4. Two more mathematical models based on different definitions of the concept of "risk": (i) variance of NPV, (ii) unattainability probability of expected NPV are proposed. Model example of a problem and the numerical solution for a variety of options for building the investment program of the enterprise are considered in section 5. Conclusion summaries the study.

2 General formulation of the problem

Main criterion for forming an optimal investment program of the enterprise is NPV of the investment program [1, 11]. Let

- $P = \{p_1, p_2, \dots, p_n\}$ be set of n of investment projects that can be included to the investment program;
- $L = \{l_1, l_2, \dots, l_n\}$ be set of durations of implementation of investment projects (i. e. accounting period);
- m be planning horizon (the number of billing periods);
- $R = \{r_0, r_1, \dots, r_{m-1}\}$ be fixed financial resources or funding the company's investment program at billing periods.

Each of the investment projects p_j, $j = 1, 2, \dots, n$ can be characterized by two parameters:

- value C_{js} of the net presents value of the project p_j that started during s-th period;
- need volume I_{jsi} to finance the investment project p_j launched at the period s over current period i.

Indicators of income and expenditure are predictable values. They depend on a number of factors. Therefore it is advisable to consider C_{js} and I_{jst} as random variables. We receive interval estimations of the net present value $[\underline{C}_{js}, \overline{C}_{js}]$, needs $[\underline{I}_{jst}, \overline{I}_{jst}]$, and financial resources of the enterprise $[\underline{r}_i, \overline{r}_i]$ based on a retrospective analysis for each project p_j, $j = 1, 2, \dots, n$, and for all settlement periods i, $s = 1, 2, \dots m$.

Let us introduce the boolean variables

$$x_{js} = \begin{cases} 1, & \text{beginning of the project } p_j \text{ is period } s, \\ 0, & \text{beginning of the project } p_j \text{ is not period } s. \end{cases} \tag{1}$$

Realizable subset of projects of the set P is a subset of projects that can be financed within the available financial resources for all settlement periods $i, s = 1, 2, \ldots m$. NPV of the investment program is the sum of discounted net income of projects included into the investment program [11, 13].

Since the implementation of the investment project p_j may begin no later than at the period $m - l_j$, then the following condition

$$\sum_{s=0}^{m-l_j} x_{js} \leq 1, \quad j = 1, 2, \ldots, n \tag{2}$$

holds. Conditions for realization of the investment program may be written in the form

$$\sum_{j=1}^{n} \sum_{s=0}^{i} I_{jsi} x_{js} \leq r_i, \quad i = 0, 1, 2, \ldots, m-1. \tag{3}$$

If investment project p_j may be included into the investment program then, in view of (2), its NPV be

$$C_j = \sum_{s=0}^{m-l_j} C_{js} x_{js}. \tag{4}$$

NPV of the whole investment program is equal to

$$C = \sum_{j=1}^{n} C_j = \sum_{j=1}^{n} \sum_{s=0}^{m-l_j} C_{js} x_{js}. \tag{5}$$

3 The maximin strategy

Application cautious strategy aimed at maximizing the guaranteed NPV is reduced to the solution of the problem

$$\underline{C}(x) = \sum_{j=1}^{n} \sum_{s=0}^{m-l_j} \underline{C}_{js} x_{js} \to \max_{x \in D}, \tag{6}$$

here $x = \{x_{js} : j = 1, 2, \ldots, n, \ s = 0, 1, 2, \ldots m - l_j\}$, admissible set B satisfies the constraints

$$\sum_{j=1}^{n} \sum_{s=0}^{i} \overline{I}_{jsi} x_{js} \leq \underline{r}_i, \quad i = 0, 1, 2, \ldots, m-1; \tag{7}$$

$$\sum_{s=0}^{m-l_j} x_{js} \leq 1, \quad j = 1, 2, \ldots, n; \tag{8}$$

$$x_{js} \in \{0, 1\}, \quad j = 1, 2, \ldots, n, \quad s = 0, 1, .., m - l_j. \tag{9}$$

We reach the warranty of optimal value of the problem (6)–(9) due to the use of lower bounds $\underline{C}_{js}$ for NPV, and $\underline{r}_{js}$ for volumes of financing for periods, and $\overline{I}_{jst}$ for upper bounds for all financing needs.

The problem (6)–(9) is the boolean linear programming problem with a non-negative matrix of conditions, so it can be solved by pseudo polynomial algorithm based on dynamic programming.

4 Building effective investment programs under risk

It is possible to look for optimal investment program of the enterprise in set of effective investment programs (i. e. Pareto set in the space of criteria "risk – NPV"). Intelligent decision support systems allow to select the most suitable investment program based on the identified system decision-makers preferences.

4.1 NPV dispersion as risk measure

Let us use the expectation of NPV of the investment program as a measure of income, and its variance as the risk measure.

Assuming that the net present value of each project $p_j \in P$ is uniformly distributed in the interval $[\underline{C}_{js}, \overline{C}_{js}]$ for all $s = 1, 2, \ldots, l_j$ we find the expectation of net present value

$$\mathbf{E}\{C(x)\} = \mathbf{E}\left\{\sum_{j=1}^{n}\sum_{s=0}^{m-l_j} C_{js}x_{js}\right\} = \sum_{j=1}^{n}\sum_{s=0}^{m-l_j} (x_{js} \cdot \mathbf{E}\{C_{js}\}) =$$
$$= \sum_{j=1}^{n}\sum_{s=0}^{m-l_j} \left(\frac{\overline{C}_{js} + \underline{C}_{js}}{2} \cdot x_{js}\right).$$

We find the variance of the net present value given the nature of the Boolean variables x and independence between the net present value of the various projects

$$\mathbf{D}\{C(x)\} = \mathbf{D}\left\{\sum_{j=1}^{n}\sum_{s=0}^{m-l_j} C_{js}x_{js}\right\} = \sum_{j=1}^{n}\sum_{s=0}^{m-l_j} (x_{js} \cdot \mathbf{D}\{C_{js}\}) =$$
$$= \sum_{j=1}^{n}\sum_{s=0}^{m-l_j} \left(\frac{(\overline{C}_{js} - \underline{C}_{js})^2}{12} \cdot x_{js}\right).$$

Thus, the construction of an efficient investment program at risk is reduced to problems

$$\mathbf{E}\{C(x)\} = \sum_{j=1}^{n} \sum_{s=0}^{m-l_j} \left(\frac{\overline{C}_{js} + \underline{C}_{js}}{2} \cdot x_{js} \right) \to \max_{x \in D:\, \mathbf{D}\{C(x)\} \leq d}, \tag{10}$$

$$\mathbf{D}\{C(x)\} = \sum_{j=1}^{n} \sum_{s=0}^{m-l_j} \left(\frac{(\overline{C}_{js} - \underline{C}_{js})^2}{12} \cdot x_{js} \right) \to \min_{x \in D:\, \mathbf{E}\{C(x)\} \geq e}, \tag{11}$$

where $x = \{x_{js} : j = 1, 2, \ldots, n, \ s = 0, 1, 2, \ldots m - l_j\}$, admissible set D satisfies the constraints (7)–(9), d and e be levels of permissible dispersion and the expectation respectively.

Tasks (10) and (11), as well as the task of (6)–(9), are the problems of Boolean linear programming with non-negative conditions of the matrix, so they can be resolved by pseudopolynomial algorithm based on dynamic programming.

4.2 Probability of given NPV inaccessibility as risk measure

Let C be a predetermined level NPV. Probability of achieving a given level be

$$\mathbf{P}\left\{C(x) \geqslant C\right\} = \mathbf{P}\left\{ \sum_{j=1}^{n} \sum_{s=0}^{m-l_j} C_{js} x_{js} \geqslant C \right\}. \tag{12}$$

Let us introduce events

$$E_j : \sum_{s=0}^{m-l_j} C_{js} x_{js} = y_j, \quad j = 1, 2, \ldots, n, \quad \sum_{j=1}^{n} y_j \geqslant C$$

consist of the fact that the income from the project p_j will not be less y_j. These events are used by methods of reduction problems with probabilistic criteria to a deterministic view [15].

It follows from (8) and (9) that

$$\mathbf{P}\left\{E_j\right\} = \sum_{s=0}^{m-l_j} x_{js} \mathbf{P}\left\{C_{js} \geqslant y_j\right\} = \sum_{s=0}^{m-l_j} \left(x_{js} \frac{\overline{C}_{js} - y_j}{\overline{C}_{js} - \underline{C}_{js}} \right).$$

Hence, given the independence of the NPV value of the different projects, we have,

$$\mathbf{P}\left\{C(x) \geqslant C\right\} = \prod_{j=1}^{n} \mathbf{P}\left\{E_j\right\} = \prod_{j=1}^{n} \sum_{s=0}^{m-l_j} \left(x_{js} \frac{\overline{C}_{js} - y_j}{\overline{C}_{js} - \underline{C}_{js}} \right).$$

Further instead of maximizing the probability of $\mathbf{P}\{\cdot\}$ we consider the problem of maximizing its logarithm. Due to the monotony of the logarithmic function the optimal solutions to both problems are the same. We have

$$\ln \mathbf{P}\left\{C(x) \geqslant C\right\} = \sum_{j=1}^{n} \ln\left(\sum_{s=0}^{m-l_j}\left(x_{js}\frac{\overline{C}_{js}-y_j}{\overline{C}_{js}-\underline{C}_{js}}\right)\right) =$$

$$= \sum_{j=1}^{n}\sum_{s=0}^{m-l_j}\left(x_{js}\ln\frac{\overline{C}_{js}-y_j}{\overline{C}_{js}-\underline{C}_{js}}\right) \cong -\sum_{j=1}^{n}\sum_{s=0}^{m-l_j}\left(x_{js}\frac{y_j-\underline{C}_{js}}{\overline{C}_{js}-\underline{C}_{js}}\right).$$

The second equality is a consequence of (8) and (9), and the last equality is a consequence of approximate equality $\ln(1-\xi) \cong -\xi$.

On the other hand

$$\ln \mathbf{P}\left\{C(x) \geqslant C\right\} = \ln\left[1-\mathbf{P}\left\{C(x) < C\right\}\right] \cong -\mathbf{P}\left\{C(x) < C\right\},$$

consequently

$$\mathbf{P}\left\{C(x) < C\right\} \cong \sum_{j=1}^{n}\sum_{s=0}^{m-l_j}\left(x_{js}\frac{y_j-\underline{C}_{js}}{\overline{C}_{js}-\underline{C}_{js}}\right). \tag{13}$$

This equation determines the probability of investment program setpoint NPV inaccessibility, and later this probability is used as a measure of risk.

Let us introduce determinate variables z_{ji}, $j = 1, 2, \dots, n$, $i = 0, 1, \dots, m-1$ and let us consider events $A_{ji} = \{I_{jsi} \leq z_{ji}\}$ and $\{B_i = \sum_{j=1}^{n} z_{ji} \leq r_i\}$. Event A_{ji} means the fact that at period i the resources required by the project p_j started at any period $s \leq m - l_j$ do not exceed a value of z_{ij}. Event B_i means the fact that the resources required for all performed projects at the period i do not exceed value r_i. Probabilities of the introduced events are

$$\mathbf{P}\left\{I_{jsi} \leq z_{ji}\right\} = \frac{z_{ji}-\underline{I}_{jsi}}{\overline{I}_{jsi}-\underline{I}_{jsi}}, \quad \mathbf{P}\left\{\sum_{j=1}^{n} z_{ji} \leq r_i\right\} = \frac{\overline{r}_i - \sum_{j=1}^{n} z_{ji}}{\overline{r}_i - \underline{r}_i}.$$

Let us find the probability of the conditions 3 realizability of the investment program considering the variables z_{ji} as fixed. We have

$$\mathbf{P}\left\{r_i(x) = \sum_{j=1}^{n}\sum_{s=0}^{i} I_{jsi}x_{js} \leq r_i\right\} =$$

$$= \mathbf{P}\left\{\sum_{j=1}^{n} z_{ji} \leq r_i\right\} \cdot \prod_{j=1}^{n}\sum_{s=0}^{m-l_j}\left(x_{js}\mathbf{P}\left\{I_{jsi} \leq z_{ji}\right\}\right). \tag{14}$$

for any billing period $i = 0, 1, 2, \dots, m-1$. Finding of equation (14) logarithm and taking into account

$$\ln \mathbf{P}\left\{r_i(x) \leq r_i\right\} \cong -\mathbf{P}\left\{r_i(x) > r_i\right\},$$

$$\ln \mathbf{P}\left\{I_{jsi} \le z_{ji}\right\} \cong -\mathbf{P}\left\{I_{jsi} > z_{ji}\right\} = -\frac{\overline{I}_{jsi} - z_{ji}}{\overline{I}_{jsi} - \underline{I}_{jsi}},$$

$$\ln \mathbf{P}\left\{\sum_{j=1}^{n} z_{ji} \le r_i\right\} \cong -\mathbf{P}\left\{\sum_{j=1}^{n} z_{ji} > r_i\right\} = -\frac{\sum_{j=1}^{n} z_{ji} - \underline{r}_i}{\overline{r}_i - \underline{r}_i},$$

as well as conditions (8) and (9) we have

$$\mathbf{P}\left\{r_i(x) > r_i\right\} = \frac{\sum_{j=1}^{n} z_{ji} - \underline{r}_i}{\overline{r}_i - \underline{r}_i} + \sum_{j=1}^{n} \sum_{s=0}^{m-l_j} \left(x_{js} \frac{\overline{I}_{jsi} - z_{ji}}{\overline{I}_{jsi} - \underline{I}_{jsi}}\right),$$
$$i = 1, 2, \ldots, m. \quad (15)$$

Equation (15) defines the probability of exceeding of resources required for the calculation period $i = 0, 1, 2, \ldots, m$ with the enterprise investment program.

Thus, if α be tolerable risk unreachable investment program setpoint NPV, β_i be tolerable risk of exceeding the investment program of the enterprise, the resources required for the calculation period $i = 0, 1, 2, \ldots, m$ then the problem of defining the maximum of the expected income can be represented as follows

$$C(x, y, z) = \sum_{j=1}^{n} y_j \to \max_{x,y,z} \quad (16)$$

$$\sum_{s=0}^{m-l_j} C_{js} x_{js} = y_j, \; j = 1, 2, \ldots, n; \quad (17)$$

$$\sum_{j=1}^{n} \sum_{s=0}^{m-l_j} \left(x_{js} \frac{y_j - \underline{C}_{js}}{\overline{C}_{js} - \underline{C}_{js}}\right) \le \alpha; \quad (18)$$

$$\frac{\sum_{j=1}^{n} z_{ji} - \underline{r}_i}{\overline{r}_i - \underline{r}_i} + \sum_{j=1}^{n} \sum_{s=0}^{m-l_j} \left(x_{js} \frac{\overline{I}_{jsi} - z_{ji}}{\overline{I}_{jsi} - \underline{I}_{jsi}}\right) \le \beta_i, \quad i = 1, 2, \ldots, m; \quad (19)$$

$$x_{js} z_{ji} \le \overline{I}_{jsi}, \quad j = 1, 2, \ldots, n, \; s = 0, 1, 2, \ldots, m - l_j, \; i = 1, 2, \ldots, m; \quad (20)$$

$$\sum_{s=0}^{m-l_j} x_{js} \le 1, \quad j = 1, 2, \ldots, n; \quad (21)$$

$$x_{js} \in \{0, 1\}, \quad j = 1, 2, \ldots, n, \quad s = 0, 1, .., m - l_j. \quad (22)$$

5 Example

Let us consider the application of the above mathematical models to the next task. To implement proposed $n = 7$ investment projects, all projects according to preliminary calculations are cost-effective. The planning horizon of the investment program is $m = 11$ billing periods. The duration of projects $j = 1, 2, \ldots, 7$

is the same and amounts to $l_j = 8$ billing periods, thus beginning of any project is possible only in the billing period $s = 0, 1, 2, 3$.

Table 1 contains the interval estimations of projects NPV depending on the time of s start implementation.

Table 1. NPV of the projects

Project	$s=0$		$s=1$		$s=2$		$s=3$	
j	$\underline{C}_{j0}$	$\overline{C}_{j0}$	$\underline{C}_{j1}$	$\overline{C}_{j1}$	$\underline{C}_{j2}$	$\overline{C}_{j2}$	$\underline{C}_{j3}$	$\overline{C}_{j3}$
1	655	850	585	780	522	717	466	661
2	246	441	220	415	196	391	175	370
3	164	359	146	341	131	326	117	312
4	383	578	342	537	305	500	272	433
5	334	529	298	493	266	461	237	964
6	972	1167	867	1063	774	970	691	887
7	414	609	369	565	330	525	294	490

Table 2 contains interval estimations of the allowed amount of financing the company's investment program for the calculation period i.

Table 2. Estimates of acceptable amounts of funding

i	0	1	2	3	4	5	6	7	8	9	10
$\underline{r}_i$	1800	1800	1800	1800	1800	1800	1800	1800	1800	1800	1800
$\overline{r}_i$	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000

Table 3 contains interval estimations I_{jsi} costs of project $j = 1, 2, \ldots, 7$ at calculation periods $i = 0, 1, \ldots, 10$ under start period s.

All valid risks β_i of exceeded the resources required for the calculation period $i = 0, 1, 2, \ldots, m$ of investment program of the company are equal to the risk tolerance α to get the unreachable investment program setpoint NPV.

Results of the solution received by using MS Excel are presented in Table 4.

Columns from the second to the eighth of Table 4 correspond to the seven best programs of investment companies that satisfy given constraints on the type and amount of risk. This columns contain billing period number $s: \; x_{js} = 1$ of implementation beginning for each project $j = 1, 1, \ldots, n$ included in the investment program.

Guaranteed NPV equals to 2644 and its variance is equal to 1058 with maximin strategy optimal investment program. The highest expected NPV for the variance of $D_0 = 1058$ is equal to 2916 and implements the investment program, which differs from the maximin. Levels of allowable variance $1.5D_0$ and $3D_0$

Table 3. Costs of project $j = 1, 2, \ldots, 7$ at periods $i = 0, 1, \ldots, 10$, $s = 0, 1, 2, 3$ (thousands of rubles)

j	$i = s$		$i = 1 + s$		$i = 2 + s$		$i = 3 + s$		$i = 4 + s$		$j = 5 + s$		$i = 6 + s$		$i = 7 + s$	
	$\underline{I}_{jsi}$	$\overline{I}_{jsi}$	$\underline{I}_{jsi}$	$\overline{I}_{jsi}$	$\underline{I}_{jsi}$	$\overline{I}_{jsi}$	$\underline{I}_{jsi}$	$\overline{I}_{jsi}$	$\underline{I}_{jsi}$	$\overline{I}_{jsi}$	$\underline{I}_{jsi}$	$\overline{I}_{jsi}$	$\underline{I}_{jsi}$	$\overline{I}_{jsi}$	$\underline{I}_{jsi}$	$\overline{I}_{jsi}$
1	100	120	100	119	140	150	120	132	88	100	72	80	50	58	40	50
2	300	320	300	303	300	400	80	100	80	100	80	100	90	100	90	100
3	88	99	120	144	130	155	88	100	75	80	59	60	58	60	55	60
4	400	500	680	700	199	215	140	150	140	150	140	150	140	150	140	150
5	530	600	530	600	530	600	70	80	70	80	70	80	70	80	70	80
6	680	850	680	850	390	500	390	500	350	400	180	200	180	200	180	200
7	480	500	380	385	330	380	320	350	300	330	300	330	300	330	300	330

Table 4. Competitive investment programs

Project j	Maximin $s : x_{js} = 1$	NPV dispersion $s : x_{js} = 1$			Probability of accessibility NPV: $s : x_{js} = 1$		
		$3D_0 = 3174$	$1.5D_0 = 2116$	$D_0 = 1058$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.03$
1	0	0	1	0	0	1	1
2	–	0	3	3	–	0	0
3	3	1	0	0	0	–	–
4	–	3	–	–	2	2	3
5	3	–	0	1	0	–	–
6	0	0	1	2	0	0	0
7	0	0	3	0	0	0	3
Project	1	2	3	4	5	6	7
NPV	2644	3782	3297	2916	4044	3588	2944
D	$D_0 = 1058$	3174	2116	1058	3372	2253	1103
α	0	0.104	0.051	0.031	0.096	0.049	0.026

correspond to different Pareto optimal investment programs having an average NPV equal to 3297 and 3782 respectively. Increasing the level of α unreachable probability of possible expected value of NPV in stochastic models also leads to various investment programs with increasing average expected value of NPV.

Since the stochastic model (16)–(22), in contrast to the model (10)–(11), allows the risk of exceeding the investment program of the enterprise resources required for the calculation period, then the potential average expected NPV in the stochastic model are higher.

Images of all the projects in Table 4 for the coordinate systems “variance NPV” – “expected NPV” and “probability unreachable” – “expected NPV” are shown in Figure 1.

Investment programs built in the example are effective (i. e. belong to the Pareto set in the space of criteria).

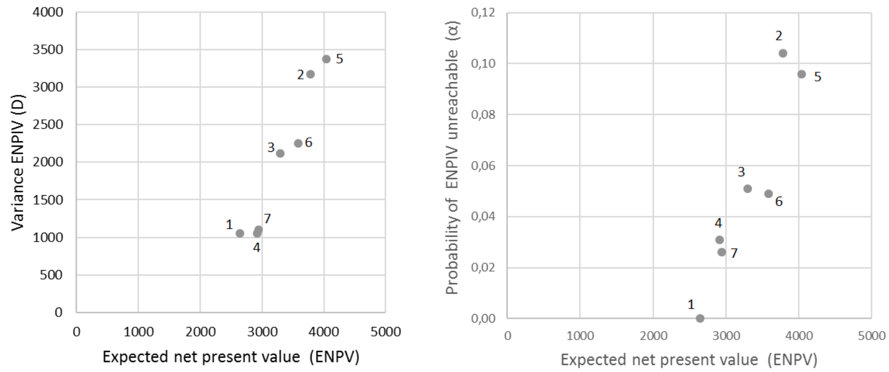

Fig. 1. Samples in different spaces criteria projects

Conclusion

Considered models of optimal investment program with known distribution of funds for each period allow to shape the Pareto-optimal investment programs. Presented modification of this model which takes into account the uncertainty of financial resource volumes to support investment projects.

Solutions of the respective tasks provide systematic assessment of investment attractiveness of the enterprise can be used by intelligent supporting systems for choice of efficient portfolio based on derivative criteria of performance: (i) the payback period of the investment, (ii) the rate of return on capital, (iii) the difference between the amount of income and investment costs (non-recurring expenses) for the entire useful life of the investment project, (iv) reduced production costs, and (v) risk appetite of decision makers.

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