Optimal Scanning of Gaussian and Fractal Brownian Images with an Estimation of Correlation Dimension

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Abstract. The paper considers the influence of the image pixels position in a one-dimensional sequence on the result of correlation dimension evaluation. The sequence formed as a result of reading of the two-dimensional pixel image. Dimension evaluation is performed by maximum likelihood method, using image elements ordering and creating vectors in pseudophase space by Takens theorem, and is used as a texture feature if texture processing and detection of objects. The two different scanning methods are considered. There is a problem of optimization of the scan path in order to maintain correlations between pixels in sequence. The first method is to scan on the criterion of maximum correlation between the adjacent sets of pixels. The second method deals with choice of scan direction by criterion of scalar product maximum of chosen vector and previous one. An estimator of correlation dimension is evaluated.

Keywords: optimal scanning, correlation dimension, maximum likelihood estimator, fractal analysis, Gaussian and Fractal Brownian images.

1 Introduction

Problem of detection of objects and its edges, as well as the separation of areas at images is one of the most actual in modern thematic image processing. The distinction between areas and objects from each other is carried out by means of evaluation of certain parameters — textural features. We can make a decision about difference of objects on the basis of the difference of characteristic values. One of textural attributes are statistical features, which are determined by taking into account the statistical properties of a sequence of pixels. Among them there is a correlation dimension, which is formally defined as the probability that the distance between the vectors formed from samples in the time series in pseudophase space is less than a predetermined constant value.

One of the problems of dimension estimation of the image is a violation of correlations between pixels during ordering them into a one-dimensional sequence after reading. Radar images usually have a complex structure, which means that the relationship of pixels is in different directions. Conventional methods perform scanning reading pixels horizontally, vertically or diagonally. In addition, there are space-filling curves, which allow to read all of the pixels in a selected area once, sequentially selecting only adjacent pixels. Such a curve is, for example, the Peano-Hilbert curve [1], which has a fractal structure and allows to completely read the pixels in a square area. This curve is good for scan an image with fractal properties of objects. Fractal properties and parameters, representing it, are promising among different textural features. Such properties are quantitatively estimated by value of fractal dimension [2, 3]. Hilbert curve has predetermined trajectory; therefore, it does not change depending on pixels correlation. Moreover, all of the proposed methods do not have the property of adaptability, that is, are universal for all the images and do not consider their statistical properties. The sequence of pixels is weakly correlated, that makes identification of relationships more difficult. Thus, there is the task of finding the optimal scanning method that is able to perform the adaptation of the image with respect to the properties.

The aim of investigation is to develop a method of scanning an image, providing more efficient textural processing and evaluation of the correlation dimension by usage of fractal properties of the analysable image fragment as well as algorithms of the fractal dimension estimation that are optimal by the maximum likelihood criterion. An improvement of methods and algorithms is performed by taking into account statistical and geometric dependency of data. An algorithm for data scan within image frame is proposed as well.

2 A maximum likelihood estimation of correlation dimension

The most precise estimates of correlation dimension are obtained by maximum likelihood algorithm [4, 5]. Basic iterations for the dimension evaluation are presented in reference [6]. When distances normalization $r_m = l_m/l_{\max}$, $m = 1, .., M$, and correlation dimension equals D a probability distribution law for distances between vectors $\mathbf{V} = \{V_1, .., V_{NE}\}$ in pseudophase space is set by power law $F(r) = r^D$ and probability density function is as follows: $w(r) = \dfrac{dF(r)}{dr} = D \times r^{D-1}, 0 < r < 1$ [4].

In paper [4] it is discussed the problem when all M distances $\mathbf{r} = \{r_m, m = 1, .., M\}$ between vectors $\mathbf{V}$ are independent, a correlation dimension equals to D. Then a combined probability density function of distances is as follows:

$$w(\mathbf{r}/D) = \begin{cases} \prod\limits_{m=1}^{M} D r_m^{D-1}, \ \mathbf{r} \in [0, 1), \\ 0, \ \mathbf{r} \notin [0, 1). \end{cases} \tag{1}$$

Dependent distances are formed in accordance with the following rule [6]:

1) $N - 1$ independent random numbers are generated with the power law distribution of probabilities within the range of values $(0; 1)$; these numbers set distances from the first vector to all other $N-1$ vectors. A combined probability density function of these $N-1$ independent distances between vectors is obtained from (1) by substitution N instead of M:

$$w_1(\mathbf{r}_1/D) = \begin{cases} \prod\limits_{m=1}^{N-1} Dr_{1m}^{D-1}, \ \mathbf{r}_1 \in [0,1), \\ 0, \ \mathbf{r}_1 \notin [0,1). \end{cases} \tag{2}$$

2) $N-2$ independent random numbers are generated with the power law distribution of probabilities within the range of values $(r_{\max k}; r_{\min k})$, $k = 3, .., N$; these numbers set conditionally independent distances from the second vector to other $N-2$ vectors except the 1^{st} vector. Minimum and maximum values are defined by the triangle rule:

$$r_{\min k} = |r_{12} - r_{1k}|, r_{\max k} = r_{12} + r_{1k}, k = 3, .., N.$$

Combined probability density function of these $N-2$ independent distances between vectors is as follows:

$$w_2(\mathbf{r}_2/\mathbf{r}_1, D) = \begin{cases} \prod\limits_{m=N}^{2N-3} \dfrac{D}{r_{\max k} - r_{\min k}} (r_{\max k} - r_{\min k})^{D-1}, \\ r_m \in [r_{\min k}, r_{\max k}), k = 3, .., N, \\ 0, r_m \notin [r_{\min k}, r_{\max k}), k = 3, .., N, \\ m = N + k - 3. \end{cases} \tag{3}$$

As minimum and maximum values $r_{\min k}, r_{\max k}$ depend on distances with numbers $1, .., N-1$, then distances $\mathbf{r}_1, \mathbf{r}_2$ are also statistically dependent and their combined probability density function is derived subject to (2), (3) :

$$w_{12}(\mathbf{r}_1, \mathbf{r}_2/D) = w_1(\mathbf{r}_1/D) w_2(\mathbf{r}_2/\mathbf{r}_1, D). \tag{4}$$

3) Coordinates of the first and second vectors in space of embeddings $D_E = 2$ are $x_1 = 0, y_1 = 0, x_2 = r_{12}, y_2 = 0$. Coordinates of other $i = 3, .., N$ vectors are defined by geometry of their position using the cosine theorem and distances from i-th vector to the first and second vectors: $x_i = \dfrac{r_{12}^2 + r_{1i}^2 - r_{2i}^2}{2r_{12}}, y_i = \pm\sqrt{r_{1i}^2 - x_i^2}$. For convenience, sign of coordinates y_i is chosen positive.

4) As a result of coordinates evaluation of all vectors we can calculate other $\dfrac{N^2}{2} - \dfrac{5N}{2} - 3$ dependent distances $\mathbf{r}_3 = \begin{cases} r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \\ i = 3, .., N, j = i+1, .., N \end{cases}$. As distances $\mathbf{r}_3$ are absolutely defined by distances $\mathbf{r}_1, \mathbf{r}_2$, than they do not contain additional information for the correlation dimension estimation.

Optimal estimates of the correlation dimension should take into account statistical and geometrical dependences of vectors, which are represented in the likelihood function. A maximum likelihood estimate is obtained as a result of solving of the optimization problem:

$$D_{est} = \arg \max_{0<D<D_E} w_{12}(\mathbf{r}_1, \mathbf{r}_1 | D). \tag{5}$$

The solving of equation (5) is realized by setting equal to zero of first order derivative of $w_{12}(\mathbf{r}_1\mathbf{r}_2|D)$ with respect to D. A maximum likelihood estimate of correlation dimension is obtained:

$$\hat{D} = -(3N-3)\left[\sum_{m=1}^{N-1} \ln r_{1m} + \sum_{m=N}^{2N-3} \ln(r_m - r_{\min k})\right]. \quad (6)$$

3 Optimal scanning by maximum correlation coefficient

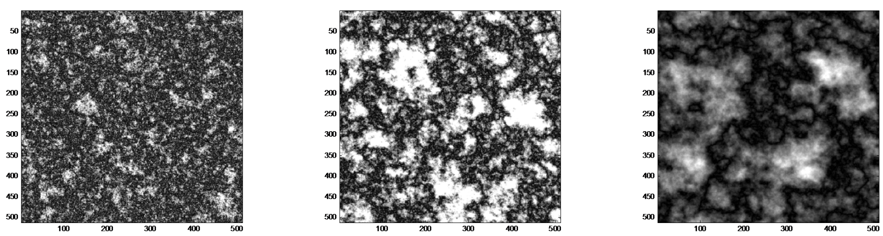
It is necessary to choose an approximating model for processing of signals and images as well. The main principle of model choosing is to provide a maximum conformance of model and real signal parameters. A 2D Fractional Brownian motion is widely used as a model of a chaotic fractal image. Samples of 2D fBm are formed $X(k,m), k,m = 1,..,N$, by one of famous methods [7, 8] and characterized by intensity σ^2 and Hurst exponent H. Variance of fBm increments in the space interval $\Delta x, \Delta y$ equals $\sigma^2 \left|\Delta x^2 + \Delta y^2\right|^H$, and its correlation within disjoint intervals of space equals [7]:

$$\begin{aligned} \mathbf{M} &= \{(X(k_1,m_1) - X(k_3,m_3)) \times (X(k_2,m_2) - X(k_4,m_4))\} = \\ &= \frac{\sigma^2}{2}\left|k_1^2+m_1^2\right|^H + \frac{\sigma^2}{2}\left|k_2^2+m_2^2\right|^H - \frac{\sigma^2}{2}\left|(k_1-k_2)^2+(m_1-m_2)^2\right|^H - \\ &- \frac{\sigma^2}{2}\left|k_1^2+m_1^2\right|^H - \frac{\sigma^2}{2}\left|k_4^2+m_4^2\right|^H + \frac{\sigma^2}{2}\left|(k_1-k_4)^2+(m_1-m_4)^2\right|^H - \\ &- \frac{\sigma^2}{2}\left|k_3^2+m_3^2\right|^H - \frac{\sigma^2}{2}\left|k_2^2+m_2^2\right|^H + \frac{\sigma^2}{2}\left|(k_3-k_2)^2+(m_3-m_2)^2\right|^H + \\ &- \frac{\sigma^2}{2}\left|k_3^2+m_3^2\right|^H + \frac{\sigma^2}{2}\left|k_4^2+m_4^2\right|^H - \frac{\sigma^2}{2}\left|(k_3-k_4)^2+(m_3-m_4)^2\right|^H. \end{aligned} \quad (7)$$

Two-dimensional Gaussian process $X(k,m)$ is used for representing of image with zero mathematical expectation, dispersion D_x and correlation matrix

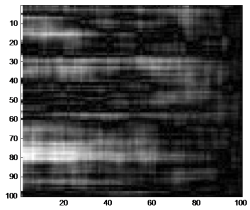
$$\mathbf{R} = \{R(i_1,j_1,i_2,j_2) = D_x \exp\left[-(\alpha_x|i_1-i_2| + \alpha_y|j_1-j_2|)\right], i_1,i_2,j_1,j_2 = 1,..,N\}. \quad (8)$$

Figures 1-2 represent image samples according to above-mentioned models.

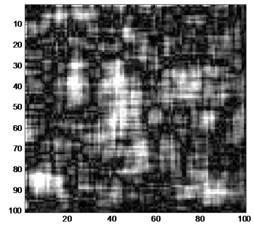


a) $H = 0.1$ b) $H = 0.5$ c) $H = 0.9$

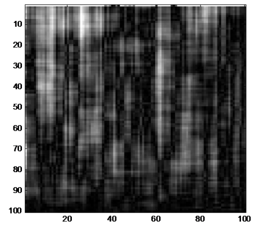
Fig. 1. Image of 2D Fractal Brownian motion.



a) $\alpha_x = 0.01, \alpha_y = 0.1$



b) $\alpha_x = 0.1, \alpha_y = 0.1$



c) $\alpha_x = 0.1, \alpha_y = 0.01$

Fig. 2. Image of 2D Gaussian process.

In a review paper [1] various methods of the scan path optimization are considered, based on the correlation property of pixels. It is supposed that the construction of the scan path, taking into account the correlation property of pixels, allows to most fully represent dynamic properties of the image and improve the accuracy of correlation dimension estimation

In this paper two method of image scanning is discussed. The first method deals with choice of scan direction by criterion of correlation coefficient maximum (7), (8) of each subsequent pixel. This method uses image characteristics, averaged by significant ensemble of samples, but it doesnt consider properties of particular image. The second method deals with choice of scan direction by criterion of scalar product maximum of chosen vector **X** and previous one **Y**. This method doesnt require a priori information about statistical properties of image. Data of particular image are most fully used for construction of optimal scan path. The optimal scanning algorithm consists of a following steps:

- first vector consists of M image pixels from first line of current frame;
- one of Q predefined directions from last pixel to end of next vector is considered;
- standard scalar product is evaluated for pair of vectors previous **X** and each of Q considered directions **Y** — as follows:

$$R_{X,Y} = \frac{(\mathbf{X}, \mathbf{Y})}{||\mathbf{X}|| \times ||\mathbf{Y}||} \tag{9}$$

- maximum value of scalar product defines a pair of vectors and thereby the direction of scan trajectory;
- scan trajectory moves to new pixel, which is last for chosen vector;
- the algorithm is repeated until majority pixels are covered.

Figures 3-6 represent optimal scan paths for Gaussian and Fractal Brownian images, obtained by two proposed methods.

4 Correlation dimension evaluation

Dimension evaluation is performed by maximum likelihood method [4], with image elements ordering and vectors formation in pseudophase space by Takens

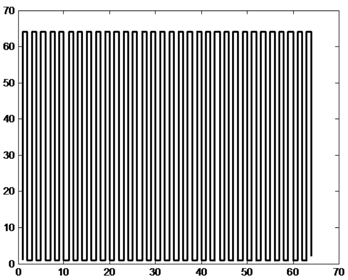


Fig. 3. Optimal scanning trajectory by 1-st method, image of figure 1c.

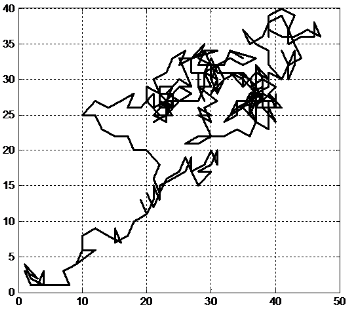


Fig. 4. Optimal scanning trajectory by 2-st method, image of figure 1c.

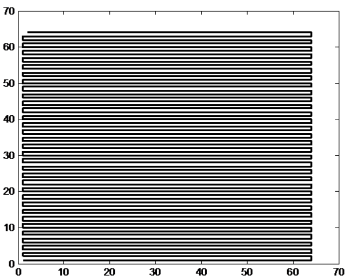


Fig. 5. Optimal scanning trajectory by 1-st method, image of figure 2a.

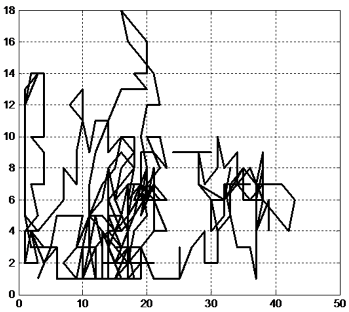


Fig. 6. Optimal scanning trajectory by 2-st method, image of figure 2a.

theorem [9]. An estimator of correlation dimension of Gaussian random image is investigated. Then we change value of α_y and estimate a correlation dimension in two case: scan trajectory is vertical (figure 7) and scan trajectory is horizontal (figure 8). The evaluated estimates are averaged by 20 samples, the vector size is equal $N_E = 3$.

5 Conclusions

Results of analysis of proposed algorithm show, that appliance of maximum likelihood algorithm demands provision of maximum correlations between image pixels in one-dimension sequence. Scanning by maximum correlation coefficient criterion provides bigger difference between values of correlation dimension estimates and therefore simplifies solution of object detection problem.

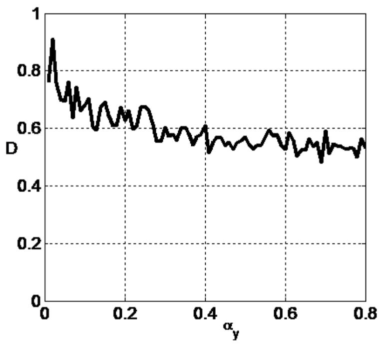


Fig. 7. Dependency of correlation dimension on α_y, $\alpha_x = 0.7$, vertical scan.

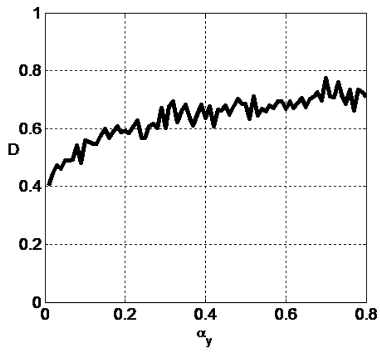


Fig. 8. Dependency of correlation dimension on α_y, $\alpha_x = 0.7$, horizontal scan.

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References

1. T. Ouni, M. Abid, Scan Methods and their Application in Image Compression, International Journal of Signal Processing, Image Processing and Pattern Recognition, vol. 5, No. 3, 2012, pp. 49-63.
2. Yu.G. Sosulin, A.B. Russkin, Fractal Detection of Extensive Low in Contrast Objects in Images, Radiotechnika, 2009, 12, pp. 48-57.
3. A.A. Potapov, Fractals in Radio Physics and Radiolocation: Selection Topology. oscow: Universitetskaya kniga, 2005.
4. Luciana De Luca, Dario Luzio, Massimo Vitale, A ML Estimator of the Correlation Dimension for Left-hand Truncated Data Samples,, Pure and Applied Geophysics, vol.159, pp. 2789-2803, 2002.
5. A. Yu. Parshin, Yu. N. Parshin, Usage of non-Gaussian statistics for RF Signals Detection by Complex Energy and Fractal Detector, International Radar Symposium, IRS 2013, Proceeding, vol. I, II, pp. 779-784, Drezden, Germany, German institute of navigation, 2013.
6. A. Yu. Parshin, Yu. N. Parshin, Analysis of Maximum Likelihood Estimation of Fractal Dimension by Independent and Dependent Data, 5th Mediterranean Conference on Embedded Computing (MECO), 2016, pp: 172 175.
7. R.M. Crownover, Introduction to Fractals and Chaos. Jones and Bartlett, 1995.
8. Wood A. and Chan G., Simulation of stationary Gaussian processes in [0, 1]d, Journal of computational and graphical statistics, 1994, vol. 3, No.4, p. 409-432.
9. F. Takens, Detecting Strange Attractors in Turbulence, Lecture Notes in athematics, vol. 898, 1981, pp. 366-381.