Methods of estimating competence as a system of fuzzy statements

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Abstract. The work is based on estimation of competence as a fuzzy event, the reliability of which is determined by the membership function; the method of competence evaluation based on fuzzy statements is proposed, which creates a system of compositional rules of fuzzy products. The final verbal assessment of competence is proposed to determine basing on the linguistic variable "Competence", defined on the set of values of membership function of competence.

Keywords: Competence, fuzzy set, fuzzy statements, fuzzy logic, linguistic variable, linguistic expressions, membership functions, terms of linguistic variables.

Keyterms: Competence Formation Process**,** Estimating competence, Mathematical modeling, Formal Method, Estimating Competence

1 Introduction

Training specialists that meet modern requirements, promotion of individuals to positions should be based on competence approach, which must consider not only the availability of necessary knowledge, habits and skills but also such psychological peculiarities of subjects as their creativity, ability to work in a team and so on. The quantitative estimation of competence necessary for making appropriate decisions is a multicriterial problem, and therefore we need here to derive an integral estimation of competence. Since there is no generally accepted methodology of working out this problem, it makes the article topical.

Published works analysis. In article [1] the problem of assessing competence was resolved on the basis of defining a competence as a fuzzy discrete set, the values of which are the required competences. But this approach is less adequate to forms of experts' knowledge representation regarding competence, which leads to difficulties of its application.

The aim of this paper is to develop a methodology for the quantitative assessment of competence based on fuzzy statements [2,3], which is a more adequate form of representation of the knowledge base of experts that is necessary to create the methodology for the quantitative assessment of competence of the subject.

2 Main results

Hereinafter the paper presents and will further use the following definitions and their characteristics.

Definition 1. Competence is a fuzzy event, characterized by the membership function, the completion of which designates the ability of a subject to effectively perform his duties on a certain position which is a result of his professional training and lifelong self-learning.

Definition 2. Quantitative evaluation of competence is an integral characteristic of knowledge, skills, motivation of a subject from the totality of the abilities, defined by the membership function $\mu_k \in [0;1]$, and it characterizes the degree of ability to effectively perform the responsibilities of a certain office position.

Definition 3. Verbal evaluation of competence is an integral characteristics of knowledge, skills, motivation of a subject, which is defined by the terms of a linguistic variable "Competence", that is set on a multitude of values of the membership function $\mu_k \in [0;1]$, and it characterizes a degree of his ability to effectively perform the duties of a certain position.

Terms of this linguistic variable can be: «Incompetent person»; «Little competent person»; «Average competent person»; «Well competent person»; «Highly competent person».

Definition 4. Ability is one of many components of competence, that is characterized by a dynamic combination of a special job knowledge, habits and skills of a subject. Obtaining the integral characteristics of competence (its membership function $\mu_k \in [0;1]$) is offered to base on the system of compositional rules of fuzzy production F_i, $i=\overline{1,n}$ in the form of implications [1]:

$$F_i : \text{IF } A_i, \text{THEN } B_i; i=\overline{1,n}, \quad (1)$$

where $A_i; i=\overline{1,n}$ - is a fuzzy statement that is a condition (antecedent); B_i, $i=\overline{1,n}$ - is a fuzzy statement, that is a conclusion (the consequent) of fuzzy production.

For example, the system of compositional rules can be the following:

- F_1: «IF the subject possesses required knowledge(A_{11}), habits and skills (A_{12}), and is motivated to perform his responsibilities(A_{13}), that are related to corresponding office positions, THEN he is competent (B_1)»;
- F_2: «IF the subject graduated from the university with an appropriate specialization (A_{21}) and has good grades (A_{22}), THEN he possesses required knowledge ($B_2=A_{11}$)»;

- F_3: «IF the subject knows how to use a personal computer for tackling essential job tasks (A_{31}), and he has an experience for relevant position not less than a year (A_{32}), THEN he has required skills (B_3=A_{12})»;

- F_4: «IF the subject likes his profession (A_{41}) and earns a good salary (A_{42}), THEN he is motivated (B_4=A_{13})»;

- F_5: «IF the subject is communicative (A_5), THEN he has one of the required skills (B_5=A_{12})»; and so on.

In some rules the conditions and conclusions can be complex (consist of simple fuzzy statements connected by logical operations of negation, conjunction and disjunction). For instance, in rules F_1,F_2,F_3,F_4 the conditions are complex: they are composed of simple conditions connected by logical conjunction (^): A_1= A_{11}^A_{12}^A_{13}; A_2=A_{21}^A_{22}; A_3=A_{31}^A_{32}; A_4=A_{41}^A_{42}. The conclusion can also be complex: consist of simple conclusions often connected by logical disjunction operations. In some compositional rules both the condition and the conclusion may be in a form of fuzzy linguistic statements [3] (for example, statement A_{42}, which uses the term “good salary” of the linguistic variable (LV) "salary level"). At this, the consequent of one fuzzy production can be the antecedent of another fuzzy production and vice versa (for example, B_3=A_{12}); some compositional rules can have either the same conditions, or the same conclusions.

The system of composition rules is created by relevant experts. Such rules can range from tens to more than a hundred.

To find the membership functions of complex statements the following formulas can be applied [3]:

1. The membership function of negation $\bar{A}$ of the fuzzy statement A is found by formula

$$\mu_{\bar{A}} = 1 - \mu_A \tag{2}$$

2. The membership function of disjunction A_1+A_2 of fuzzy statements A_1 and A_2 can be found by algebraic formula:

$$\mu_{A_1+A_2} = \mu_{A_1} + \mu_{A_2} - \mu_{A_1}\mu_{A_2} \tag{3}$$

or the formula of maximum:

$$\mu_{A_1+A_2} = \max(\mu_{A_1}; \mu_{A_2}) \tag{4}$$

3. The membership function of conjuction A_1^A_2 of fuzzy statements A_1 and A_2 can be found by algebraic formula:

$$\mu_{A_1A_2} = \mu_{A_1}\mu_{A_2} \tag{5}$$

or the formula of minimum:

$$\mu_{A_1 A_2} = \min(\mu_{A_1}; \mu_{A_2}) \quad (6)$$

The condition of some of the compositional rules is defined by core abilities: knowledge, capabilities and skills, which characterize the competence. The condition of other rules is defined by position or professional abilities, and their consequences can be core abilities.

Competences have hierarchical structure (fig.1). One competence might be general for one position, and special or professional for another, at the same time. Understanding the hierarchical character of competences allows you to specify the compositional rules that are necessary for defining the membership function of a corresponding conclusion. This process is called activation of the compositional rules.

In the example that is considered, for quantitative assessment of the competence it is necessary to activate compositional rules F_1, F_2, F_3, F_4, F_5. For each fuzzy compositional rule F_i the experts define a degree of importance μ_{Fi}, $i=\overline{1,n}$. As in the system of compositional rules it is necessary to apply important rules, so their membership functions are usually close to one. This corresponds to the Pareto principle.

Fig.1. The hierarchical structure of abilities

For each subject of evaluation the membership function μ_{Aj} of fuzzy statements of conditions A_{ij} is determined by analyzing resumes, interviewing, testing and setting professional problem tasks. To find the membership functions of complex statements of conditions A_i formulas (2-6) can be applied.

The central place in competence assessment is taken by a method of conclusion: finding the membership functions of conclusions of compositional rules μ_{Bj}.

In the direct method of drawing a conclusion, based on using a fuzzy generalization of the rule of fuzzy modus ponens [3], the classical implication A → B is replaced by the rule of fuzzy production «IF x is A, THEN y is B», where A and B – are fuzzy discrete sets on universums X and У, where $A_i=x_i \in X$, $B_i=y_i \in Y$.

As several conclusions of compositional rules determine the conclusion of the compositional rule that is located on the upper hierarchical level, then competence will be determined by the product of fuzzy binary relations, which is a fuzzy binary relation on the Decart product of conditions and conclusions of corresponding compositional rules.

In the example that is being considered, the competence (conclusion B_1) is determined by condition A_1, which in turn, is determined by conditions (abilities) B_2; B_3 and B_4. Condition A_5 determines conclusion $B_4=A_{12}$. The hierarchical of fuzzy statements, that determine competence, are shown in fig.2.

According to this hierarchy, first the membership functions of conditions A_2, A_3, A_4 and A_5 are found. On the basis of them the membership functions of the conclusions B_2, B_3, B_4 and B_5 are found. The membership functions of these conclusions determine the membership functions of the condition A_1, on the basis of which the membership function of conclusion B_1 is found.

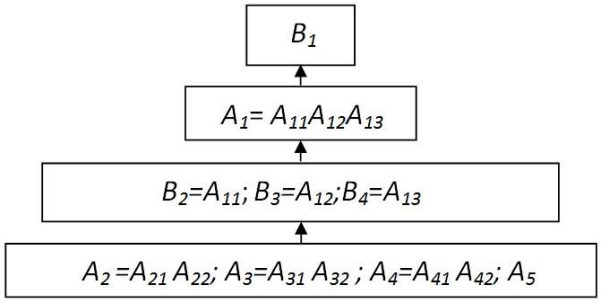

Fig.2. The example of fuzzy hierarchical statements of the system of compositional rules

The membership function) μ_{Bi} (y) of the fuzzy conclusion B_i is frequently found by max- min composition [3]:

$$\mu_{B_i}(y) = \max_x \left[\min(\mu_A(x); \mu_R(x; y))\right] \tag{7}$$

where $\mu_{Ai}(x)$ - the membership function of conditions; $\mu_R(x;y)$ - the membership function of fuzzy relation R of the Decart product of all conditions $A_i=x_i \in X$, and conclusions $B_i=y_i \in Y$.

Sometimes, the max-prod composition is used [3], which gives a more pessimistic assessment:

$$\mu_{B_i}(y) = \max_x \left[\mu_A(x) * \mu_R(x; y)\right] \tag{8}$$

To account for a degree of a risk, that is taken by a person, who uses competence assessment, it is proposed to apply the derived max – min composition of the type:

$$\mu_B(y) = \max_{x \in X} \left\{(1-\lambda) \min_x \left(\mu_A(x); \mu_R(x, y)\right) + \lambda \max_x (\mu_A(x); \mu_R(x, y))\right\}$$

$$\forall(x_i, y_j) \in X^T * Y \quad (9)$$

where $\lambda \in [0;1]$ - the coefficient that characterizes the degree of a risk that is taken when applying the competence assessment.

With zero degree of the risk ($\lambda = 0$) statement (9) transforms into statement (7). The maximum degree of the risk is $\lambda = 1$.

In the problem considered the membership functions of importance of compositional rules μ_{Fi} act as the membership function $\mu_R(x;y)$ of a fuzzy relation R. As psychologists established quantitative information is verbally encoded and stored in a human's brain in the form of terms of linguistic variables (LV) [2, 3]. To get competence assessment [5] in the convenient form the fuzzy statement E – "Competence" should be defined as a linguistic variable. Let us define linguistic variable "Competence" with a tuple $< E, E_j, j = \overline{1,5}; \mu_{E_j}(x) \in [0;1]; x \in [0;1]; >$, where E – is a name of LV (in this problem E– is «Competence»), whose range of values lies in the interval [0;1]; E_j – terms of LV; $\mu_{Ei}(x)$– the membership function of term E_j ; $x=\mu_{Bi}$. Terms of LV "Competence" can be: E_1 – very low competence; E_2 – low competence; E_3 – medium competence; E_4 – good competence; E_5 – high competence.

Using trapezoidal membership functions of terms and considering Harrington's scale it is possible to specify LV "Competence" as follows [4]: $E_1 = \langle 0;0;0;1;0.2 \rangle$; $E_3 = \langle 0.3;0.4;0,6;0,7 \rangle$; $E_4 = \langle 0.6;0.7;0,8;0,9 \rangle$; $E_5 = \langle 0.8;0.9;1;1 \rangle$.

Trapezoidal membership functions of terms can be defined by experts by means of four numbers <a; b; c; d>, which define each term.

Thus, the main stages for evaluating the competence is shown in fig.3. In the evaluation there should be two main stages: the stage of obtaining knowledge and the stage of evaluation of competence.

The preparatory stage of obtaining knowledge involves the implementation of items 1; 2; 3; 4. At this stage the experts in the field, in which competence of the subject will be evaluated, are used.

At the next stage (performing item 5) in order to evaluate each subject, the membership function of fuzzy statements of conditions, that are arguments in multicriterial evaluation of competence, are determined by analyzing resumes, interviewing, testing and giving problem professional tasks.

Design stage of assessment of competence includes items 6;7;8. The given method of assessment of experts' competence, which is based on the use of fuzzy logic, allows to overcome such problems as multicriteriality of the task to evaluate the competence of the subject; incomplete character of necessary information for this task; the impossibility to measure quantitatively certain partial indicators of competence. This method is more adequate to represent the knowledge base of corresponding experts.

Let us imagine we received the following assessments for a particular subject: $\mu_{A21}=1$; $\mu_{A22}=0,8$; $\mu_{A31}=0,9$; $\mu_{A32}=1$; $\mu_{A41}=0,6$; $\mu_{A42}=0,65$; $\mu_{A5}=0,5$; $\mu_{A6}=1$.

The reliability of the compositional rules, which are considered in the example, was estimated by the experts as the following values of the membership functions: $\mu_{F1}=0,5$; $\mu_{F2}=0,8$; $\mu_{F3}=0,9$; $\mu_{F4}=0,9$; $\mu_{F5}=1$; $\mu_{F6}=0,85$.

It is needed to find the membership function μ_{Bi} of conclusion μ_{Bi} regarding the competence of the subjection the basis of which a verbal evaluation of his competence can be made.

When using the formula (6) to find the membership functions of complex fuzzy statements of conditions, we have μ_{A2}=min (1;0,8)=0,8; μ_{A3}=0,9; μ_{A4}=0,6.

1. Specifying the system of rules of fuzzy production $F_i; i=\overline{1,n}$ in the form [1]: F_i : IF A_i , THEN B_i ; $i=\overline{1,n}$,
2. Specifying the degree of truthfulness (or importance) of every rule of fuzzy production.
3. "A person's competence" LV assessment; $<E, E_j, j=\overline{1,5}; \mu_{E_j}(x) \in [0;1]; x \in [0;1];>$.
4. Specifying formulas that will be used for calculating corresponding membership functions.
5. Getting membership functions of conditions of the system of compositional rules for the subject of assessment.
6. Calculating membership functions of complex conditions with formula for conjunction (6).
7. Calculating membership functions of a person's competence.
8. The verbal assessment of competence on the basis of linguistic variable "Competence"

Fig.3. The main stages for evaluating the competence

Conditions A_{11}; A_{12} and A_{13} are determined (are concluded) by conditions A_2; A_3; A_4 and A_5, that can be defined by the matrix of fuzzy relation R, the elements of which are the membership functions μ_{Fi}:

$$\mu_R(x, y) = \begin{array}{c|ccc} x_i \setminus y_j & A_{11} & A_{12} & A_{13} \\ A_2 & 0{,}8 & 0 & 0 \\ A_3 & 0 & 0{,}9 & 0 \\ A_4 & 0 & 0 & 0{,}9 \\ A_5 & 0 & 1 & 0 \end{array}$$

When applying the derived max – min composition (9), and λ =0.6, we get:

$$\mu_{A_{11}} = \max((1-0.6)\min\left(|0{,}8;\ 0{,}9;\ 0{,}6;\ 0{,}5| \otimes \begin{vmatrix} 0{,}8 \\ 0 \\ 0 \\ 0 \end{vmatrix}\right) + 0.6\max\left(|0{,}8;\ 0{,}9;\ 0{,}6;\ 0{,}5| \otimes \begin{vmatrix} 0{,}8 \\ 0 \\ 0 \\ 0 \end{vmatrix}\right) = 0{,}8,$$

where symbol $\otimes$ means some pair – wise operation between the corresponding elements vector of the raw and vector of the column.

Accordingly, μ_{A12}=0,74; μ_{A13}=0,78.

The membership function of complex fuzzy statement of condition A_1 is

$$\mu_{A2}=\min (0,8;0,74;0,78)=0,74$$

Then the conclusion B_1 is determined by the membership function

$$\mu_{B1}=\min (\mu_{A1}; \mu_{F1})=\min(0,74;0,95)=0,74,$$

that corresponds to term E_4 that is high competence with the degree of confidence 1.

3 Conclusions

1. Assessment of a person's competence is proposed as a fuzzy event, realization of which means the ability of the subject of assessment to effectively perform responsibilities of a certain office position that is a result of his professional training and lifelong self-learning. Quantitative evaluation of competence is an integral characteristic of knowledge, skills, motivation of the subject from the totality of the abilities, defined by the membership function $\mu_k \in [0;1]$, and it characterizes the degree of person's ability to effectively perform responsibilities of a certain office position. Verbal evaluation of competence is an integral feature of knowledge, skills, motivation of the subject, which is defined by the terms of a linguistic variable "Competence», which is set on the multitude of values of the membership function, and characterizes a degree of his ability to effectively perform the duties of the certain office position.
2. A methodology of quantitative estimation of a person's competency is proposed. The method of quantitative evaluation of competence is proposed on the basis of fuzzy statements that create a system of compositional rules. The paper shows the use of production algorithms on the basis of max-min and multiplicative convolution of membership functions and the algorithm that account for the risk taken by a person who uses this competence assessments. The method of assessment of experts' competence that is based on the application of fuzzy logic allows to overcome such problems as multicriteriality of the task of evaluating competence of the subject, incomplete character of necessary information for this task, impossibility to measure quantitatively certain partial indicators of competence; and is more adequate to represent the experts' knowledge base.

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