Descriptive Modelling of System Dynamics at Different Stages

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Abstract. The characteristic problem of modern time that appears before researchers in the various areas of knowledge is the informative supersaturating that is largely caused by the systematic use of internet-technologies. Which mean that modern researcher, regardless of object of his researches, that studies the difficult system, runs into the enormous volumes of data, that require rapid treatment, analysis and practical conclusions.

Keywords: dynamical systems, mathematical modelling, descriptive dynamics

1 Introduction

The vast majority of data that are accessible via the Internet do not have the formalised structure, contain errors or are the results of incomplete observations. This means that analysis of large data arrays, their organisation, proceeding in the skipped information and construction of the model, is an intricate scientific and technical problem. It is well-known that the problem to expose terms of stability in at the natural and artificial systems, which are needed to provide existence and development of these systems is a very important problem. Unfortunately, we often do not have any formal theory when we study complex systems, and we are forced to build qualitative dynamic models based on available observational data and then to determine terms of system stability.

Fortunately, today there is a lot of datasets for the different areas of science (biology, medicine, ecology and other) and these datasets are accessible to the researchers. For example, data from International Council for the Exploration of the Sea (ICES) [2] that were used in previous researches conducted by Department of Theoretical and Applied Computer Science at V.N. Karazin Kharkiv National University were sent to the study of the indicated task.

The problem of homoeostasis (aspiring of the system is to keeping balance) and stability is closely constrained with the problem of dynamic firmness of this system. Study of this problem is closely constrained with the research of relations that determine the dynamics of system states that is described by the complexes of system parameters and their changes (transitions) in dependence on interrelations between these system parameters.

Despite of the different ways to find out the connections between parameters of the system in biology and ecology a general method offers based on the binary relations of next types : $(+;+)$, $(-;-)$, $(-;+)$, $(-;0)$, $(+;0)$, $(0;0)$. Such description suits to many systems. For the systems with many parameters this set describes all pair wise relationships between them. This approach gives to us an opportunity to show the structure of relations in the obvious form of intercommunications between the parameters of the natural system.

It is important to understand that not always is possibility to find the marked dependences by means of statistical methods. For example, a cross-correlation analysis is used for the estimation of connections between two or more variables, but covers only statistical dependences and does not give to description of case- consequence relations.

A perspective method for description of the system is descriptive modelling of system dynamics. It is a convenient method for the description of the difficult systems, that consists from solving two important tasks.

The first task is the construction of descriptive model of dynamics of the system. A construction gives an empiric idea about the state of the system transitions. It‘s only a description that based on the discrete eventual scale of the states of the system, but not exact presentation of the system. For example: the degenerate state, low-spirited state, normal state, state higher norms, state of complete satiation. A scale gives us an opportunity to describe the value of parameters of the system and consider time-histories of vector of the states of the system. A solution of this problem is useful and practical, because a descriptive model does not need high-cube of data and time for the construction and gives an opportunity to lay down an idea about tendencies state of the system transition, to check and cast aside inadequate hypotheses.

The second task is a problem of authentication, or, in other words, task of synthesis of the system. Based on the conclusions that we got at the first step we can assume there is a mechanism of state of the system transition to that, to try to build him self-reactants description, to pick up to basis parameters, to expect the error of our suppositions.

2 Modelling Framework

The study is based on the modelling framework proposed in [4].

2.1 States, System Trajectories, and Dynamics

Practically it looks like the following. Consider that the biological or economic system can be described through N components like $A_1, A_2, \ldots, A_N$. These components can be of different nature. For example, they can be number of animals or amount of biomass of different species, etc. Discrete values are assumed for components, such as $1, 2, \ldots, K$. Here 1 is the least meaning, and K is the greatest. Thus, the value 1 means the minimum amount of a component and the value K means the maximum amount. The value of each component is observed and measured at discrete instant of time $t = 0, 1, \ldots$. Thus, the value of the component A_i at the instants of time $t = 0, 1, \ldots$ is numbered as $A_i(0), A_i(1), \ldots$.

The trajectory of the system is described by an infinite-right matrix as

$$\begin{pmatrix} A_1(0) & A_1(1) & A_1(2) & \ldots \\ A_2(0) & A_2(1) & A_2(2) & \ldots \\ \vdots & \vdots & \vdots & \vdots \\ A_N(0) & A_N(1) & A_N(2) & \ldots \end{pmatrix} \tag{1}$$

This trajectory includes all states of the system at the moments $t = 0, 1, \ldots$. Hence, the state of the system at the instant of time t is represented by the vector $(A_1(t), A_2(t), \ldots, A_N(t))^\mathsf{T}$ where the sign T is the sign of the matrix transposition. It is supposed that the system is strictly determined, and it's state at the moment of time t is completely determined by the state at the moment $t-1$.

The system has only finite number of states, namely, K^N. In this case, there exists $\tau > 0$ such that $A_i(t+\tau) = A_i(t)$ for all $t \geq t_0$ where t_0 is some positive integer. This τ is called a period of the trajectory.

We can extract a minor from the matrix (1) formed by the columns t^{th} $(t \geq t_0)$, $(t+1)^{\text{th}}$, and up to $(t+\tau-1)^{\text{th}}$. The obtained minor

$$\begin{pmatrix} A_1(t) & A_1(t+1) & A_1(t+2) & \ldots & A_1(t+\tau-1) \\ A_2(t) & A_2(t+1) & A_2(t+2) & \ldots & A_2(t+\tau-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_N(t) & A_N(t+1) & A_N(t+2) & \ldots & A_N(t+\tau-1) \end{pmatrix} \tag{2}$$

presents full description of the behaviour of the system.
In this context, we say that dynamics is the complex of $\{1, 2, \ldots, K\}$-

valued mappings $f_i(s, a_1, \ldots, a_N)$ where $i = 1, 2, \ldots, N$, $a_1, \ldots, a_N$ belong to $\{1, 2, \ldots, K\}$, and $s = t, t+1, \ldots, t+\tau-1$ such that

$$A_i(s+1) = f_i(s, A_1(s), \ldots, A_N(s)) \quad \text{for } t \le s < t+\tau-1. \tag{3}$$

We study only the case when all f_i in (3) do not depend on s and call this case the case of stationary dynamics. In this case, the equations Eq (3) take the form

$$A_i(s+1) = f_i(A_1(s), \ldots, A_N(s)) \quad \text{for } t \le s < t+\tau-1. \tag{4}$$

2.2 Relationships between Components

In physics, the dynamics of a system is usually determined by the sum of contributions corresponding to the dynamics of pairwise interactions of system components. Now, we give some generalisation of this idea for our study.

Introduce the concept of relationships between components. Let $\Omega = \{=, 0, +\}$ then the relationship between components A_i and A_j is determined as a member from the set $\Omega \times \Omega$ and denoted as $\Lambda(A_i; A_j) = (\omega_1, \omega_2) \in \Omega \times \Omega$.
If $\Lambda(A_i; A_j) = (\omega_1, \omega_2)$ then the mean of the relation is the following

1. if $\omega_1 = -$ then the larger value of the component A_j is, the lower value of the component A_i would be;
2. if $\omega_1 = 0$ then the value of the component A_j would not influence on value of the component A_i;
3. if $\omega_1 = +$ then the lower value of the component A_j is, the larger value of the component A_i would be.

We require also antisymmetric of the relationship Λ, i.e. we claim the satisfaction of the condition

$$\Lambda(A_i, A_j) = (\omega_1, \omega_2) \text{ implies } \Lambda(A_j, A_i) = (\omega_2, \omega_1).$$

All the combinations (ω_1, ω_2) correspond to relationships (interspecific interactions) of neutralism, amensalism, predation, commensalism, and mutualism widely used in ecology and biology [3].

We can associate each matrix Λ with the $N \times N$-matrix S consisting of elements equals ether -1, or 0, or $+1$ in the following manner

$$S_{ij} = \begin{cases} -1 & \text{if } \Lambda(A_i, A_j) = (\omega_1, \omega_2) \text{ and } \omega_1 = - \\ 0 & \text{if } \Lambda(A_i, A_j) = (\omega_1, \omega_2) \text{ and } \omega_1 = 0 \\ +1 & \text{if } \Lambda(A_i, A_j) = (\omega_1, \omega_2) \text{ and } \omega_1 = + \end{cases}$$

Below we denote by $\mathcal{S}_N$ the class of such $N \times N$-matrices.

Now we describe some general approach to determine system dynamics based on a matrix from the class $\mathcal{S}_N$.

To do this let us consider some positive real-valued function ψ_{ij} of two arguments from $\{1, 2, \ldots, K\}$ that specifies the influence degree of component A_j onto component A_i in the corresponding states. We require only that each function ψ_{ij} does not decrease in the second argument. Also, let us consider some folding function $\pi\colon \mathbb{R}^N \to \mathbb{R}$ and determine

$$f_i(a_1, \ldots, a_N) = \\ \begin{cases} \operatorname{inc} a_i & \text{if } \pi(S_{i1} \cdot \psi_{i1}(a_i, a_1), \ldots, S_{iN} \cdot \psi_{iN}(a_i, a_N)) > \delta \\ a_i & \text{if } |\pi(S_{i1} \cdot \psi_{i1}(a_i, a_1), \ldots, S_{iN} \cdot \psi_{iN}(a_i, a_N))| \le \delta \\ \operatorname{dec} a_i & \text{if } \pi(S_{i1} \cdot \psi_{i1}(a_i, a_1), \ldots, S_{iN} \cdot \psi_{iN}(a_i, a_N)) < -\delta \end{cases} \tag{5}$$

where $\operatorname{inc} x = \min(x+1, K)$ and $\operatorname{dec} x = \max(x-1, 0)$ for $1 \le x \le K$.

Let us set the following problem.

Problem 1. Let ψ_{ij}, $\delta > 0$, and π be given then for each matrix S of class $\mathcal{S}_N$ and a dataset to find an operator $T = (f_1, \ldots, f_N)^{\mathsf{T}}$ such that the trajectory obtained with using the operator T is well coincide with the given dataset.

2.3 Weight Function Approach

This approach assumes a special form of functions ψ_{ij} and π. Namely, let $\psi_{ij}(a_i, a_j) = \psi_{ij}^{(k)}$ where $k = |a_i - a_j|$ and $\pi(x_1, \ldots, x_N) = \sum_{s=1}^{N} x_s$. In this case, Eq (5) is rewritten in the following form

$$f_i(a_1, \ldots, a_N) = \begin{cases} \operatorname{inc} a_i & \text{if } \sum_{s=1}^{N} S_{is} \cdot \psi_{is}^{(|a_i - a_s|)} > \delta \\ a_i & \text{if } \left| \sum_{s=1}^{N} S_{is} \cdot \psi_{is}^{(|a_i - a_s|)} \right| < \delta \\ \operatorname{dec} a_i & \text{if } \sum_{s=1}^{N} S_{is} \cdot \psi_{is}^{(|a_i - a_s|)} < -\delta \end{cases} \tag{6}$$

It is easily seen that Eq (6) determine uniquely the transition operator $T = (f_1, \ldots, f_N)^{\mathsf{T}}$ by the tuple of parameters

$$\langle \delta, \psi_{ij}^{(k)} \mid 1 \le i, j \le N, 1 \le k \le K \rangle.$$

In this context, it is evident that different parameter tuples can determine the same transition operator. Therefore, we have the following problem.

Problem 2. For two tuples $\langle \delta', \psi_{ij}^{(k)} \mid 1 \leq i, j \leq N, 1 \leq k \leq K \rangle$ and $\langle \delta'', \phi_{ij}^{(k)} \mid 1 \leq i, j \leq N, 1 \leq k \leq K \rangle$ determine are the corresponding transition operators equal or no.

Decidability of this problem is ensured by the theory of linear inequalities developed by S. Chernikov [1] But the principal task is to develop effective computational method to solve the problem.

3 Conclusion

In the paper, the modelling framework has been established. This framework is appropriate for mathematical modelling of descriptive dynamics of complex natural and artificial systems. We have demonstrated that all stages of building mathematical models are too complicated, but the most difficult task among them is the model parameter estimation for identifying the structure of the studied system.

This paper starts the development of the class of mathematical models, which will be useful in solving different problems, especially some environmental and healthcare problems.

At this paper, the few problems that need to be solved have been drawn. Firstly, it is a choice of operators that are adequate to our rules of transitions. And secondly, it is a problem of data visualisation.

Both of these problems are closely related to the problem of large data, and we hope that our study will contribute to both modelling of complex systems and processing of big data areas of knowledge.

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