Green's Functions Application for Computer Modeling in Electromagnetics

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Abstract. Features of application of the Green's function method in the Cartesian, cylindrical, and spherical coordinate systems are considered. The equivalent circuit method is used to describe the layered structure. To simulate propagation of the electromagnetic waves in each layer, matrices of the layer transmission and boundaries are applied. It is shown that the transfer matrices of the boundaries in the Cartesian and spherical coordinate systems are unit matrices. Different types of load model the boundaries of areas. The proposed approach allows one to create universal algorithms with common modules for solving various electromagnetic problems. Those are excitation and propagation of waves, antenna radiation and diffraction associated with flat, cylindrical, and spherical layered structures with an arbitrary number of layers, permittivity, and permeability. Some examples of using the Green's functions in software products are presented.

Keywords: electromagnetics, Greens function, scattering, radiation, algorithms

1 Introduction

The tasks of electromagnetic modeling are the integral part of most projects related to telecommunications, navigation, and radar. These can be particular ones related to design of antennas, antenna arrays, high frequency circuits, elements of radio-electronics, etc. Tasks can be very different, such as synthesis of the non-reflective or selectively transmitting materials, solution of electromagnetic scattering problem at objects, solution of problem of radio wave propagation in inhomogeneous media, etc. At the present stage, taking into account the great capabilities of computing technology, specialized electromagnetic simulation software (such as Ansys HFSS, FEKO, CST Microwave Studio) is widely used [13]. However, solution of the electromagnetic radiation and diffraction problems is usually associated with partition of the analyzed objects into the simplest volumes or surfaces and the subsequent solution of systems of extremely high-order algebraic equations. This leads to significant costs of the CPU time. This is especially apparent when optimizing the geometry of objects or the electro-physical characteristics of the materials is used. Good initial approximation in optimizing the electrodynamic characteristics of the objects under study is actual.

Along with the use of special software, methods based on an analytical approach are also used, but recently not so wide. This fact is due to the rather complex preparatory work and high requirements for knowledge of mathematics in derivation of expressions suitable for programming. Another constraint is the limitations in the choice of the shape of the analyzed objects for compact computational formulas. The analyzed objects should be fitted to the existing coordinate systems. The preferable forms are the plane-parallel structures, parallelepipeds, cylinders, spheres and their fragments. This requirement is due to the need to perform the integration operation over the coordinate surfaces. Despite these limitations, analytical methods can be considered as a powerful tool for solving many electromagnetic problems, and, also, as a first step in solving complex problems using electromagnetic simulation software. For example, if the task is to synthesize an object with a minimum scattering diameter, the initial synthesis of the non-reflective coatings can be performed using the analytical approaches. The gain increases with increasing electrical dimensions of the objects under investigation.

One of the most common analytical methods for solving problems of electrodynamics is the Green's function method. Being a solution of the inhomogeneous Maxwell equations for a source in the form of a delta function, the Green's functions allow one to calculate a vector field at an arbitrary point in space. By taking into account all the excitation points, one can construct the pattern of the electromagnetic field distribution. Another advantage of this method is that the field can be counted only in the required areas that save the CPU time. When obtaining Green's functions, the boundary conditions at the boundaries of the regions and the radiation conditions at infinity are taken into account. The method is a rigorous electrodynamic one.

This article applies the Green's functions to layered structures in the Cartesian, cylindrical, and spherical regions and shows the features of the use of the Green's functions apparatus when using the model of equivalent electrical circuits to describe the layered structure. The results of electromagnetic modeling are given on the example of synthesis of the non-reflective coatings and solution of the diffraction problems.

2 Green's functions in generalized coordinates

In general, the task of calculating the electromagnetic field excited by the extraneous electric $\boldsymbol{J}(\boldsymbol{r}')$ and/or magnetic $\boldsymbol{M}(\boldsymbol{r}')$ currents is written in the following form:

$$\boldsymbol{E}(\boldsymbol{r}) = \int\limits_{V'} \left[\overline{\overline{\Gamma}}_{11}(\boldsymbol{r}, \boldsymbol{r}')\boldsymbol{J}(\boldsymbol{r}') + \overline{\overline{\Gamma}}_{12}(\boldsymbol{r}, \boldsymbol{r}')\boldsymbol{M}(\boldsymbol{r}') \right] dv', \tag{1}$$

where $\overline{\overline{\Gamma}}_{11}(\boldsymbol{r}, \boldsymbol{r}')$, $\overline{\overline{\Gamma}}_{12}(\boldsymbol{r}, \boldsymbol{r}')$ are the electric and "transfer" Green's function, respectively [4]. Similarly, expression for calculation of the magnetic field component is written. Vector $\boldsymbol{r}'$ defines the source point, vector $\boldsymbol{r}$ defines the observation point. Integration in (1) is performed over the region of the source.

For an unlimited homogeneous space, the Green's function for electric field excited by electric current has a simple form:

$$\overline{\overline{\Gamma}}_{11}(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{4\pi} \frac{\exp(-jk|\boldsymbol{r} - \boldsymbol{r}'|)}{|\boldsymbol{r} - \boldsymbol{r}'|}. \tag{2}$$

If the space is limited, for example, by a conducting plane, the Green's function becomes more complicated. Moreover, when constructing the functions, the used coordinate system should be taken into account.

All electromagnetic problems of excitation, radiation, and scattering are solved with a model of electric and magnetic currents. For example, the currents on the surface of the printed patch antenna are of the electric type. The currents in the slot antenna are magnetic ones. The field formed by the horn antenna is calculated as the radiation of Huygens elements in the aperture of the horn, whose amplitude and phase are determined by the parameters of the horn. The Huygens element itself is modeled by a pair of the orthogonal electric and magnetic dipoles. The diffraction problems are reduced to the problems of radiation by the currents induced on the irradiated objects.

Greens functions are especially complex in the inhomogeneous and layered media. To simplify the Green's functions, the electromagnetic field is presented as a superposition of the electric and magnetic waves [5]. Separation of wave types is carried out relatively to the selected axis of the coordinate system. In the Cartesian system, all axes are equivalent. Let's choose the z axis. The electromagnetic field is decomposed into a spatial spectrum of the plane waves. In a cylindrical system, separation occurs relatively to the z axis. The field is decomposed into a spectrum of the spatial longitudinal waves with respect to this axis: either radially propagating or azimuthally propagating waves. The choice of decomposition depends on the problem to be solved. In the spherical coordinate system, the radial axis is selected as the separation one. The field is formed by the spatial harmonics of waves propagating along the angular or radial coordinates.

Layer structure in all coordinate systems is modeled by the equivalent electrical circuits. The validity of this approach follows from solution of the inhomogeneous system of the Maxwell equations. In this case, the transverse field components are expressed in terms of the components associated with the decomposition axis. The system of 6 algebraic equations is divided into two independent systems of equations. For example, in the cylindrical coordinates, for a homogeneous layer, this system looks as follows:

$$\begin{cases} -\dfrac{d}{dr}(\dot{E}^e_{zmh}) = -j\dfrac{\gamma^2 Z_0}{\varepsilon r k_0^2}(k_0 r \dot{H}^e_{\varphi mh}) - \dfrac{h Z_0}{\varepsilon k_0}\dot{J}^{E\,ex}_{rmh} - \dot{J}^{M\,ex}_{\varphi mh}, \\ -\dfrac{d}{dr}(k_0 r \dot{H}^e_{\varphi mh}) = j\dfrac{\varepsilon k_0^2}{Z_0 \gamma^2} r \left[k^2 - h^2 - \left(\dfrac{m}{r}\right)^2\right] \dot{E}^e_{zmh} + k_0 r \dot{J}^{E\,ex}_{zmh} - \\ \qquad\qquad - \dfrac{m h k_0}{\gamma}\dot{J}^{E\,ex}_{\varphi mh} + \dfrac{\varepsilon k_0}{\gamma^2 Z_0} m \dot{J}^{M\,ex}_{rmh}. \end{cases} \tag{3}$$

The type of the reduced system of equations resembles the system of equations for the voltage and current in a long line as follows:

$$\begin{cases} -\dfrac{d}{dr}V_E = jZ_E\chi I_E + \nu_{ct}^E, \\ -\dfrac{d}{dr}I_E = jY_E\chi V_E + i_{ct}^E. \end{cases} \tag{4}$$

Comparing (3) and (4) you can get the following expressions for equivalent electric circuit:

$$V_E = \dot{E}_{zmh}^e, I_E = -k_0 r \dot{H}_{\varphi mh}^e,\ \chi^2 = k^2 - h^2 - \left(\frac{m}{r}\right)^2,\ Z_E = \frac{\gamma^2}{k_0^2 \varepsilon r} \frac{Z_0}{\sqrt{k^2 - h^2 - \left(\frac{m}{r}\right)^2}}.$$

Since we have electric and magnetic wave, the decomposition of two E and H-lines in electric equivalent circuit is used (Fig. 1). Similar operations have been carried out for other coordinate systems. The generalized coordinate τ corresponds to the coordinate z, ρ, or r in the Cartesian, cylindrical, and spherical coordinate system, respectively.

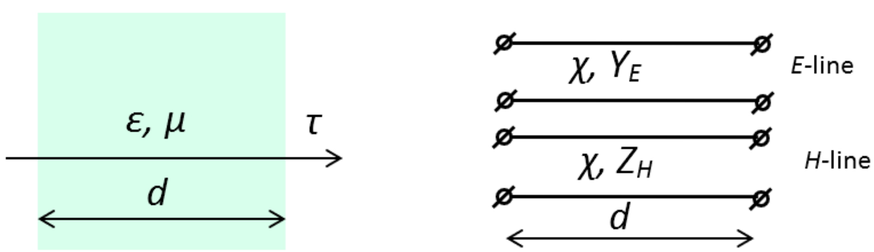

Fig. 1. A homogeneous layer and its electric circuit model

Let us note that the propagation constant and the characteristic impedance in the equivalent circuit in the cylindrical and spherical coordinate systems depend on the radial component. In the Cartesian coordinate system, they remain constant.

If the medium is heterogeneous and it is modeled by a layered structure, the equivalent circuit is the cascade connection of line segments with different parameters (Fig. 2). The directional resistances $\overleftarrow{Z}_T$, $\overrightarrow{Z}_T$ and conductivities $\overleftarrow{Y}_T$, $\overrightarrow{Y}_T$, as the terminal loads are used for the region boundaries modeling (Table 1 and Table 2).

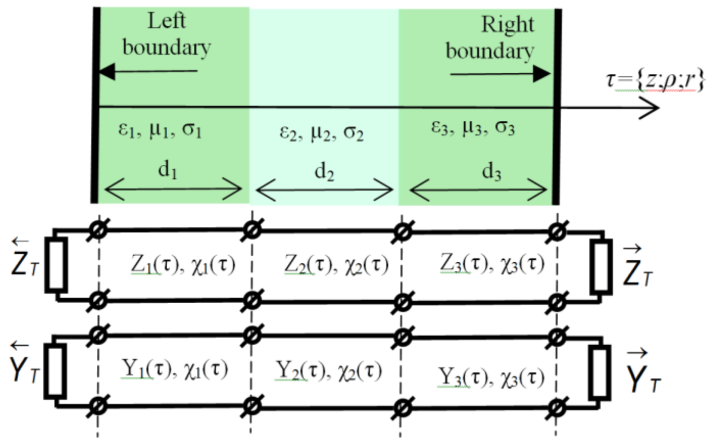

Fig. 2. The layered structure and equivalent electric and magnetic lines

In Tables 1 and 2, the index N is the number of the last layer, $H_m^{(2)}$, $H_m^{(2)'}$, J_m, J_m', $h_m^{(2)}$, $h_m^{(2)'}$, j_m, and j_m' are the cylindrical and spherical Hankel and Bessel functions and their derivatives, respectively. The functions C_{1m}, S_{2m}, S_{1m}, C_{2m}, s_n, s_n', c_n, and c_n' are combinations of the cylindrical and spherical functions of the 1st and 2nd kind, respectively [6]. The plus sign indicates that we determine the input resistance or admittance in the equivalent circuit to the right of the boundary. The free space wave impedance is $Z_0 = Y_0^{-1} = 120\pi$ Ohm.

Wave impedance and propagation constant of equivalent electric and magnetic line is shown in Table 3.

In general, the circuit in Fig. 2 can be described as a cascade connection of the 4-port devices. The layer transmission matrices $[C_i]$ and boundary transmission matrices $[\Gamma_i]$ are introduced (Fig. 3).

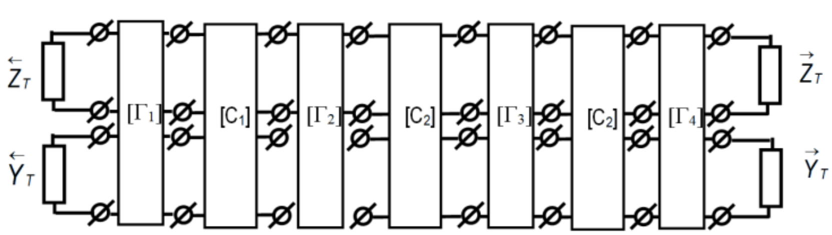

Fig. 3. The layer and boundary transfer matrices for the layered structure model

Table 1. The expressions for outer terminal loads

Region boundary	Outer Terminal Loads $\overrightarrow{Z}_T$, $\overrightarrow{Y}_T$
Free space in Cartesian system	$\overrightarrow{Y}^+_{NE} = Y_0$
	$\overrightarrow{Z}^+_{NH} = Z_0$
Layered flat screen	$\overrightarrow{Y}^+_{NE} = jY_0 \dfrac{\gamma_{N+1}}{k_{N+1}} \cot \gamma_{N+1} d_{N+1},\ \gamma_{N+1} = \sqrt{k^2_{N+1} - \xi^2 - \eta^2}$
	$\overrightarrow{Z}^+_{NE} = jZ_0 \dfrac{\gamma_{N+1}}{k_{N+1}} \tan \gamma_{N+1} d_{N+1},\ \gamma_{N+1} = \sqrt{k^2_{N+1} - \xi^2 - \eta^2}$
Free space around the cylinder	$\overrightarrow{Y}^+_{NE} = jY_0 \dfrac{k_0^2 \rho_N}{\gamma_{N+1}} \dfrac{H_m^{(2)'}(\gamma_{N+1}\rho_N)}{H_m^{(2)}(\gamma_{N+1}\rho_N)}, \gamma_{N+1} = \sqrt{k_0^2 - h^2}$
	$\overrightarrow{Z}^+_{NH} = jZ_0 \dfrac{k_0^2 \rho_N}{\gamma_{N+1}} \dfrac{H_m^{(2)'}(\gamma_{N+1}\rho_N)}{H_m^{(2)}(\gamma_{N+1}\rho_N)}, \gamma_{N+1} = \sqrt{k_0^2 - h^2}$
Inner surface of perfectly conductive cylinder	$\overrightarrow{Y}^+_{NE} = jY_0 \dfrac{\varepsilon'_{N+1}}{\gamma_{N+1}} k_0^2 \rho_N \dfrac{C_{1m}(\gamma_{N+1}\rho_N, \gamma_{N+1}\rho_{N+1})}{S_{2m}(\gamma_{N+1}\rho_N, \gamma_{N+1}\rho_{N+1})}$
	$\overrightarrow{Z}^+_{NH} = jZ_0 \dfrac{\mu'_{N+1}}{\gamma_{N+1}} k_0^2 \rho_N \dfrac{S_{1m}(\gamma_{N+1}\rho_N, \gamma_{N+1}\rho_{N+1})}{C_{2m}(\gamma_{N+1}\rho_N, \gamma_{N+1}\rho_{N+1})}$
Inner surface of an impedance cylinder	$\overrightarrow{Y}^-_{NE} = k_0 \rho_N \sqrt{\dfrac{2\sigma}{\omega\mu_0}} \dfrac{(1-j)}{2}$
	$\overrightarrow{Z}^-_{NH} = k_0 \rho_N \sqrt{\dfrac{\omega\mu_0}{2\sigma}} (1+j)$
Free space in spherical system	$\overrightarrow{Y}^+_{NE} = jY_0 \dfrac{h_m^{(2)'}(k_0 r_N)}{h_m^{(2)}(k_0 r_N)}$
	$\overrightarrow{Z}^+_{NE} = jZ_0 \dfrac{h_m^{(2)'}(k_0 r_N)}{h_m^{(2)}(k_0 r_N)}$

Table 2. The expressions for inner terminal loads

Region boundary	**Inner Terminal Loads** $\overleftarrow{Z}_T$, $\overleftarrow{Y}_T$
Free space in Cartesian system	$\overleftarrow{Y}^-_{1E} = Y_0$
	$\overleftarrow{Z}^-_{1H} = Z_0$
Layered flat screen	$\overleftarrow{Y}^-_{1E} = -jY_0 \dfrac{\gamma_1}{k_0} \cot \gamma_1 d_1,\ \gamma_1 = \sqrt{k_1^2 - \xi^2 - \eta^2}$
	$\overleftarrow{Z}^-_{1H} = -jZ_0 \dfrac{\gamma_1}{k_0^2} \tan \gamma_1 d_1,\ \gamma_1 = \sqrt{k_1^2 - \xi^2 - \eta^2}$
Dielectric cylinder	$\overleftarrow{Y}^-_{1E} = jY_0 \dfrac{\varepsilon_1'}{\gamma_1} k_0^2 \rho_1 \dfrac{J_m'(\gamma_1\rho_1)}{J_m(\gamma_1\rho_1)},\ \gamma_1 = \sqrt{k_1^2 - h^2}$
	$\overleftarrow{Z}^-_{1H} = jZ_0 \dfrac{\mu_1'}{\gamma_1} k_0^2 \rho_1 \dfrac{J_m'(\gamma_1\rho_1)}{J_m(\gamma_1\rho_1)},\ \gamma_1 = \sqrt{k_1^2 - h^2}$
Layered perfectly conductive cylinder	$\overleftarrow{Y}^-_{1E} = jY_0 \dfrac{\varepsilon_1'}{\gamma_1} k_0^2 \rho_1 \dfrac{C_{1m}(\gamma_1\rho_1, \gamma_1\rho_0)}{S_{2m}(\gamma_1\rho_1, \gamma_1\rho_0)}$
	$\overleftarrow{Z}^-_{1H} = jZ_0 \dfrac{\mu_1'}{\gamma_1} k_0^2 \rho_1 \dfrac{S_{1m}(\gamma_1\rho_1, \gamma_1\rho_0)}{C_{2m}(\gamma_1\rho_1, \gamma_1\rho_0)}$
Impedance cylinder	$\overleftarrow{Y}^-_{1E} = \overleftarrow{Y}^+_{1E} = -k_0\rho_1 \sqrt{\dfrac{2\sigma}{\omega\mu_0}} \dfrac{(1-j)}{2}$
	$\overleftarrow{Z}^-_{1H} = \overleftarrow{Z}^+_{1H} = -k_0\rho_1 \sqrt{\dfrac{\omega\mu_0}{2\sigma}} (1+j)$
Dielectric sphere	$\overleftarrow{Y}^-_{1H} = jY_0 \sqrt{\dfrac{\varepsilon_1'}{\mu'}} \dfrac{j_m'(k_1, r_1)}{j_m(k_1, r_1)}$
	$\overleftarrow{Z}^-_{1H} = jZ_0 \sqrt{\dfrac{\mu'}{\varepsilon_1'}} \dfrac{j_m'(k_1, r_1)}{j_m(k_1, r_1)}$
Layered perfectly conductive sphere	$\overleftarrow{Y}^-_{1E} = -jY_0 \sqrt{\varepsilon_1'} \dfrac{s_n'(k_1r_1, k_1r_0)}{s_n(k_1r_1, k_1r_0)}$
	$\overleftarrow{Z}^-_{1H} = -j \dfrac{Y_0}{\sqrt{\varepsilon_1'}} \dfrac{c_n'(k_1r_1, k_1r_0)}{c_n(k_1r_1, k_1r_0)}$

Table 3. Equivalent line parameters in different coordinate systems

Coordinate system	**Equivalent line Characteristic**	
	Parameter	*Expression*
Cartesian	Admittance of E-line	$Y_i = Y_0\sqrt{\varepsilon'_i/\mu'_i}$
	Admittance of H-line	$Z_i = Z_0\sqrt{\mu'_i/\varepsilon'_i}$
	Propagation constant	$\chi_i = k_i$
Cylindrical	Admittance of E-line	$Y_i = Y_0\frac{k_0^2\varepsilon'_i r}{\gamma_i^2}\sqrt{\gamma_i^2-(\frac{m}{r})^2}$
	Admittance of H-line	$Z_i = Z_0\frac{k_0^2\mu'_i r}{\gamma_i^2}\sqrt{\gamma_i^2-(\frac{m}{r})^2}$
	Propagation constant	$\chi_i = \sqrt{\gamma_i^2-(\frac{m}{r})^2}$, $\gamma_i = \sqrt{k_i^2-h^2}$
Spherical	Admittance of E-line	$Y_i = Y_0 k_0\varepsilon'_i/\chi_i$
	Admittance of H-line	$Z_i = Z_0 k_0\mu'_i/\chi_i$
	Propagation constant	$\chi_i = \sqrt{k_i^2-\left(\frac{m(m+1)}{r}\right)^2}$

Wave propagation in each layer is described with the 4-port network and transmitting matrix $[C_i]$. There is no interconnection between E- and H-lines in homogeneous medium and between inner the boundaries of each layer; so, the transmitting matrix can be simplified:

$$[C_i] = \begin{bmatrix} [C_{iE}] & [0] \\ [0] & [C_{iH}] \end{bmatrix},$$

where [0] is the null matrix of order 2. Two ports layer network with the transfer matrix $[C_{iE}]$ is associated with the equivalent electric line, and the network with the transfer matrix $[C_{iH}]$ is associated with the equivalent magnetic line.

The boundary transmission matrix in the Cartesian and spherical coordinate systems is the identity matrix. But in the cylindrical coordinates, these matrices can be complicated:

$$[\Gamma_i] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -N_i \\ N_i & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where $N_i = \dfrac{mh}{k_0}\left(\dfrac{1}{\gamma_i^2} - \dfrac{1}{\gamma_{i+1}^2}\right)$, $\gamma_i = \sqrt{k_i^2 - h^2}$.

The suggested layer structure model allows one to develop algorithms for the electromagnetic field calculation based on common modules for different kind of radiation, propagation, and scattering problems.

3 Green's functions in basic coordinate systems

The Greens functions according to the electromagnetic field decomposition onto the electric and magnetic waves have two parts. The first one takes into account the layer structure and it is characteristic part of the functions. The second part describes field in the perpendicular (to the layers) directions. These surfaces may be limited or unlimited by conducting surfaces.

In the Cartesian coordinates, the Greens functions are as follows:

$$\overline{\overline{\Gamma}}_{ij}(x, y, z, x', y', z') = j\omega\mu_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F\left\{g(z; z'), f(z; z')\right\} G_t(x, y; x', y') d\xi d\theta, \quad (5)$$

where for unlimited space along the x and y coordinates

$$G_t(x, y; x', y') = \frac{1}{4\pi} e^{-i\xi(x-x')} e^{-i\theta(y-y')}.$$

In (5), the characteristic functions and are solutions of the differential equations with the proper boundary conditions:

$$\frac{\partial^2 f(z, z')}{\partial z^2} + \gamma^2 f(z, z') = \delta(z - z'),$$
$$\frac{\partial^2 g(z, z')}{\partial z^2} + \gamma^2 g(z, z') = \delta(z - z').$$

If the boundary is the perfect conductor, we have at this section $f(z, z') = \dfrac{\partial f(z, z')}{\partial z} = 0$.

For the cylindrical coordinate system, the Greens functions are defined as follows:

$$\overline{\overline{\Gamma}}_{ij}(\rho, \varphi, z, \rho', \varphi', z') = -j\frac{\omega\mu_0}{2\pi} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} F\left\{g_m(\rho; \rho'), f_m(\rho; \rho')\right\} G_t(m, h, \varphi, z; \varphi', z') dh,$$

where $G_t(m,h,\varphi,z;\varphi',z') = \dfrac{1}{2\pi} e^{-jm(\varphi-\varphi')} e^{-jh(z-z')}$.

The characteristic functions $g_m(\rho;\rho')$ and $f_m(\rho;\rho')$ are solutions of the differential equations:

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dg_m(\rho;\rho')}{d\rho}\right) + \left(k^2 - h^2 - \frac{m^2}{\rho^2}\right) g_m(\rho;\rho') = -\delta(\rho-\rho'),$$
$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{df_m(\rho;\rho')}{d\rho}\right) + \left(k^2 - h^2 - \frac{m^2}{\rho^2}\right) f_m(\rho;\rho') = -\delta(\rho-\rho').$$

If the spherical structures are under consideration, the Greens functions are defined as:

$$\overline{\overline{\Gamma}}_{ij}(\theta,\varphi,r;\theta',\varphi',r') = j\omega\mu_0 \sum_{n=0}^{\infty}\sum_{m=-n}^{n} F\{g_n(r;r'), f_n(r;r')\} G_t(m,n,\theta,\varphi;\theta',\varphi'),$$

where $G_t(m,n,\theta,\varphi;\theta',\varphi') = \dfrac{1}{4\pi}\varPhi\left\{P_n^m(\cos\theta)e^{-jm\varphi}, P_n^m(\cos\theta')e^{jm\varphi'}\right\}$.

The spherical harmonics are calculated using the associated Legendre polynomials $P_n^m(\cos\theta)$.

The characteristic functions are solutions of the inhomogeneous equations:

$$\frac{\partial^2 g_n(r,r')}{\partial r^2} + k^2 g_n(r,r') = -\delta(r-r'),$$
$$\frac{\partial^2 f_n(r,r')}{\partial r^2} + k^2 f_n(r,r') = -\delta(r-r').$$

Thus, the Greens functions for three main coordinate systems (Cartesian, cylindrical and spherical) are given. A wide class of electromagnetic excitation, radiation, and diffraction problems can be solved using the appropriate distribution of extraneous currents. Using the analytical approaches allows one significantly speed up the calculation in electromagnetics.

4 Greens function application in electromagnetic software development

We applied the Greens function method to spherical and cylindrical Luneburg lens investigation [7]. It was successfully used for spherical and geodesic antenna radomes analysis [8]. Application of Greens functions to diffraction problems solving is described in this part. The spherical Luneburg lens is used with low directional antennas to increase the antenna gain. The Luneburg lens consists of inhomogeneous dielectric medium. The dielectric constant is changed from $\varepsilon = 2$ in the lens center to $\varepsilon = 1$ at the outer surface. This kind structure is fabricated as the layered one. It may be easy modeled by our approach.

The interface of the Luneburg lens design software is shown in Fig. 4.

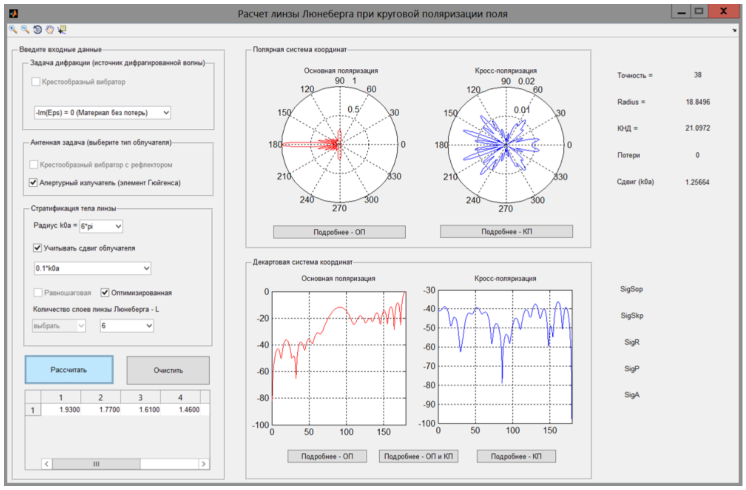

Fig. 4. The interface of software for the Luneburg lens radiation analysis

Radiation pattern for the main and cross-polarized components are calculated. Several types of low directional antennas as a Huygens element, a dipole with conducting screen, and crossed dipoles may be used as the lens irradiator. The high speed of calculations is confirmed by Table 4, in which the processing time for radiation pattern computation and different electrical outer radius k_0a is shown. In comparison with the traditional software, the calculation is performed by 2–3 orders faster.

Table 4. Processing time of the Luneburg lens antenna radiation calculation

k_0a	t, sec
2π	1.8
4π	2.3
6π	3.1
12π	12.2
20π	30.3
28π	43.5

For scattering electromagnetic waves by conducting and dielectric cylinder, a special software was developed. This kind of problems is applied to the non-reflecting cover design. The two-layer structures as a cover were analyzed. The interface of the software is shown in Fig. 5.

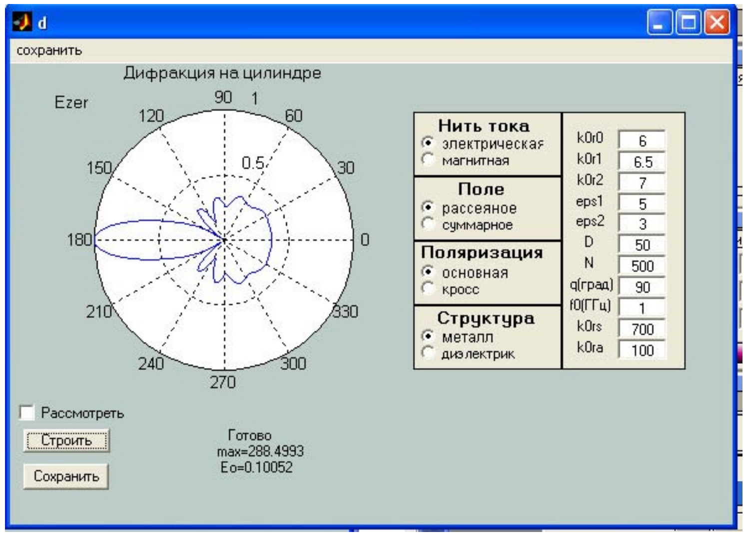

Fig. 5. The interface of software for scattering by covered cylinder

5 Conclusion

TThe spectral-domain full-wave approach based on Greens function method for layered magneto-dielectric structure analysis is described. The Green's functions for electromagnetic excitation problems in Cartesian, cylindrical and spherical coordinate systems are presented. The equivalent circuit model for layered structures analysis is applied. The 4-port network with transmitting matrix is used for boundary between layers description. The same kind of matrices was applied for wave propagation analysis in each layer. The solution remains correct for any type of isotropic magneto-dielectrics. The suggested approach was used for electromagnetic software developing. The procedure of calculation and optimization electromagnetic radiation and scattering is significantly accelerated in comparing with commercial software.

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