# Mathematical Method of Mapping Configuration Space for Manipulator Master-Slave Teleoperation

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## Abstract

The aim of the article is to expand the functionality of the copy control of an anthropomorphic manipulator using an exoskeleton master device. To achieve the goal, the article proposed a mathematical method for mapping the configuration space of the master manipulator into the configuration space of the slave manipulator. The developed mathematical method is based on the requirements of maximizing the working space of the slave manipulator, the uniqueness of the correspondence of the position of the slave manipulator to the position of the master manipulator and the human-like movements.

Keywords: anthropomorphic manipulator, mathematical method, copying control.

## 1 Introduction

Currently, many areas of human activity are intensively implemented robotic systems. One of the promising areas of robotics is anthropomorphic robots. The capabilities of anthropomorphic robots depend on the type of control used. Simple and effective type of control robots of this type is copying control. Copying control implements the virtual presence of the operator in the robot, which allows the robot to solve complex problems in non-deterministic environments: rescue operations, space exploration, military operations. The basis of this type of control is the simultaneous formation of the motion laws for all mobility degrees of the anthropomorphic robot (anthropomorphic manipulator) through the master device.

Copy control is one of the first ways to control robots. This is explained by the coincidence of the kinematic schemes of the Master and the Slave with an accuracy of the scale factor. In this case, the measured values of the generalized coordinates of the Master can be used as Slave control signals.

A new stage in the development of copy control systems began with the advent of master devices in the form of exoskeleton. These devices have mobility, as well as prospects for the implementation of force-moment sensing. The decisive factor when using exoskeleton is the convenience and freedom of movement, which determines their adjustment to the anthropometric parameters of a person and the difference between the Master kinematic scheme and the Slave kinematic scheme. The development of methods for calculating the rotation angles of

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In: Jože Rugelj, Maria Lapina (eds.): Proceedings of SLET 2019 – International Scientific Conference Innovative Approaches to the Application of Digital Technologies in Education and Research, Stavropol – Dombay, Russia, 20-23 May 2019, published at http://ceur-ws.org

the operator's hand based on the rotation angles of the driver [**Tebueva2018**, **Teb2018**, **Pet2018**] allows the capture of the operator's hands movements directly.

The developed exoskeleton complexes [**Kutlubaev2016**, **Kut2016**] for control of anthropomorphic manipulators (AM) in the majority represent the lever mechanism consisting of three rigid links (shoulder, elbow, hand) and the gripper identical to a hand of the person. In working condition, the exoskeleton is put on the operator, while the lever system is located parallel to the human hand. Registration of the rotation angles of the exoskeleton is carried out with the help of built-in encoders, the information from which is transmitted to the actuator or simulation program. At the same time, the centers of the end of the links and the rotation angles of the exoskeleton and the operator's hands do not coincide, since they are in different positions in space. Different anthropometric parameters between the human hand and the manipulator create errors in the formation of control laws that affect on the accuracy of the target operations of the actuator.

The existing approach to the control of anthropomorphic manipulator [**Tebueva2018**, **Teb2018**, **Pet2018**]] using a copy-type master device involves the use of generalized coordinates of the operator's hand as control signals, rather than the master device. The scheme of the generalized coordinates calculation of the operator's hand is shown in figure 1.

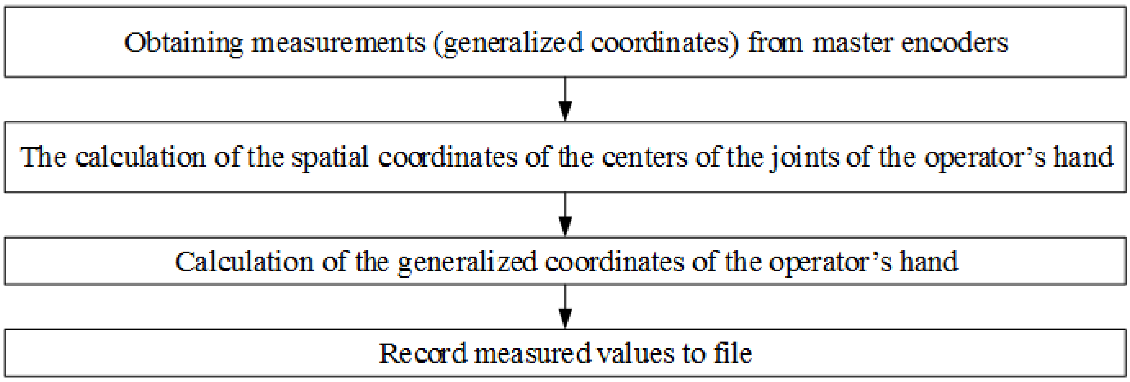


Figure 1: Scheme of generalized coordinates calculation of the operator's hand

The most complex computational problem is the calculation of generalized coordinates of the operator's hand on the basis of data on the position of the operator's hand joints, for which the methods of solving direct and inverse problems of kinematics are used [**Tebueva2018**, **Teb2018**].

The solution of the direct kinematics problem is necessary to convert the information about the position of the manipulator from its own coordinate system to the working (absolute) coordinate system to determine the coordinates of the links of the manipulator. The solution of the inverse kinematics problem is designed to calculate the spatial configuration of the manipulator by the position of its links [**Tebueva2018**, **Teb2018**]. This solution requires a description of the overall characteristics of the manipulator in a form convenient for their analysis and recording the equations of coordinate transformation. Of the existing approaches to the description of the dimensional characteristics of the manipulator, the main ones are their expression in the form of a system of linear or matrix equations [**Jaz2007**].

The basis for solving the problems of kinematics is the choice of parameters that uniquely determine the orientation of a solid in space. For this purpose, there are a number of kinematic parameters: guiding cosines and Euler angles [**Tebueva2018**, **Teb2018**, **Per2019**, **Mar2014**], Rodrigue-Hamilton parameters (quaternions) [**Rad2012**, **Isa2018**, **Gou2012**, **Gor2016**], Cayley-Klein parameters [**Oni2006**, **Jaz2007**] and the Denavit-Hartenberg representation [**Sic2009**].

The possibility of using a geometric approach to solving the inverse problem of kinematics [**Sha2018**] and the high prevalence of using the Denavit-Hartenberg apparatus provides advantages when it is used in describing the kinematics of the master device and calculating the generalized coordinates of the operator's hand.

When calculating the rotation angles of the operator's hand, the task of determining the coordinates of the elbow joint of the operator's hand arises, since there is an infinite number of positions of this joint. The solution of this problem is proposed in the work [**Pet2018**]. Another important factor is the difference between the lengths of the manipulator links from the lengths of the operator's arm parts, as well as the different "shoulder width" of the operator and the anthropomorphic robot. This leads to various kinds of problems in the transmission of the rotation angles of the operator's hand as the rotation angles of the Slave. For example, even with the coincidence of the links lengths, with a different distance between the shoulders, a situation is possible when two Masters joined their palms, but Slaves did not. In this case, the connection of the palms of the two Slaves becomes impossible. Creating anthropomorphic manipulators with adjustable length links is possible, but it is a complex task. A simpler task is to map the configuration space of the Master to the configuration space of the Slave. This article proposes a method that solves this problem.

## 2 Proposed method

### 2.1 System description

As an operator's hand model, we will consider an anthropomorphic manipulator. In this case, the hand of the operator can be considered as a virtual master manipulator. Since the Master and Slave are anthropomorphic manipulators, one formalism can be used to describe them. Further we will designate the values relating to the Master, without a stroke, and relating to the slave - with a stroke. The kinematic scheme of the Master is shown in Figure 2.

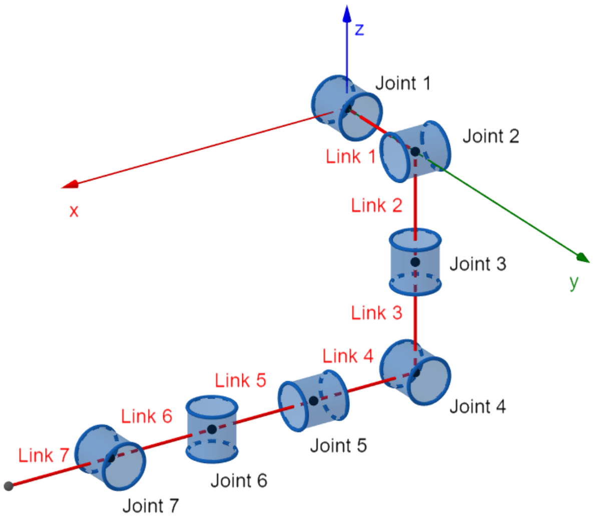


Figure 2: Kinematic scheme of anthropomorphic manipulator

To describe the parameters of the kinematic scheme, we use the Denavit-Hartenberg representation. In accordance with the Denavit-Hartenberg representation, each $i$-th link is described by a set of four parameters: $a, d, \alpha, \theta$, one of which characterizes the movement of the manipulator in the $i$-th joint and is the generalized coordinate of the manipulator. A system of seven links is respectively described by parameter vectors.$\mathbf{a}, \mathbf{d}, \alpha, \theta$. Since all kinematic pairs of a manipulator are rotational, the vector of generalized coordinates that uniquely describes the position of the manipulator is $\theta$.

In accordance with the Denavit-Hartenberg representation, each link of the manipulator is associated with the associated coordinate system according to a certain rule. The direction and location of the associated coordinate systems of the manipulator under consideration is shown in Figure 3.

The matrix of homogeneous transformation from the $i$-th coordinate system to the $j$-th can be found by the formulas [**Yan2016**]:

$${}^{\boldsymbol{i}}\boldsymbol{T_j}=\prod_{k=i+1}^{j} {}^{\boldsymbol{i-1}}\boldsymbol{A_i},\ i<j,$$

$${}^{\boldsymbol{i-1}}\boldsymbol{A_i}=\boldsymbol{T_{z,\theta}}\left(\theta_i\right)\boldsymbol{T_{z,d}}\left(d_i\right)\boldsymbol{T_{x.a}}\left(a_i\right)\boldsymbol{T_{x,\alpha}}\left(\alpha_i\right),$$

$$\boldsymbol{T_{z,\theta}}\left(\theta\right)=\begin{bmatrix} cos\left(\theta\right) & -sin\left(\theta\right) & 0 & 0 \\ sin\left(\theta\right) & cos\left(\theta\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{T_{z,d}}\left(d\right)=\left[\begin{array}{cccc} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & d\\ 0 & 0 & 0 & 1 \end{array}\right],$$

$$\boldsymbol{T_{x.a}}\left(a\right)=\left[\begin{array}{cccc} 1 & 0 & 0 & a\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{array}\right],$$

$$\boldsymbol{T_{x,\alpha}}\left(\alpha\right)=\left[\begin{array}{cccc} 1 & 0 & 0 & 0\\ 0 & cos(\alpha) & -sin(\alpha) & 0\\ 0 & sin(\alpha) & cos(\alpha) & 0\\ 0 & 0 & 0 & 1 \end{array}\right],$$

${}^{\boldsymbol{i}}\boldsymbol{T_j}$ – transformation matrix from $j$-th to $i$-th coordinate system;
${}^{\mathbf{i-1}}\mathbf{A_i}$ – homogeneous matrix of complex transformation for adjacent coordinate systems;
$\mathbf{T_{z,\boldsymbol{\theta}}}\left(\theta\right)$ – is a homogeneous matrix of elementary rotation around the $z$-axis by the angle $\theta$;
$\mathbf{T_{z,d}}\left(d\right)$ – homogeneous matrix of elementary shift along the $z$-axis by distance $d$;
$\mathbf{T_{x.a}}\left(a\right)$ – homogeneous matrix of elementary shift along the $x$-axis by distance $a$;
$\mathbf{T_{x,\boldsymbol{\alpha}}}\left(\alpha\right)$ – is a homogeneous matrix of elementary rotation around the $x$-axis by an angle $\alpha$;
$\mathbf{a}, \mathbf{d}, \boldsymbol{\alpha}, \boldsymbol{\theta}$ – Denavit-Hartenberg parameters describing the kinematic structure of the manipulator.

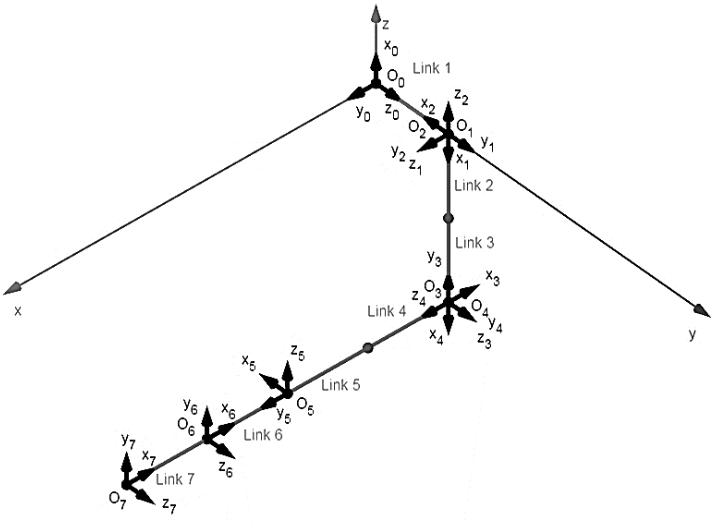


Figure 3: The coordinate system associated with the links of the anthropomorphic manipulator

The matrix of the uniform transformation $\mathbf{T_i}$ from the$i$-th coordinate system to the global coordinate system associated with the humeral articulation can be found using the formula:

$$\mathbf{T_i}=\boldsymbol{T_0}{}^{\mathbf{0}}\boldsymbol{T_i},\ i>0,$$

$$\mathbf{T_0}=\boldsymbol{T_{x.a}}\left(-90^{\circ}\right)\boldsymbol{T_{z,\theta}}\left(-90^{\circ}\right).$$

Solution of the direct kinematics problem, i.e. determining the position of points $\mathbf{O_i}$ in the Cartesian coordinate system for a given value of $\boldsymbol{\theta}$ has the form:

$$\mathbf{O_i}=\mathbf{T_i}\left(\begin{array}{c} 0\\ 0\\ 0\\ 1 \end{array}\right),$$

$\mathbf{O_i}$ – is the beginning of the $i$-th coordinate system.

The left upper submatrix $\mathbf{R_i}$ of size 3x3 of the matrix $\mathbf{T_i}$ describes the rotation of the $i$-th coordinate system relative to the global coordinate system. This rotation can also be described using Euler angles. Euler angles $\alpha_i, \beta_i, \gamma_i$ can be calculated by the rotation matrix $\mathbf{R_i}$ as follows:

$$\mathbf{R_i} \equiv \left[ \begin{array}{cccc} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{array} \right],$$

$$\alpha_i = ATAN2\left(a_x, -a_y\right),$$

$$\beta_i = ATAN2\left(-\cos\left(\alpha_i\right) s_x - \sin\left(\alpha_i\right)\ s_y\ , \cos\left(\alpha_i\right) n_x + \sin\left(\alpha_i\right)\ n_y\ \right),$$

$$\gamma_i = ATAN2\left(\sin\left(\alpha_i\right) a_x - \cos\left(\alpha_i\right)\ a_y\ , a_z\right),$$

$\mathbf{R_i}$ – is the rotation matrix of the $i$-th coordinate system relative to the global coordinate system;
$n_i$,$s_i$,$a_i$,$p_i$ –$\mathbf{R_i}$ matrix elements;
$ATAN2$ – arctangent function, taking into account the angle's quadrant.
All expressions in this subsection are also valid for Slave.

### 2.2 Problem statement

The inputs to the developed method are:

- invariant Master parameters of Denavit-Hartenberg $\mathbf{a}, \mathbf{d}, \boldsymbol{\alpha}$;
- distances $m$ between Master shoulder joints;
- rotation angles $\theta$ of the Master device;
- invariant Slave parameters of Denavit-Hartenberg $\mathbf{a}^{'}, \mathbf{d}^{'}, \boldsymbol{\alpha}^{'}$;
- distances $m^{'}$ between Slave shoulder joints;

The output of the method is the angles of rotation $\boldsymbol{\theta}^{'}$ of the Slave.
In this case, the following requirements for copying control should be met:

- the movements similarity of the Master and Slave;
- the coincidence of the characteristic positions of the Master and Slave;
- maximization of the Slave working space involved;
- avoid self-collision of the Slave manipulator links.

### 2.3 Proposed solution

Consider the radius vector $\mathbf{r_e}$, connecting the origin and center of the manipulator effector. $|\mathbf{r_e}|$ reaches the maximum value of $r_m$ with full straightening of the manipulator (Figure 3), zero - with the coincidence of the effector position with the origin, and intermediate in all other cases.

We introduce the value of $k_r$ – the coefficient of directness of the manipulator:

$$\mathrm{k}_r = \frac{|\mathbf{r_e}|}{r_m}.$$

To maximize the involved working area of the operating space, it is necessary for $\mathrm{k}_r$ and $\mathrm{k}_r^{'}$ reach their maximum and minimum values simultaneously. Otherwise, this means that a bent executive arm will correspond to a fully extended master manipulator and vice versa. In the first case, it will be impossible to rectify the executive manipulator. In the second case, further straightening of the master manipulator will not have any effect on the executive. To solve this problem, it is proposed to observe the following relationship in the process of copying control:

$$\mathrm{k}_r^{'} = \mathrm{k}_r.$$

Then $\left|\mathbf{r}_\mathbf{e}^{'}\right|$ can be found by the formula:

$$\left|\mathbf{r}_\mathbf{e}^{'}\right| = r_m^{'}\frac{|\mathbf{r}_\mathbf{e}|}{r_m}.$$

In addition to the module $\mathbf{r}_\mathbf{e}^{'}$ it is also necessary to know its direction in order to determine the position of the effector of the actuator arm. To describe the vector $\mathbf{r}_\mathbf{e}^{'}$ it is convenient to use a spherical coordinate system that uses its module $\left|\mathbf{r}_\mathbf{e}^{'}\right|$, azimuth angle $\alpha_a$ and zenith angle $\beta_a$ to define the end of the radius vector (Figure 4).

At first glance it may seem that the following equations can be used for copy control:

$$\mathrm{k}_r^{'} = \mathrm{k}_r,$$

$$\alpha_a^{'} = \alpha_a,$$

$$\beta_a^{'} = \beta_a,$$

however, this approach has some problems. Figure 5 shows a top view of an anthropomorphic robot. When $\mathbf{r}_\mathbf{e}^{'}$ hits in area 1, it is indeed possible to assume that $\alpha_a^{'} = \alpha_a$. The problem is connected with the connection of the "palms" of two robot manipulators in the sagittal plane. In general, the operator's shoulder width and the robot's "shoulder width" are different. If the distance between the shoulders of the robot is greater than the distance between the shoulders of the operator, then when $\alpha_a^{'} = \alpha_a$ the robot will not be able to connect the palms as an operator (Figure 5). If the distance between the shoulders of the robot is less than that between the shoulders of the operator, the robot will bring the palms together before the person connects them, which will lead to poor control.

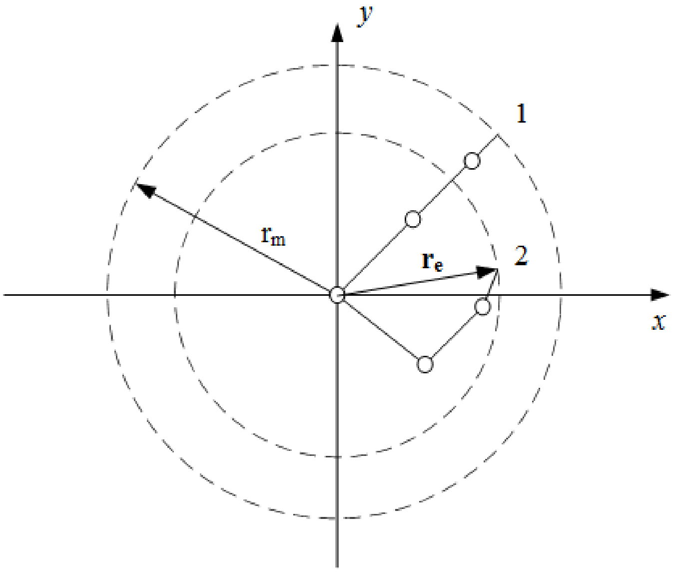


Figure 4: Radius-vector of the center of the operator's hand: 1 – the maximum extended arm of the operator; 2 – the bent arm of the operator

To solve this problem, it is proposed to use the following relationships:

$$\alpha_a^{'} = \begin{cases} \alpha_a, \text{ if } \alpha_a < \frac{\pi}{2}, \\ c_3\alpha_a^3 + c_2\alpha_a^2 + c_1\alpha_a + c_0, \text{ if } \alpha_a \geq \frac{\pi}{2}, \end{cases}$$

$$\beta_a^{'} = \beta_a,$$

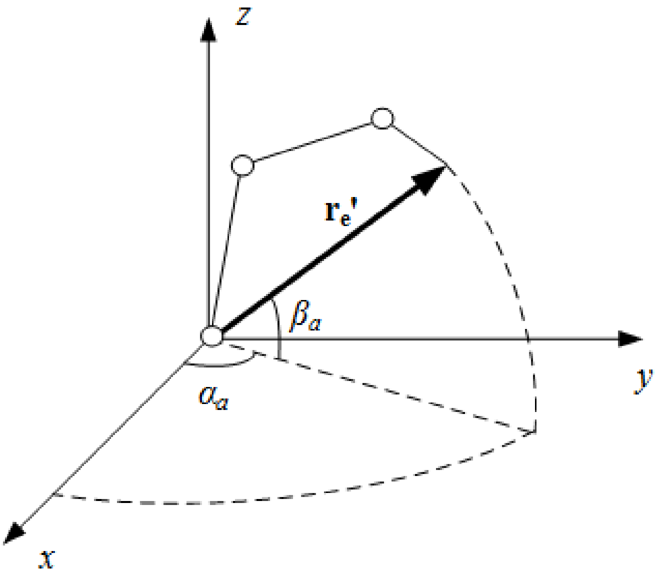


Figure 5: Radius vector of the Slave effector in a spherical coordinate system

$c_i$ – empirically determined coefficients of the polynomial based on ergonomic estimates.

The above relations completely define $\mathbf{r}_\mathbf{e}^{'}$ but do not allow to find the vector of the generalized coordinates of the executive manipulator $\theta^{'}$.

The first four degrees of mobility AM, counting from the base, are designed to move the gripper, while the other three ensure its orientation in space. The orientation of the gripper in space is determined by the triple Euler angles; we denote them as $\alpha_h,\ \beta_h, \gamma_h$. For comfortable manipulation of objects using a copy control, the following relationships must be observed:

$$\alpha_h^{'} = \alpha_h,$$

$$\beta_h^{'} = \beta_h,$$

$$\gamma_h^{'} = \gamma_h.$$

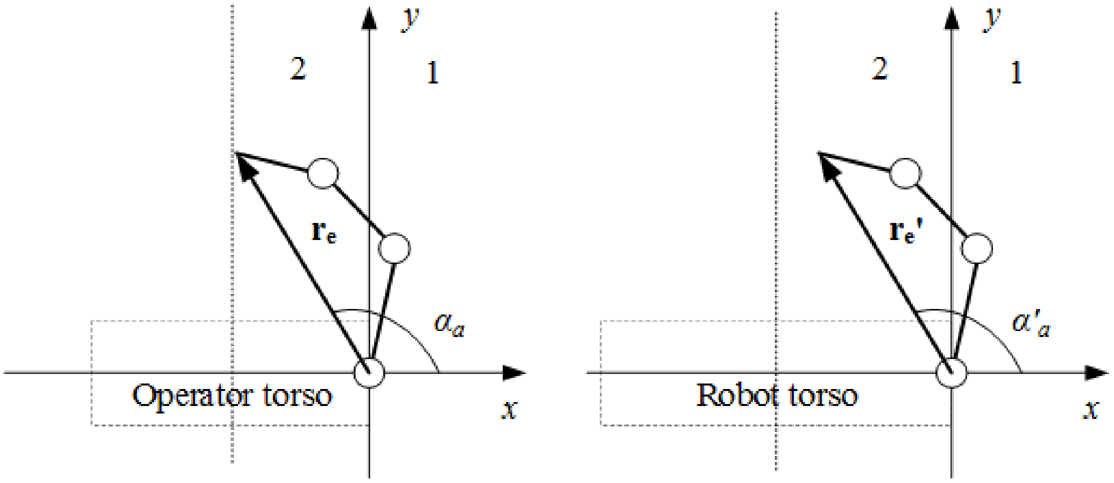


Figure 6: Radius vector of the Slave effector in a spherical coordinate system

In this case, according to the well-known position of the center of the gripper $\mathbf{r}'_\mathbf{e}$ and its orientation in space given by the three Euler angles $\alpha'_h$, $\beta'_h, \gamma'_h$, the radius vector $\mathbf{O}'_\mathbf{6}$ of the wrist joint of the executive manipulator can be found by the formula:

$$\mathbf{O}'_\mathbf{6} = \begin{bmatrix} \mathbf{R}(\alpha'_h,\ \beta'_h, \gamma'_h) & \mathbf{r}'_\mathbf{e} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} \begin{bmatrix} a_7 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

the first matrix is a homogeneous transformation matrix from the coordinate system associated with the seventh link;

the second matrix describes the coordinates of the wrist joint $\mathbf{O}'_\mathbf{6}$ in the seventh coordinate system;

$a_7$ – is the Denavit-Hartenberg parameter of the 7th link.

Due to the kinematic redundancy of the anthropomorphic manipulator (Figure 6), to solve the inverse problem of kinematics and calculate the desired vector $\boldsymbol{\theta}'$, there is not enough knowledge of the Cartesian coordinates of the $\mathbf{r}'_\mathbf{e}$ effector and the wrist joint $\mathbf{O}'_\mathbf{6}$ of the Slave. The Cartesian coordinates $\mathbf{O}'_\mathbf{3}$ of the Slave elbow joint are also required. The kinematic redundancy of AM lies in the fact that the given position $\mathbf{O}'_\mathbf{6}$ of the wrist joint corresponds in general to an infinite set of possible positions $\mathbf{O}'_\mathbf{3}$ of the elbow joint.

It is proposed to choose one of the possible positions on the basis of the Slave's human-like requirement. To enhance human-like, let the orientation in space of the plane formed by the Slave shoulder joint $\mathbf{O}'_\mathbf{1}$, the ulnar joint $\mathbf{O}'_\mathbf{3}$ and the wrist joint $\mathbf{O}'_\mathbf{6}$ coincides with the similar plane formed by the joints of the Master As a coordinate system associated with this plane, a coordinate system associated with the elbow joint can be taken. In this case, we can assume that:

$$\mathbf{R}'_\mathbf{3}{=}\mathbf{R_3}.$$

The position of the elbow joint $\mathbf{O}'_\mathbf{3}$ can be found from the matrix equation:

$$\mathbf{O}'_\mathbf{1} = \begin{bmatrix} \mathbf{R_3} & \mathbf{O}'_\mathbf{3} \\ \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} -a_3 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$\begin{bmatrix} \mathbf{R_3} & \mathbf{O}'_\mathbf{3} \\ \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} & \mathbf{1} \end{bmatrix} = \mathbf{O}'_\mathbf{1} \begin{bmatrix} -a_3 \\ 0 \\ 0 \\ 1 \end{bmatrix}^{\mathbf{-1}}.$$

Based on the expressions obtained, the rotation angles of the Slave can be calculated using the method given in [**Pet2018**].

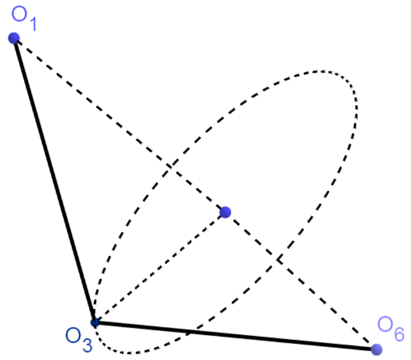
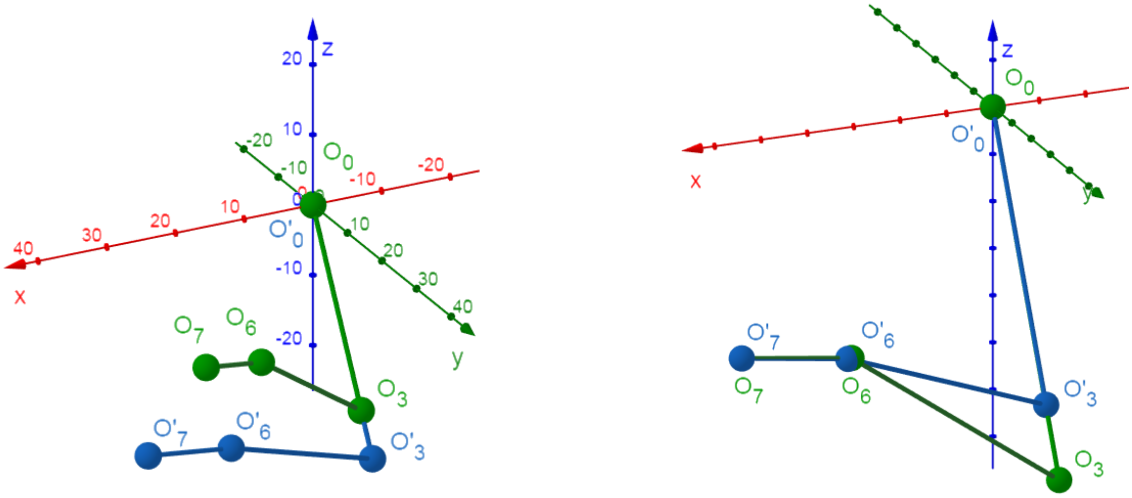


Figure 7: Radius vector of the Slave effector in a spherical coordinate system

## 3 Results

To test the effectiveness of the proposed method, a numerical simulation of the process of copying control and mapping the Master configuration space to the Slave configuration space was performed. An example of the correspondence between the position of the Master and the Slave is shown in Figure 7. Figure 7a shows the position of the Master and the Slave in a single coordinate system, and in Figure 7b in two coordinate systems scaled for clarity and superimposed on each other. As can be seen from the simulation results, when applying the proposed method, there is a clear similarity between the position of the Master and the Slave.



a) b)

Figure 8: Numerical simulation results: a – same scale; b – relative scale

## 4 Discussion

The proposed method of mapping the configuration space of the Master configuration space to the Slave configuration space allows copy control to be performed even with significant differences in the lengths of the links of the Master and the Slave. This feature allows to effectively use promising exoskeleton Master devices. The proposed method is based on the requirements of safety and human-like movements, maximizing the use of the AM workspace. It is planned to introduce the proposed method into existing copy management systems.

## 5 Acnowlegments

The research was carried out within the framework of the implementation of a research project on the development of a software and hardware control system based on solving the inverse problem of dynamics and kinematics within the framework of FCNIR 2014-2020 (unique identifier RFMEFI57517X0166) with the financial support of the Ministry of Science and Higher Education of the Russian Federation.