Stability Analysis of a Model with General Retrials and Constant Retrial Rate*

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Abstract. The paper deals with a single-server model with constant retrial rate. If an arrival meets the server busy, it joins the infinite-capacity orbit and then tries to occupy the server again after generally distributed time interval. Unlike the classical retrial policy, the intensity of orbit customers does not depend on its number. To derive the stability condition for such a model, we construct less complicated for analysis dominating system and illustrate its stability condition, basing on Markov chain approach. Then, relying on heuristic method and using some results from renewal theory, we make a basic assumption about stability condition of the model under consideration. Simulation results for Pareto and Weibull distribution of retrial time confirm, that assumed stability condition gives a good approximation of theoretical stability region.

Keywords: Retrial System · Constant Retrial Rate · Stability Analysis · General Retrials · Heuristic Approach · Renewal Theory.

1 Introduction

We study a single-server retrial system with Poisson input. If an arrival finds the server busy, he joins a virtual orbit and after a *retrial time* attempts to enter server again. We consider a constant retrial rate policy, in which the total rate of secondary (retrial) attempts does not depend on the orbit size. Unlike the most of existing works in which exponential retrials are studied, we consider a general distribution of the retrial times. The main purpose of the research is to find the stability conditions of the system under consideration.

For the retrial systems with *classic retrial discipline* (when the retrial rate increases with the orbit size) the tight sufficient stability condition has been established for the New-Better-Than-Used (NBU) retrials in [1], and this condition

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indeed coincides with the stability criterion of the corresponding buffered classic system.

The known sufficient stability conditions for the constant rate retrial systems are definitely superficial, and there is a gap between them and necessary stability conditions even for the exponential retrials (in this regard see stability analysis of a multiclass retrial system in [2]). To find more tight stability condition in the system with non-exponential retrials, we construct a dominating retrial system which is more easy to be analyzed. This domination property is intuitive and we do not provide the proof in this work. Thus our analysis in this point is heuristic. Our main idea is to approach the remaining retrial times by the stationary overshoot in the renewal process generated by the sequence of the independent identically distributed (iid) retrial times. This assumption seems plausible since an arrival instant of a Poisson customer can considered as a random instant in a retrial interval. Indeed, as our simulation shows, this assumption is confirmed for a few specific non-exponential distributions of the retrial times. This analysis allows to suggest more tight stability condition of the retrial systems and by this to extend the stability region of these systems.

To motivate our research, we outline the applicability of the retrial systems. A constant retrial rate system has been introduced in [3] to describe the behavior of telephone exchange centers. Later, the multi-server extension of such a model was analysed in [4]. Constant retrial rate systems are successfully used in simulation of multi-access protocols, in particular, see applications to TCP in [5], to CSMA/CD (Carrier Sense Multiple Access with Collision Detection) protocol in [6] and to ALOHA-type multiple access systems in [7], etc.

The most of stability results are obtained for the system with exponential retrial times. For instance, a multiserver bufferless system $M/M/c/0$ was analyzed by matrix analytic method in [8, 9]. Stability of a retrial system $GI/M/1$ with a renewal input has been investigated in [10], where the generating function method s applied. Stability conditions for a general $GI/G/c/K$ system with exponential retrials have been obtained by the regeneration method in [11]. Just a few papers consider retrial systems with non-exponential retrials. In particular, the papers [12, 13] deal with *PH- retrial times* and provide some approximations and simulation results. Finally, the detailed overviews of the retrial systems can be found in the well-known monographs [14], [15] and in the paper [16].

The paper is organized as follows. Section 2 contains the description of the system. In Section 3, an auxiliary dominating system with the known stability condition is constructed. In Section 4, based on ideas from renewal theory, we make and advocate an assumption which allows to formulate a tighter stability condition of the system under consideration. Section 5 contains simulation results for the systems with *Pareto and Weibull* retrial times.

2 Description of the Model

We consider a single-server bufferless retrial system with *constant retrial rate* denoted by Σ, which is fed by *Poisson* input of (primary) customers with arrival instants $\{t_n,\, n \geq 1\}$ and rate λ. Thus exponential interarrival times $\tau_n = t_{n+1} - t_n$ are iid. (Here and in what follows we omit serial index to denote generic element of an iid sequence.) The service times $\{S_n,\, n \geq 1\}$ are assumed to be iid as well with a general distribution F_S. Denote the traffic intensity $\rho = \lambda \mathsf{E} S$. If a primary customer meets the server busy, he joins the infinite-capacity virtual *orbit* and then attempts to occupy the server after a random time ξ with a general dsitribution F_ξ. Denote by $N(t)$ the number of customers in orbit (secondary customers) at instant t. (All continuous-time processes are assumed to be defined at instant t^-.) By definition, the retrial rate does not depend on the orbit size $N(t)$.

This system is *regenerative* and it is important to emphasize that the number of customers in the system and in particular, the orbit-size process $\{N(t)\}$ are both regenerative processes, and regenerations occur when a new arrival sees an empty system. We call the process $\{N(t)\}$ and the system *positive recurrent* if the mean regeneration period is finite. Positive recurrence means that the system possesses *stationary regime* [17].

3 A Dominating System

First we present some known stability results for particular systems with constant retrial rate. In the paper [11] the following stability condition of a single-server system with exponential retrials is obtained:

$$\lambda \rho < \mu_0 (1 - \rho), \tag{1}$$

where $\mu_0 = 1/\mathsf{E}\xi$ is the retrial rate. Because ρ is the stationary busy probability of the server (again see [11]), then condition (1) has an evident probabilistic interpretation: input rate $\lambda\rho$ *to the orbit* must be less that the successful rate *from the orbit*. We note that a similar interpretation can be applied to the stability condition of the retrial system with renewal input and exponential retrials considered in [10]. Assume first that the retrial time ξ is NBU, that is for arbitrary $x,\, y \geq 0$

$$\mathsf{P}(\xi > x + y | \xi > y) \leq \mathsf{P}(\xi > x).$$

Note that in this case the *tail of the retrial time* is *stochastically* less or equal than ξ. A sufficient stability condition of a model with NBU retrials and classical retrials, has been obtained in the recent work [1]. Denote by $\{D_k,\, k \geq 1\}$ the departure instants of the customers from the system, and define $N_k = N(D_k^+)$, the orbit size just after the k-th departure, $k \geq 1$. An important observation is that interval $[D_k,\, D_{k+1})$, $k \geq 1$ contains two phases: an *idle period* I_k which appears after each departure, and the actual service time S_{k+1}. (By assumption, we assign service times in the order the customers *enter server*.) Now we construct a new single-server retrial system $\hat{\Sigma}$ with constant retrial rate and the same input and service times as in the system Σ, in which the retrial times after departures are constructed as follows. If the orbit is not empty after a departure, then the residual retrial time is replaced by an independent variable distributed as generic retrial time ξ in the original system Σ. Note that if the orbit is idle after a departure, then the idle period distributed as interarrival time τ. Also we note that the retrial times in $\hat{\Sigma}$ between (unsuccessful) attempts which see server busy are distributed as generic variable ξ. (Indeed we could ignore such attempts because they do not affect the state of the system.) We denote the departure instants in the system $\hat{\Sigma}$ by $\{\hat{D}_k\}$. (In fact the system $\hat{\Sigma}$ is a particular case of a system considered in [18].) It follows from construction that the following recursions hold:

$$D_{k+1} =_{st} D_k + \min\big(\xi(D_k),\, \tau\big) + S_{k+1},$$
$$\hat{D}_{k+1} =_{st} \hat{D}_k + \min\big(\xi,\, \tau\big) + S_{k+1},\ k \geq 1, \tag{2}$$

where $\xi(D_k)$ denotes the remaining retrial time at instant D_k in the system Σ. By the NBU property $\xi \geq_{st} \xi(D_k)$, and then recursion (2) supports our
Assumption 1: the system Σ is *less loaded* than the system $\hat{\Sigma}$.
(The exact proof of this assumption could be based on the coupling arguments developed in the paper [2].) Thus the stability of $\hat{\Sigma}$ yields the stability of the original system Σ.

Now we consider the stability condition of the system $\hat{\Sigma}$. Denote by $\{\hat{N}_k,\, k \geq 1\}$ the orbit size in the system $\hat{\Sigma}$ just after departure instant $\hat{D}_k$. It is easy to see, that the sequence $\{\hat{N}_k\}$ (unlike the sequence $\{N_k\}$) defines an irreducible, aperiodic, time-homogeneous Markov chain with the state space $\{0,\, 1,\, 2, \dots\}$. Now we find the *ergodicity* condition of this Markov chain, which is equivalent to the positive recurrence (stability) of the system $\hat{\Sigma}$.

First assume that, at some instant $\hat{D}_k$, the orbit is empty, that is $\hat{N}_k = 0$. Then the server can be captured by a primary customer only, and the following idle period is distributed as exponential variable τ. When server becomes busy, arrival input (with rate λ) joins the orbit. Thus, in the interval $[\hat{D}_k,\, \hat{D}_{k+1})$, the sequence $\{\hat{N}_k\}$ transits from state 0 to a state i with probability (w. p.)

$$p_i = \int_0^\infty e^{-\lambda t}\frac{(\lambda t)^i}{i!}dF_S(t),\ \ i \geq 0.$$

If $\hat{N}_k > 0$, then the the two following alternative events may happen: i) the primary customer occupies the server with w.p. $\mathsf{P}(\tau < \xi)$, or ii) a retrial attempt is successful w. p. $\mathsf{P}(\tau \geq \xi)$. (In the original system this happens w. p. $\mathsf{P}(\tau < \xi(D_k))$ and $\mathsf{P}(\tau \geq \xi(D_k))$, respectively, which are not analytically available unless ξ is exponential.) It then follows that $\hat{N}_{k+1} = \hat{N}_k + i$ w.p.

$$\mathsf{P}(\tau < \xi;\ i \text{ arrivals during } S_{k+1}) = p_i \int_0^\infty (1 - e^{-\lambda x})dF_\xi(x)$$

and $\hat{N}_{k+1} = \hat{N}_k + i - 1$ w.p.

$$\mathsf{P}(\tau \geq \xi;\, i\,\text{arrivals during } S_{k+1}) = p_i \int_0^\infty e^{-\lambda x}dF_\xi(x).$$

Now we apply the well-known sufficient (negative drift) condition for the Markov chain to be ergodic [17,19] which in our case can be formulated as $\sup_k \mathsf{E}|\hat{N}_{k+1} - \hat{N}_k| < \infty$ and

$$\mathsf{E}[\hat{N}_{k+1} - \hat{N}_k | \hat{N}_k = j] < 0, \;\; j > 0. \tag{3}$$

A simple algebra gives, regardless of k,

$$-1 \le \mathsf{E}[\hat{N}_{k+1} - \hat{N}_k] \le \sum_i i p_i = \rho < \infty,$$

and to check (3), we have

$$\begin{aligned}
\mathsf{E}[\hat{N}_{k+1} - \hat{N}_k | \hat{N}_k = j] &= \mathsf{P}(\tau < \xi) \sum_{i\ge 1} (i + j) p_i \\
&+ \mathsf{P}(\tau \ge \xi) \sum_{i \ge 1} (i + j - 1) p_i - j \\
&= \sum_{i \ge 1} i p_i - \mathsf{P}(\tau \ge \xi) = \lambda \mathsf{E} S - \mathsf{P}(\tau \ge \xi).
\end{aligned}$$

Thus the system $\hat{\Sigma}$ is stable if the following condition holds:

$$\rho < \mathsf{P}(\tau \ge \xi), \tag{4}$$

which has a clear probabilistic interpretation. Namely, the probability ρ that an arriving customer joins the orbit must be less than the probability $\mathsf{P}(\tau \ge \xi)$ that an attempt happens earlier than the next primary customer arrives. Note that

$$\mathsf{P}(\tau \ge \xi) = \int_0^\infty e^{-\lambda x} dF_\xi(x) = \mathcal{L}_\xi(\lambda),$$

is Laplace-Stieltjes transform of the retrial time ξ. Remark that condition $\rho < \mathcal{L}_\xi(\lambda)$ coincides with the stability condition, obtained in [18] for a general case of retrial system with balking and repairable server, where retrial time begins only when the server is idle.

4 Stability Analysis

In previous section we have found a sufficient stability condition for a dominating system $\hat{\Sigma}$. One may expect that this condition is not tight enough for the original system, and in this section we use an idea from renewal theory to formulate more tight stability condition for the system Σ. First, we construct a renewal process generated by the iid copies $\{\xi_i,\, i \ge 1\}$ of the generic retrial time ξ. Denote the renewal process

$$Z_n = \sum_{i=0}^n \xi_i, \;\; n \ge 0, \;\; Z_0 := 0,$$

and define the residual renewal time at instant t as

$$\xi(t) = \min_n \{Z_n - t : Z_n - t \geq 0\}, \;\; t \geq 0. \tag{5}$$

It is well-known that if $\mathsf{E}\xi < \infty$ and ξ is *non-lattice* (it is assumed in what follows) then the convergence in distribution holds [17]:

$$\xi(t) \Rightarrow \xi_s \quad t \to \infty,$$

where ξ_s denotes the *stationary excess* of the renewal process $\{Z_n\}$. Denoting the tail distributions

$$\overline{F}_{\xi_s}(x) = \mathsf{P}(\xi_s > x), \quad \overline{F}_{\xi}(x) = \mathsf{P}(\xi > x),$$

we have (see, for instance, [17])

$$\overline{F}_{\xi_s}(x) = \frac{1}{\mathsf{E}\xi} \int_x^\infty \overline{F}_\xi(y)dy = \frac{1}{\mathsf{E}\xi} \int_0^\infty \mathsf{P}(\xi > x + y)dy.$$

If retrial time ξ is NBU, that is $\mathsf{P}(\xi > x + y) \leq \mathsf{P}(\xi > x)\mathsf{P}(\xi > y)$, then

$$\overline{F}_{\xi_s}(x) \leq \frac{\mathsf{P}(\xi > x)}{\mathsf{E}\xi} \int_0^\infty \mathsf{P}(\xi > y)dy = \overline{F}_\xi(x),$$

that is $\mathsf{P}(\xi \leq x) \leq \mathsf{P}(\xi_s \leq x)$. Because τ and ξ are independent, then

$$\mathsf{P}(\tau \geq \xi) = \int_0^\infty \lambda e^{-\lambda x} \mathsf{P}(\xi \leq x)dx \leq \int_0^\infty \lambda e^{-\lambda x} \mathsf{P}(\xi_s \leq x)dx = \mathsf{P}(\xi_s \leq \tau),$$

where, to obtain the independence between τ and ξ_s, we use the resampling of the interarrival time τ at the corresponding departure instant. Thus we have the inequality

$$\mathsf{P}(\tau \geq \xi) \leq \mathsf{P}(\tau \geq \xi_s),$$

which becomes equality for ξ being exponential. If ξ is *not exponential* then in general

$$\mathsf{P}(\tau \geq \xi) < \mathsf{P}(\tau \geq \xi_s),$$

One can expect that the actual stationary remaining retrial time at the departure instants is close to the stationary residual renewal time (5) in the renewal process $\{Z_n\}$, and it implies the following
Assumption 2: for a generally distributed (non-exponential) ξ condition

$$\rho < \mathsf{P}(\tau \geq \xi_s) \tag{6}$$

allows to delimit more exact stability region of the original system Σ than the (excessive) condition (4) obtained for the dominating system under NBU retrials. (For exponential ξ, conditions (4) and (6) coincide with stability condition (1), obtained in [11].)

In the next Section we demonstrate, for Weibull and Pareto retrials times, that condition (6) indeed provides a good approximation of stability region for the corresponding system.

5 Simulation

To verify the tightness of the new (heuristic) stability condition (6) for a non-exponential retrial time ξ, we study the dynamics of the orbit provided the traffic intensity ρ is selected to satisfy the following inequalities:

$$\mathsf{P}(\tau \geq \xi) \leq \rho < \mathsf{P}(\tau \geq \xi_s), \tag{7}$$

that is, when sufficient stability condition (4) is violated. Define the *unknown* real stability region border $\mathcal{S}$, that is condition

$$\rho < \mathcal{S}$$

is the stability (*positive recurrence*) criterion of the system. (Conversely, condition $\rho \geq \mathcal{S}$ implies *instability* of the system.) In sections 5.1 – 5.2 we obtain an approximate value of $\mathcal{S}$ by analysing the orbit behavior in simulated system as follows. First, we vary the value of load coefficient ρ and illustrate the dynamics of mean orbit size. When for some value ρ^* we obtain stable orbit if $\rho < \rho^*$, while for $\rho \geq \rho^*$ the orbit goes to infinity, we define the approximate value of $\mathcal{S}$ by ρ^*.

Below we show by simulation that the value $\mathcal{S}$ is indeed close to the probability $\mathsf{P}(\tau \geq \xi_s)$ for Weibull and Pareto retrial times.

5.1 Weibull Retrials

We consider Weibull retrial time ξ with the scale parameter 1 and the shape parameter w, that is,

$$F_\xi(x) = 1 - e^{-x^w}.$$

It is straightforward to show that

$$\mathsf{P}(\tau \geq \xi_s) = \int_0^\infty e^{-\lambda x} dF_{\xi_s}(x) = \frac{1}{\Gamma(1+\frac{1}{w})} \int_0^\infty e^{-(\lambda x + x^w)} dx,$$

where Γ is Gamma function. If $w = 2$, then ξ has NBU property and

$$\mathsf{P}(\tau \geq \xi_s) = \frac{1}{\Gamma(1.5)} \int_0^\infty e^{-\lambda x} e^{-x^2} dx = \frac{\sqrt{\pi}}{2\Gamma(1.5)} e^{\lambda^2/4}\, \mathsf{erfc}\Big(\frac{\lambda}{2}\Big),$$

where

$$\mathsf{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is complementary error function [20]. Next, we fix $\lambda = 1$ in which case

$$\mathsf{P}(\tau \geq \xi) = 0.454, \qquad \mathsf{P}(\tau \geq \xi_s) = 0.616.$$

Figure 1 presents the orbit dynamics for a few values of $\mathsf{E}S = \rho$ for exponential service time. As Figure 1 shows, when ρ satisfies inequalities (7) (solid lines) then orbit demonstrates stability behaviour, while when ρ violates (7) (dash line) the orbit becomes unstable. Note that in these experiments

$$\mathcal{S} \approx 0.593 < \mathsf{P}(\tau \geq \xi_s) = 0.616,$$

and we conclude, at least for the studied NBU Weibull retrials, that the bound $\mathsf{P}(\tau \geq \xi_s)$ indeed slightly violates the border of stability region. However the proximity between $\mathsf{P}(\tau \geq \xi_s)$ and (in general unknown) actual border $\mathcal{S}$ can be used to approach $\mathcal{S}$ effectively in simulation.

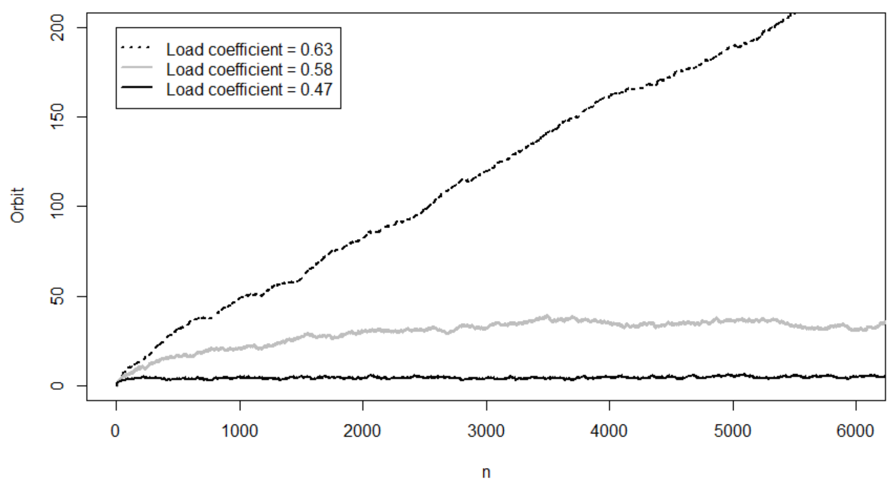

Fig. 1. Orbit dynamics for NBU Weibull retrials, $w = 2$, $\lambda = 1$.

Now we consider Weibull retrials with $w < 1$, in which case ξ has the so-called *New-Worse -Than-Used* (NWU) property meaning that

$$\mathsf{P}(\xi > x + y | \xi > y) \geq \mathsf{P}(\xi > x), \; x \geq 0, \, y \geq 0.$$

For arbitrary $\lambda > 0$ and $w = 1/2$ we have

$$\begin{aligned} \mathsf{P}(\tau \geq \xi_s) &= \frac{1}{\mathsf{E}\xi} \int_0^\infty e^{-(\lambda x + \sqrt{x})} dx \\ &= \frac{1}{2\sqrt{\lambda^3}\Gamma(3)} \Big[\sqrt{\pi} e^{\frac{1}{4\lambda}} \Big(\mathsf{erf}(0.5/\sqrt{\lambda}) - 1 \Big) + 2\sqrt{\lambda} \Big], \end{aligned}$$

where

$$\mathsf{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is Gauss error function [20]. (Note, that in the NWU case, we can not claim that the corresponding system $\hat{\Sigma}$ dominates the system Σ.) Selecting $w = 1/2$ and $\lambda = 1$, we have

$$\begin{aligned} \mathsf{P}(\tau \geq \xi_s) &= \frac{1}{\Gamma(3)} \int_0^\infty e^{-(x+\sqrt{x})} dx \\ &= \frac{1}{\Gamma(3)} \Big[\frac{\sqrt{\pi} e^{\frac{1}{4}}}{2} \Big(\mathsf{erf}\big(1/2\big) - 1 \Big) + 1 \Big] \\ &= \frac{0.454}{2} = 0.227. \end{aligned}$$

The orbit dynamics for $w = 1/2$ is presented on Figure 2, where the solid grey

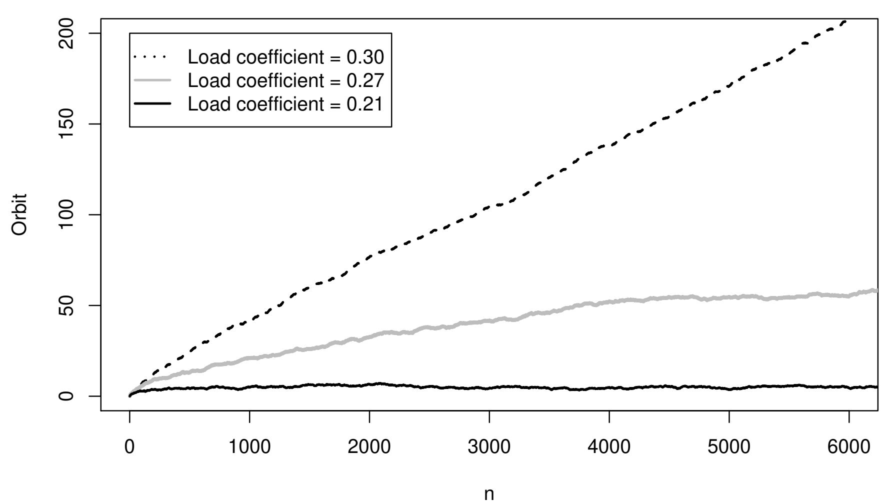

Fig. 2. Orbit dynamics for NWU Weibull retrials, $w = 0.5$, $\lambda = 1$.

line corresponds to $\rho = 0.27$. In this case condition (6) is violated and orbit demonstrates "less" instability (in comparison with the case $\rho = 0.30$, dashed line). Numerical results also show that in this case

$$\mathcal{S} \approx 0.265 > \mathsf{P}(\tau \geq \xi_s) = 0.227,$$

and thus condition $\mathsf{P}(\tau \geq \xi_s)$ narrows the actual stability region.

5.2 Pareto Retrials

Now we consider retrial time ξ with Pareto distribution

$$F_\xi(x) = 1 - x^{-\alpha}, \;\; \alpha > 1, \qquad x \geq 1.$$

In this case

$$\begin{aligned}\mathsf{P}(\tau \geq \xi_s) &= \frac{1}{\mathsf{E}\xi}\Big(\int_0^1 e^{-\lambda x} dx + \int_1^\infty e^{-\lambda x} x^{-\alpha} dx\Big) \\ &= \frac{\alpha-1}{\alpha}\Big(\frac{1-e^{-\lambda}}{\lambda} + \int_1^\infty e^{-\lambda x} x^{-\alpha} dx\Big).\end{aligned}$$

For instance, for $\lambda = 1$ and $\alpha = 2$,

$$\mathsf{P}(\tau \geq \xi_s) = (0.632 + 0.148)/2 = 0.390.$$

Simulation shows that in this experiment,

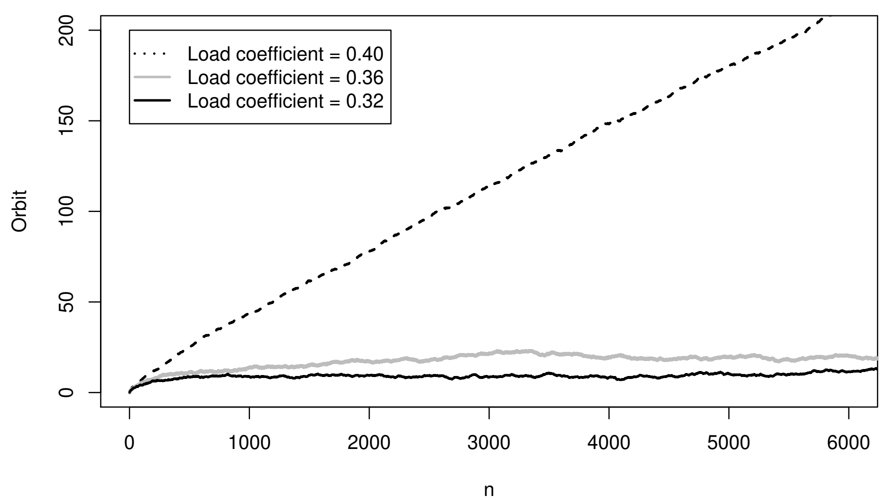

Fig. 3. The orbit dynamics for Pareto retrials, $\alpha = 2$, $\lambda = 1$.

$$\mathcal{S} \approx 0.385 < \mathsf{P}(\tau \geq \xi_s) = 0.390,$$

and thus the border $\mathsf{P}(\tau \geq \xi_s)$ “extend” actual stability region a little. The results of simulation are illustrated by Figure 3.

6 Conclusion

In this paper we develop a heuristic approach, using an idea from renewal theory, to construct more tight stability condition for a single-server retrial system with Poisson input, general iid service times and general (iid) retrial times. It is assumed that the system follows a constant retrial rate policy. We construct a dominating retrial system and study an embedded Markov chain at the departure instants to find sufficient stability condition for the NBU retrials. Then we formulate a more tight stability condition and verify its accuracy by simulation in the systems with Weibull (both NBU and NWU retrials) and Pareto retrials. As experiments show, this new condition allows to approach the actual border of the stability region but it must be used carefully.

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