On Stochastic and Failure Rate Orderings in Systems with Two-Component Service Time Mixture *

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Abstract. In this paper, we discuss stochastic and failure rate comparisons of two-component mixture distributions and the properties of *conditional excess distribution* of two-component mixture. We consider the uniform distance between conditional excess mixture distribution and it's parent distribution. Then we apply the *failure rate comparison* and *stochastic ordering* techniques to construct the upper and lower bounds for the steady-state performance indexes of a multiserver model. This theoretical analysis is further illustrated by the comparison of conditional excesses of service times, the waiting times and queue sizes in the queueing systems with mixed service time distribution.

Keywords: Conditional Excess Distribution, Failure Rate Comparison, Multiserver System, Performance Analysis, Finite Mixture Distribution

1 Introduction

The analysis of the behaviour of mixtures of random variables has a long history, see for instance, [4,6]. The mixtures arise in many applications, for example in biology, when population consists of several subpopulations referred to a different components of mixture. In the communication networks they can be used to model queueing systems with several classes of customers.

This paper is dedicated to the properties of conditional excess distribution of two-component mixtures. The excesses over given and increasing thresholds play a fundamental role in many applications when we study the asymptotic behaviour of the performance indexes describing queueing systems. For instance, the conditional distribution F_u (defined by (5) is known as the *excess–life or residual lifetime* distribution function in reliability theory and also in medical statistics [7]. In the insurance context, F_u is usually referred to as the *excess–of–loss* [7]. In the analysis of communications systems the conditional excess

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distribution is often used to estimate the probability that a performance measure (for instance, the waiting time) exceeds a high threshold [12].

In previous research, we developed the comparison of the performance measures (rather than excess) in the systems with different service time distribution and with (two-component) mixture of given distributions. In particular, we have considered the following two-component distributions: *Hyperexponential* distribution, *Pareto* distribution, Exponential-Pareto mixture distribution. The corresponding stationary measures in such systems have been compared with the corresponding measures in the systems with (one-component) Exponential distribution and Pareto distribution. As a result, some useful bounds for the performance measures have been obtained, see [14,13].

The main new contribution of this research is as follows. Using the *stochastic and failure rate ordering*, monotonicity properties (with respect to the interarrival time and service times [1,17]) we can compare the *conditional excesses of service times* and performance indexes in queueing system with the *two-component mixture* service time and the corresponding measures in the system in which service time distribution coincides with the distribution of a mixture component.

The structure of the paper is as follows. In Section 2, we define finite two-component mixture distributions and consider properties of stochastic and failure rate comparison of mixture components. In Section 3, we introduce the conditional excess distribution F_u over the threshold u and discuss some its properties. In Section 4, we consider the uniform distance between the conditional excess mixture distribution and it's parent distribution which is illustrated by two examples: for the Hyperexponential distribution and two-component Pareto distribution. These results are further applied in Section 5 to the conditional excess distributions of service times and distributions of queue size and waiting time in the multiserver systems.

2 Stochastic and Failure Rate Ordering

In this section we give some basic definitions which are used below. Let X be a non-negative random variable with distribution function F and density f. For each x such that the tail distribution $\overline{F}(x) = \mathsf{P}(X > x) > 0$, we define the *failure rate* function as

$$r_F(x) = \frac{f(x)}{\overline{F}(x)}, \quad x \geq 0.$$

An absolutely continuous distribution F with density f is said to have an *increasing failure rate (IFR)* if $r_f(x)$ is an increasing function. Analogously, the distribution F (with density f) has *decreasing failure rate* (DFR) if the function $r_F(x)$ is decreasing.

The distribution function F is said to be *new better than used* (NBU) if for $x,\ u \geq 0$

$$\overline{F}(x+u) \leq \overline{F}(x)\overline{F}(u).$$

We say that df F is *new worse than used* (NWU) if for $x,\ u \geq 0$

$$\overline{F}(x+u) \geq \overline{F}(x)\overline{F}(u).$$

One can show that the IFR (DFR) property of a distribution function implies the NBU (NWU) property of the corresponding failure rate function, see [4].

Consider two non-negative random variables X and Y with distribution functions F and G, respectively. We say that X *is less than* Y *stochastically*, and denote it as $X \underset{st}{\leq} Y$, if

$$\overline{F}(x) \leq \overline{G}(x),\ x \geq 0.$$

We say that X *is less than* Y in *failure rate*, and denote it $X \underset{r}{\leq} Y$, if

$$r_F(x) \geq r_G(x), \quad \text{for all } x \geq 0.$$

It is well-known [15] that the failure rate ordering implies stochastic ordering, that is,

$$X \underset{r}{\leq} Y \ \Rightarrow \ X \underset{st}{\leq} Y\ .$$

Distribution function

$$H(x) = pF(x) + (1-p)G(x), \ \ x \geq 0, \tag{1}$$

where a constant $p \in (0,\, 1)$, is said to be a *two-component mixture* of distributions F and G. The constant p is called mixture parameter. Suppose that the random variables X, Y with distribution functions F, G, respectively, are independent, and let I be indicator function independent of $X,\ Y$, taking value 1 with probability p (value 0 with probability $1-p$). Then it is said that the variable

$$Z = I\,X + (1-I)\,Y \tag{2}$$

has *two-component mixture distribution* (1).

It is proved in [13] that if the components X, Y are ordered stochastically (in failure rate) then the mixture Z has the following natural stochastic (failure rate) bounds:

$$X \underset{st}{\leq} Y \ \Rightarrow \ X \underset{st}{\leq} Z \underset{st}{\leq} Y; \tag{3}$$

$$X \underset{r}{\leq} Y \ \Rightarrow \ X \underset{r}{\leq} Z \underset{r}{\leq} Y. \tag{4}$$

3 Conditional Excess Distribution of Two-Component Mixture

Let X be a *non-negative* random variable with distribution function F and *right endpoint* x_r, defined as

$$x_r = \sup\{x \geq 0 : F(x) < 1\} \leq \infty.$$

For a fixed $u < x_r$ denote, conditionally on the event $\{X > u\}$, the excess $X_u := X - u$. Then conditional distribution

$$F_u(x) = \mathsf{P}(X_u \le x) = \mathsf{P}(X - u \le x | X > u),\ u < x_r,\ x \ge 0, \tag{5}$$

is called the *conditional excess distribution of X over the threshold u* [3]. The tail of conditional excess distribution, defined as

$$\overline{F}_u(x) = \mathsf{P}(X - u \ge x | X > u) = \frac{\overline{F}(x+u)}{\overline{F}(u)}, \quad u < x_r,\ x \ge 0,$$

plays an important role in the *reliability theory* and called the *residual lifetime*. It represents the survival function of a unit of age u, i.e., the conditional probability that a unit of age u will survive for an additional x units of time [2]. The failure rate of F_u given by

$$r_{F_u}(x) = \frac{f(x+u)}{\overline{F}(x+u)} = r_F(u+x).$$

For a given two-component mixture distribution

$$H(x) = pF(x) + (1-p)G(x),$$

we define the tail of conditional excess distribution over the threshold u:

$$\overline{H}_u(x) = \frac{\overline{H}(x+u)}{\overline{H}(u)} = \frac{p\overline{F}(x+u) + (1-p)\overline{G}(x+u)}{p\overline{F}(u) + (1-p)\overline{G}(u)},\ u < x_r,\ \ x \ge 0. \tag{6}$$

Theorem 1. *If the components of* (2) *are ordered in failure rate, $X \underset{r}{\le} Y$, then, for each $u \ge 0$,*

$$X_u \underset{r}{\le} IX_u + (1-I)Y_u \underset{r}{\le} Y_u. \tag{7}$$

Proof. The proof of theorem follows from the preservation property of the failure rates order for conditional excess distribution, that is $X \underset{r}{\le} Y$ implies $X_u \underset{r}{\le} Y_u$ [7]. Then ordering (4) implies (7). □

Theorem 2. *If the components of* (2) *are stochastically ordered, $X \underset{st}{\le} Y$, and X is NBU, Y is NWU, then*

$$X_u \underset{st}{\le} IX_u + (1-I)Y_u \underset{st}{\le} Y_u. \tag{8}$$

Proof. It is enough to prove that $X_u \underset{st}{\le} Y_u$, that is follows from NBU and NWU properties of X and Y, respectively:

$$\frac{\overline{F}(x+u)}{\overline{G}(x+u)} \le \frac{\overline{F}(x)\overline{F}(u)}{\overline{G}(x)\overline{G}(u)} \le \frac{\overline{F}(u)}{\overline{G}(u)},$$

and then

$$\overline{F}_u(x) = \frac{\overline{F}(x+u)}{\overline{F}(u)} \le \frac{\overline{G}(x+u)}{\overline{G}(u)} = \overline{G}_u(x).$$

Now the statement of the theorem follows from (3). □

We note that if X or Y or both have an exponential distribution and $X \underset{st}{\leq} Y$, then relations (8) hold. As an example we consider the Exponential-Pareto mixture distribution with tail distribution function

$$\overline{H}(x) = pe^{-\lambda x} + (1-p)\left(\frac{x_0}{x_0+x}\right)^{\alpha}, \ \alpha, \lambda, x_0 > 0, \ x \geq 0$$

and failure rate

$$r_H(x) = \frac{p\,\lambda b(x) + (1-p)\,\alpha/(x_0+x)}{p\,b(x) + (1-p)},$$

where

$$b(x) = e^{-\lambda x}\left(1 + \frac{x}{x_0}\right)^{\alpha}, \quad x \geq 0.$$

It can be verified that, under condition

$$\lambda \geq \frac{\alpha}{x_0},$$

the relations (8) hold for this distribution.

It is known that the mixture of two DFR distributions is DFR and F is DFR if and only if $F_u(x)$ is increasing in u for all $x \geq 0$ [7]. Then we immediately obtain the following statement.

Theorem 3. *Let F and G be DFR distributions. Then for all u such that*

$$\overline{H}(u) = p\overline{F}(x) + (1-p)\overline{G}(x) > 0,$$

the tail

$$\overline{H}_u(x) = \frac{\overline{H}(x+u)}{\overline{H}(u)} \quad \text{is increasing in } u \text{ for all } x \geq 0.$$

We note that mixtures of IFR distributions need not be IFR and can even be DFR [7]. An important source of DFR mixtures is the mixture of exponential distributions, which arises in the real applications. For instance, consider the Hyoerexponential distribution with parameters $\lambda_1, \ \lambda_2, \ \lambda_1 \neq \lambda_2$ and tail

$$\overline{H}(x) = pe^{-\lambda_1 x} + (1-p)e^{-\lambda_2 x}, \ \lambda_1, \ \lambda_2, \ x \geq 0. \tag{9}$$

Then the tail of conditional excess distribution H_u is increasing and, for each x, satisfies

$$\frac{\overline{H}(x+u)}{\overline{H}(u)} \to e^{-\min(\lambda_1,\lambda_2)\,x} \quad \text{as } u \to \infty.$$

4 Uniform Distance Between Conditional Excess Mixture and Parent Distributions

First we define *the uniform distance between two distributions* F and G, as [5]

$$\Delta(F,G) = \sup_x |F(x) - G(x)|,$$

which is used in the sensitivity analysis measures. The uniform distance between conditional excess mixture distribution tail (6) and it's parent distribution tail $\overline{F}_u$ is

$$\begin{aligned}\Delta(\overline{H}_u, \overline{F}_u) &= \sup_x |\overline{H}_u(x) - \overline{F}_u(x)| \\ &= (1-p)\sup_x \left| \frac{\overline{G}(x+u)\overline{F}(u) - \overline{G}(u)\overline{F}(x+u)}{\overline{F}(u)(p\overline{F}(u) + (1-p)\overline{G}(u))} \right|. \end{aligned} \tag{10}$$

If the densities for distribution functions F and G exist, and there exists x^* that delivers the supremum in equation (10), then x^* satisfies the equality

$$\frac{r_G(x^*+u)}{r_F(x^*+u)} = \frac{\overline{F}_u(x^*)}{\overline{G}_u(x^*)}. \tag{11}$$

For example, for Hyperexponential distribution (9) solution of equation (11) has the following form

$$x^* = \frac{\log\lambda_2 - \log\lambda_1}{\lambda_2 - \lambda_1},$$

and coincides with the solution x^* obtained for the uniform distance $\Delta(H,F)$ between Hyperexponential distribution H with $\lambda_1 > \lambda_2$ and the parent (Exponential) distribution F with parameter λ_1 [12]. The expression (10) in this case becomes

$$\begin{aligned}\Delta(\overline{H}_u, \overline{F}_u) &= \frac{1-p}{pe^{-(\lambda_1-\lambda_2)u} + (1-p)} \frac{|\lambda_2 - \lambda_1|}{\lambda_2} \left(\frac{\lambda_1}{\lambda_2}\right)^{-\frac{\lambda_1}{\lambda_1-\lambda_2}} \\ &= \frac{1}{pe^{-(\lambda_1-\lambda_2)u} + (1-p)} \Delta(\overline{H}, \overline{F}),\end{aligned}$$

and it follows that

$$\Delta(\overline{H}_u, \overline{F}_u) \underset{u\to\infty}{\to} \frac{\Delta(\overline{H}, \overline{F})}{1-p}. \tag{12}$$

For Pareto mixture

$$\overline{H}(x) = p\left(\frac{x_0}{x_0+x}\right)^{\alpha_1} + (1-p)\left(\frac{x_0}{x_0+x}\right)^{\alpha_2}, \ \alpha_1 > \alpha_2, \ x_0 > 0, \ x \geq 0,$$

we find from (11)

$$x^* = (x_0+u)\left(\frac{\alpha_2}{\alpha_1}\right)^{1/(\alpha_2-\alpha_1)} - x_0 - u.$$

Then, according to (12), we obtain

$$\begin{aligned}
\Delta(\overline{H}_u,\overline{F}_u) &= \sup_x \left| \frac{p\left(\frac{x_0}{x_0+x+u}\right)^{\alpha_1} + (1-p)\left(\frac{x_0}{x_0+x+u}\right)^{\alpha_2}}{p\left(\frac{x_0}{x_0+u}\right)^{\alpha_1} + (1-p)\left(\frac{x_0}{x_0+u}\right)^{\alpha_2}} - \left(\frac{x_0+u}{x_0+x+u}\right)^{\alpha_1} \right| \\
&= \sup_x \left| \left(\frac{p + (1-p)\left(1+\frac{x+u}{x_0}\right)^{\alpha_1-\alpha_2}}{p + (1-p)\left(1+\frac{u}{x_0}\right)^{\alpha_1-\alpha_2}} - 1 \right) \left(\frac{x_0+x+u}{x_0+u}\right)^{-\alpha_1} \right| \\
&= \frac{(x_0+u)^{\alpha_1-\alpha_2}}{p x_0^{\alpha_1-\alpha_2} + (1-p)(x_0+u)^{\alpha_1-\alpha_2}} \Delta(\overline{H},\overline{F}) \\
&\to \frac{1}{1-p}\Delta(\overline{H},\overline{F}), \quad u\to\infty,
\end{aligned}$$

where, as it is shown in [12],

$$\Delta(\overline{H},\overline{F}) = (1-p)\frac{\alpha_1-\alpha_2}{\alpha_2}\left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha_1-\alpha_2}}.$$

It then follows that

$$\lim_{u\to\infty} \Delta(\overline{H}_u,\overline{F}_u) = \frac{\alpha_1-\alpha_2}{\alpha_2}\left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha_1-\alpha_2}}.$$

5 Application to Queueing Systems with Service Time Mixture

In this section, we first compare the steady-state excesses of performance measures in the buffered multiserver queueing systems with renewal input flow. Consider two systems with the same number N of servers working in parallel. (In what follows the superscript (i) denotes the index of system i.) The service discipline is assumed to be First-Come-First-Served. We denote by $S_n^{(i)}$ the service time of customer n, and by $t_n^{(i)}$ his arrival instant. The sequence of the independent identically distributed (iid) interarrival times $\tau_n^{(i)} = t_{n+1}^{(i)} - t_n^{(i)}$, $n \geq 1$, and the sequence of the iid service times $\{S_n^{(i)},\ n\geq 1\}$ are assumed to be independent, $i=1,2$. Denote by $S^{(i)}$ the generic service time, and by $\tau^{(i)}$ the generic interarrival time, $i=1,2$. At the arrival instant $t_n^{(i)}$ of customer n, denote by $Q_n^{(i)}$ the *queue size*, by $\nu_n^{(i)}$ the *number of customers* and by $W_n^{(i)}$ the *waiting time* of customer n. Denote, when exists, the limits (in distribution)

$$Q_n^{(i)} \Rightarrow Q^{(i)},\ W_n^{(i)} \Rightarrow W^{(i)},\ n\to\infty, \quad i=1,2.$$

These limits exist, in particular, when the interarrival times $\tau^{(i)}$, $i=1,2$ are *non-lattice* (for instance, when input is Poisson) and the following negative drift assumption holds [1]:

$$\mathsf{E}S^{(i)} < N\mathsf{E}\tau^{(i)}.$$

Now we compare the steady-state queue size $Q^{(i)}$ and $W^{(i)}$ in the given systems $i = 1, 2$, with the corresponding indexes Q and W in the system with the mixture service time S defined as

$$S = IS^{(1)} + (1 - I)S^{(2)}.$$

The following statement contains conditions implying an ordering of conditional excess of service time and performance indexes in given systems and the system with mixed service time.

Theorem 4. *Assume that the following conditions hold:*

$$\nu_1^{(1)} \underset{st}{=} \nu_1^{(2)} = 0,\ \tau^{(1)} \underset{st}{=} \tau^{(2)},\ S^{(1)} \underset{r}{\leq} S^{(2)}. \tag{13}$$

Then the excess service times are ordered in failure rate as follows:

$$S_u^{(1)} \underset{r}{\leq} IS_u^{(1)} + (1 - I)S_u^{(2)} \underset{r}{\leq} S_u^{(2)},\ u \geq 0, \tag{14}$$

and queue sizes and waiting times are stochastically ordered:

$$\begin{aligned} Q^{(1)} &\underset{st}{\leq} Q \underset{st}{\leq} Q^{(2)}, \\ W^{(1)} &\underset{st}{\leq} W \underset{st}{\leq} W^{(2)}. \end{aligned} \tag{15}$$

If additionally, $S^{(1)}$ is NBU and $S^{(2)}$ is NWU then also

$$S_u^{(1)} \underset{st}{\leq} IS_u^{(1)} + (1 - I)S_u^{(2)} \underset{st}{\leq} S_u^{(2)}. \tag{16}$$

Proof. Under conditions (13) it follows from relation (4) that $S^{(1)} \underset{r}{\leq} S \underset{r}{\leq} S^{(2)}$. Then Theorem 5 in [17] implies (15). The inequalities (14) and (16) are the direct corollaries of the Theorems 1 and 2, respectively. □

It is worth mentioning that indeed the stochastic ordering of service times $S^{(1)} \underset{st}{\leq} S^{(2)}$ in (13) is sufficient for inequalities (15). However we use failure rate ordering because, for some distributions, it is more easy to find conditions implying this ordering. Also we note that we can replace stochastic ordering in (15) by the ordering w.p.1, using a coupling technique, see [16].

The analysis of performance indexes in multiserver systems with mixtures of service times is usually a complicated problem which, as a rule, has not analytical solution. An estimation of these indexes by simulation is often also a hard problem. In such cases we may separately analyze the systems with component service times to construct the upper and lower bounds for the target indexes based on the results of Theorem 4 and simulation method of *regenerative envelops*, recently developed in the works [8,9,10].

Indeed the mixture service time distributions naturally arise in the analysis of the multiclass queueing systems. In such systems, there are K classes of arrivals, and class-i customers have the iid service times $\{S_n^{(i)}, n \geq 1\}$ with generic element $S^{(i)}$. Assume (for simplicity only) that class-i customers follow Poisson input with rate λ_i. Then the total input rate is $\lambda = \sum_{i=1}^{K} \lambda_i$, and each new customer is class-i one with the probability $p_i = \lambda_i/\lambda$, $i = 1, \ldots, K$. In the multiserver queuing system with FCFS service discipline and stochastically equivalent servers, each new customer entering each server is class-i one with the same probability p_i. Then the (class-independent) service time of the nth customer entering the system (or arbitrary server) can be written as the mixture

$$S_n =_{st} \sum_{i=1}^{K} I_n^{(i)} S_n^{(i)}, \; n \geq 1, \tag{17}$$

where indciator $I_n^{(i)} = 1$ if the nth customer is class-i (and $I_n^{(i)} = 0$ otherwise). Thus the service time in the multiclass system has mixture distribution, and the representation (17) can be used, for instant, to study the asymptotic behaviour of the remaining service time of the customer being in the server at instant t as $t \to \infty$. It is worth mentioning that in such an analysis we can obtain not only some bounds but explicit asymptotic expressions as well. For instance, one prove, using coupling argument, that the *stationary remaining service time* in such an N-server system has the following explicit distribution

$$F(x) = 1 - \sum_{i=1}^{K} \frac{\lambda_i}{N} \int_x^{\infty} (1 - F_i(u))du, \quad x \geq 0,$$

where F_i is the distribution of $S^{(i)}$. For more details see [11].

6 Conclusion

In this paper, we study the applicability of the *failure rate ordering and stochastic comparison* to the steady-state of performance measures in the multiserver systems with two-component mixture service time distributions. For such systems, we consider the conditions imposed on service time distributions implying monotonicity properties of the failure rate functions. Also we discuss how mixture service time distribution arises in the multiclass systems. Some particular examples are considered as well. The interesting problem for future research is the preservation property of stochastic ordering for conditional excesses of performance indexes.

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