Unified Field Theory of Electroweak, Strong, and Gravitational Interactions*

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Abstract. In this paper, we are given a new interpretation of a simple gravity and gauge fields as the connections to the fiber bundle, a given change in the basis vectors, and the base layer with an infinitely small displacement in the database. Introduction of the interaction of gauge fields and the quarks and leptons based on the interpretation of leptons and quarks as vectors and tensors in the space layer. The dynamics of gauge fields and gravity introduced based on the equations of mechanics Cartan using a symplectic metric that unites gravity and gauge fields together.

Keywords: Gauge Theory, Electroweak Interactions, Metric, Gravitation, connectivity.

1 Introduction

Attempts to build a unified theory of all interactions, a unified field theory, began before Einstein created the General theory of relativity. The first of the interaction theories was the theory of electromagnetism, created by Maxwell in 1863. In 1915, Einstein formulated the General theory of relativity describing the gravitational field. Just as Maxwell was able to create a General description of electrical and magnetic phenomena, the idea of building a unified theory of fundamental interactions arose. This problem was posed by D. Gilbert. Einstein proposed a large number of options for unification gravity and the electromagnetic field [1-7]. In the first half of the twentieth century, numerous attempts were made to create such a theory. However, no satisfactory models were put forward. This is because in General relativity, gravity is a curvature of space-time, and electromagnetism has all the attributes of matter. However, in our opinion, this unification was most successfully achieved in [8], [9-11], where gravity and electromagnetism were combined in a pseudo-Riemannian metric of a 5-dimensional differentiable manifold, and in a more modern version of the union in [12], where gravity

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and electromagnetism were combined into a single electro-gravitational connection also in the space of 5-dimensions.

In the second half of the XX century, due to the discovery of weak and strong interactions, the task of building a unified theory became more complicated. In 1967, Salam and Weinberg created the theory of electroweak interaction, combining electromagnetism and weak interactions. In 1973, a theory of strong interactions (quantum chromodynamics) was proposed. On their basis, a Standard model of elementary particles was built, describing the electromagnetic, weak, and strong interactions.

After Weinberg, Salam, and Glashow created the theory of electro-weak interactions, as well as quantum chromodynamics, the need for a new approach to a unified field theory became clear, consisting of combining electro-weak interaction with quantum chromodynamics and the theory of gravity [13-16]. The possibility of such unification lies in the analogy that exists between the Yang-Mills theory and the relativity theory, and the fact that both the theory of electro-weak interaction and quantum chromodynamics contain the Yang-Mills theory [17-20]. It is not possible to generalize the works [8] and [12] to electroweak interaction and chromodynamics.

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However, in 5-dimensional space, you can easily introduce electro-gravitational connectivity, namely, by requiring:

$$d\partial_\alpha = \omega_\alpha^\beta \partial_\beta = \Gamma_{\alpha\mu}^\beta dx^\mu \partial_\beta, \tag{1}$$

where $\alpha = \overline{0,3}$; $\beta = \overline{0,3}$, $\mu = \overline{0,3}$ [18-20],

$$d\vec{e}_4 = A\vec{e}_4, \tag{2}$$

thus, we set the electro-gravitational connectivity in 5-dimensional space (ω_β^α, A), $\omega_\alpha^\beta = F_{\alpha\mu}^\beta dx^\mu$ – describes infinitesimal changes in the reference vectors of space-time, that is, gravity, when moving from one point of space-time to another infinitely close point. It is assumed that a four-dimensional space-time is embedded in a 5-dimensional manifold. At each point of this manifold, a linearly independent with $\{\partial_\alpha\}$, $\alpha = \overline{0,3}$, the vector $\vec{e}_4$ is chosen, whose changes during the transition from one point of space-time to another infinitely close point are described by the 1 - form of electromagnetic field *A*, *d* – external differential [20]. Externally differentiating the equalities (1) and (2), we get an expression for the curvature of space-time, that is, gravity and the 2-form of the electromagnetic field:

$$d^2\partial_\alpha \equiv R_\alpha^\beta \partial_\beta = d\omega_\alpha^\beta \partial_\beta - \omega_\alpha^\beta \wedge d\partial_\beta = \{d\omega_\alpha^\beta + \omega_\mu^\beta \wedge \omega_\alpha^\mu\}\partial_\beta, \tag{3}$$

$$d^2\vec{e}_4 = dA\vec{e}_4 - A \wedge A\vec{e}_4 = F\vec{e}_4, \tag{4}$$

in this way

$$R_\alpha^\beta = d\omega_\alpha^\beta + \omega_\mu^\beta \wedge \omega_\alpha^\mu, \tag{5}$$

$$F = dA \text{ [20]} \tag{6}$$

Thus, in space-time embedded in a 5-dimensional manifold, connectivity is introduced that describes both gravity and the electromagnetic field. The Hilbert-Einstein equation in GRT, as well as the Maxwell equations, are reduced to a single system of Cartan equations for the symplectic metric [21-23]:

$$\begin{aligned}
\Omega = \int \sqrt{-g}\,dx \Big\{ & \left(dg_{\alpha\beta} - \omega_{\alpha\beta} - \omega_{\beta\alpha}\right) \wedge \left[d\theta_1^{\alpha\beta} - \left(R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R - \chi T^{\alpha\beta} \right) dx^4 \right] + \\
& + \left(d\omega_\beta^\alpha + \omega_\rho^\alpha \wedge \omega_\beta^\rho - R_\beta^\alpha \right) \pi_\alpha^\beta(x) + d\theta_1^{\alpha\beta} \wedge d\pi_{\alpha\beta}(x) + \\
& + \left[d\theta_2^\alpha - \left(F_{,\beta}^{\alpha\beta} + \frac{4\pi}{c} J^\alpha \right) dx^4 \right] \wedge \\
& \wedge d\pi_\alpha(x) + d\theta_2^\alpha \wedge d\pi_\alpha^1 + \left(dA - \frac{1}{2} F_{\beta\alpha} dx^\beta \wedge dx^\alpha \right) \pi(x) \Big\}.
\end{aligned} \tag{7}$$

Here: $\alpha, \beta, \rho = \overline{0{,}3}$; dx^4 – volume differential in our 5-dimensional space; $dx^4(\partial_\alpha) = 0$, $\alpha = \overline{0{,}3}$; $dx^4(\vec{e}_4) = 1$; $dx = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$, $\wedge$ – E. Cartan's external multiplication sign; $g = Det\|g_{\mu\nu}\|$, $T^{\mu\nu}$ – dark matter and dark energy energy-momentum tensor; $\theta_1^{\alpha\beta}, \theta_2^\alpha, \pi_\beta^\alpha, \pi_\alpha, \pi, \pi_\alpha^1$ – auxiliary tensor fields; $F_{,\beta}^{\alpha\beta}$ – the covariant divergence of a tensor $F^{\alpha\beta}$; J^α – 4-vector of electric current.

Cartan's equations for this symplectic metric have the form [19]:

$$\begin{aligned}
0 &= \frac{\delta\Omega}{\delta dg_{\alpha\beta}} = \frac{\delta\Omega}{\delta d\theta_1^{\alpha\beta}} = \frac{\delta\Omega}{\delta d\pi_{\alpha\beta}} = \frac{\delta\Omega}{\delta d\pi_\alpha^1} = \frac{\delta\Omega}{\delta d\pi_\alpha} = \frac{\delta\Omega}{\delta \pi_\alpha^\beta} = \frac{\delta\Omega}{\delta\pi} = \\
&= d\theta_1^{\alpha\beta} - \left(R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R - \chi T^{\alpha\beta} \right) dx^4 = \\
&= dg_{\alpha\beta} - \omega_{\alpha\beta} - \omega_{\beta\alpha} = d\omega_\beta^\alpha + \omega_\rho^\alpha \wedge \omega_\beta^\rho - R_\beta^\alpha = d\theta_1^{\alpha\beta} = \\
&= d\theta_2^\alpha - \left(F_{,\beta}^{\alpha\beta} + \frac{4\pi}{c} J^\alpha \right) dx^4 = d\theta_2^\alpha = \\
&= dA - \frac{1}{2} F_{\beta\alpha} dx^\beta \wedge dx^\alpha.
\end{aligned} \tag{8}$$

From Cartan's system of equations (8), a system of equations describing both gravity and electromagnetism is obtained.

Hilbert-Einstein equation for the gravitational field:

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = \chi T^{\alpha\beta}; \tag{9}$$

a condition that the connectivity ω_α^β is metric:

$$dg_{\alpha\beta} = \omega_{\alpha\beta} + \omega_{\beta\alpha}; \tag{10}$$

connection of connectivity ω_α^β with the curvature of space-time R_β^α :

$$R_\beta^\alpha = d\omega_\beta^\alpha + \omega_\rho^\alpha \wedge \omega_\beta^\rho; \tag{11}$$

Maxwell equation:

$$dA = \frac{1}{2} F_{\beta\alpha} dx^\beta \wedge dx^\alpha, \tag{12}$$

$$F_{,\beta}^{\alpha\beta} = -\frac{4\pi}{c} J^\alpha. \tag{13}$$

In contrast to [2], this version of the theory of electro-gravitational connectivity can be generalized to a Unified Field Theory, which includes, in addition to the theory of gravity, the theory of electro-weak interactions and quantum chromodynamics.

To prove this, consider combining the theory of gravity with two gauge fields into a unified gravitational-gauge connection. To do this, put a 4-dimensional GRT space-time in the space of 4+2n dimensions. On the four space-time reference points, we set the connectivity (1), and on the 2n vectors that come out of space-time, we set the connectivity:

$$d\vec{e}_s = \tilde{\omega}_1{}^m{}_s \vec{e}_m, \tag{14}$$

$$\tilde{\omega}_1{}^m{}_s = -i\hat{g}_1 \tilde{A}^a M^{1s}_{am}, \tag{15}$$

where $m, s = \overline{1, n}$, $a = \overline{1, \mu}$, $\hat{g}_1 -$ calibration field charge operator $\tilde{A}^a M_a^1$,

$M^{1m}_{as} - \mu$ square matrices $n \times n$.

$$d\vec{e}_{n+s} = -i\hat{g}_2 \tilde{B}^b M^{2n+m}_{b\,n+s} \vec{e}_{n+m}, \tag{16}$$

here

$$\tilde{\omega}_2{}^{n+m}{}_{n+s} = -i\hat{g}_2 \tilde{B}^b M^{2\,n+m}_{b\,n+s}, \tag{17}$$

where $\hat{g}_2$ – calibration field charge operator $\tilde{B}^b M^2_b$,

$M^{2\,n+m}_{b\,n+s}$ – μ square matrices $n \times n$, $b = \overline{1,\nu}$.

Thus, calibration fields, with accuracy to a constant, are a connectedness that describes the change of reference vectors $\vec{e}_s$, $s = \overline{1,2n}$ when moving from one point in space-time to another infinitely close point.

The curvature of connectivity (15) and (17) is calculated in the same way as connectivity (1) using the formula (5):

$$\begin{aligned}
R_1{}^m{}_n &= d\tilde{\omega}^m_{1n} + \tilde{\omega}^n_{1s} \wedge \tilde{\omega}^s_{1n} = -i\hat{g}_1 d\tilde{A}^a M^{1m}_{an} - \hat{g}_1^2 \tilde{A}^a \wedge \tilde{A}^b M^{1m}_{as} M^{1s}_{bn} = \\
&= -i\tilde{g}_1 \left\{ d\tilde{A}^d - i\hat{g}_1 \frac{1}{2} iC^{1d}{}_{ab} \tilde{A}^a \wedge \tilde{A}^b \right\} M^1_d = \\
&= -i\hat{g}_1 \frac{1}{2} \left\{ \partial_\mu A^d_\nu - \partial_\nu A^d_\mu + \hat{g}_1 C^{1d}_{ab} A^a_\mu A^b_\nu \right\} dx^\mu \wedge dx^\nu M^{1m}_{dn} = \\
&= -\frac{i\hat{g}_1}{2} G^d_{\mu\nu} dx^\mu \wedge dx^\nu M^{1m}_{dn} = -i\tilde{g}_1 G^d M^{1m}_{dn}.
\end{aligned} \tag{18}$$

Here $G^d = \frac{1}{2} G^d{}_{\mu\nu} dx^\mu \wedge dx^\nu$ are 2-form of the calibration field, we have taken into account the relations between operators of the Lie group M^1_a [24], whose representation operates in a space stretched based on $\vec{e}_m$, $m = \overline{1,n}$:

$$\left[M^1_a, M^1_b \right] = iC^{1d}{}_{ab} M^1_d. \tag{19}$$

Similarly, you can calculate the curvature of the connection (17):

$$\begin{aligned}
R_2{}^m{}_s &= -i\hat{g}_2 \frac{1}{2} \left\{ \partial_\mu B^d_\nu - \partial_\nu B^d_\mu + \tilde{g}_2 C^{2\,d}_{ab} B^a_\mu B^b_\nu \right\} dx^\mu \wedge dx^\nu M^{2m}_{ds} = \\
&= -\frac{i\hat{g}_2}{2} F^d_{\mu\nu} dx^\mu \wedge dx^\nu M^{2m}_{ds} = -i\hat{g}_2 F^d M^{2m}_{ds}.
\end{aligned} \tag{20}$$

Here $F^d = \frac{1}{2} F^d_{\mu\nu} dx^\mu \wedge dx^\nu$ – 2-form of the calibration field, we have taken into account the relations between generators of the Lie group M^2_a, the representation of which operates in a space stretched based on $\vec{e}_{n+m}$, $m = \overline{1,n}$:

$$\left[M^2_a, M^2_b \right] = iC^{2d}{}_{ab} M^2_d. \tag{21}$$

A mathematical model of the interaction of these gauge fields with leptons and quarks can be constructed if we consider vector fields in a space stretched on a basis $\{e_m\}$:

$\vec{\Psi} = \Psi^m \vec{e}_m$ as a mathematical model of leptons. Here these components of the vectors Ψ^m are bispinors. In this case

$$d\vec{\Psi} = d\Psi^m \vec{e}_m + \Psi^m d\vec{e}_m = \left\{ d\Psi^m + \tilde{\omega}_1{}^m{}_s \Psi^s \right\} \vec{e}_m .$$

Consider the values of the components of this vector, which are 1-forms, on the vector $i\gamma^\alpha \partial_\alpha$:

$$\left\{ d\Psi^m_{g_1} + \tilde{\omega}_1{}^m{}_s \Psi^s_{g_1} \right\} \left(i\gamma^\alpha \partial_\alpha \right) = i\gamma^\alpha \left(\partial_\alpha \Psi^m_{g_1} - i g_1 A^a_\alpha M^{1m}_{a\,s} \Psi^s_{g_1} \right) - m\Psi^m_{g_1} = 0. \tag{22}$$

Here g_1 and $\Psi^m_{g_1}$ are eigenvectors and operator values $\hat{g}_1$. Equation (22) describes a fermion interacting with only one calibration field. Therefore, this is a mathematical model of a lepton interacting only with an electroweak gauge field.

As a mathematical model of quarks, consider the tensor field:

$$\Psi^{ms}_{g_1 g_2} \vec{e}_m \otimes \vec{e}_{n+s} \equiv \Psi^{ms}_{g_1 g_2} \vec{e}_m \otimes \vec{e}\,'_s .$$

The external differential of this field has the form:

$$\begin{aligned} & d\Psi^{ms}_{g_1 g_2} \vec{e}_m \otimes \vec{e}\,'_s + \Psi^{ms}_{g_1 g_2} d\vec{e}_m \otimes \vec{e}\,'_s + \Psi^{ms}_{g_1 g_2} \vec{e}_m \otimes d\vec{e}\,'_s = \\ & = \left\{ d\Psi^{ms}_{g_1 g_2} + \tilde{\omega}_1{}^m{}_p \Psi^{ps}_{g_1 g_2} + \tilde{\omega}'_2{}^s{}_p \Psi^{mp}_{g_1 g_2} \right\} \vec{e}_m \otimes \vec{e}\,'_s . \end{aligned} \tag{23}$$

Consider the values of the components of this tensor, which are 1-forms, on the vector $i\gamma^\alpha \partial_\alpha$:

$$i\gamma^\alpha \left(\partial_\alpha \Psi^{ms}_{g_1 g_2} - i g_1 A^a_\alpha M^{1m}_{a\,p} \Psi^{ps}_{g_1 g_2} - i g_2 B^b_\alpha M^{2n+m}_{b\quad n+p} \Psi^{sp}_{g_1 g_2} \right) - m\Psi^{ms}_{g_1 g_2} = 0, \tag{24}$$

Where $\Psi^{ms}_{g_1 g_2}$ and g_1, g_2 are eigenvectors and values of the operators $\hat{g}_1$ and $\hat{g}_2$:

$$\left[\vec{g}^1, \vec{g}^2 \right] = 0$$

Equation (24) describes the interaction of a fermion with two calibration fields. This fermion can serve as a mathematical model of a quark interacting with two gauge fields: electroweak and gluon.

We set the change in the connectivity of (1) the gravitational field and the connectivity of (15) and (17) gauge fields when moving from one point in space-time to another using Cartan equations for the symplectic metric Ω_1:

$$\begin{aligned}
&\Omega_1 = \int \sqrt{-g}\, dx' \Big\{ \big(dg_{\alpha\beta} - \omega_{\alpha\beta} - \omega_{\beta\alpha}\big) \wedge \left[d\theta_1^{\alpha\beta} - \left(R^{\alpha\beta} \frac{1}{2} g^{\alpha\beta} R - \chi T^{\alpha\beta} \right) dx^4 \right] + \\
&+ \big(d\omega_\beta^\alpha + d\omega^\alpha{}_\rho \wedge \omega^\rho{}_\beta - R^\alpha{}_\beta\big) \pi_\alpha{}^\beta(x') + d\theta_1^{\alpha\beta} \wedge d\pi_{\alpha\beta}(x') + \\
&d\pi_{1d}{}^{\mu\nu}(x') \wedge \big(G_{\mu\nu}^d - \partial_\mu A_\nu^d + \partial_\nu A_\mu^d - \hat{g}_1 C^1{}_{ab}{}^d A^a{}_\mu A^b{}_\nu\big) dx^4 + \\
&+ d\pi_2{}^{\mu\nu}{}_d(x') \wedge \big(F_{\mu\nu}^d - \partial_\mu B_\nu^d + \partial_\nu B_\mu^d - \hat{g}_2 C^2{}_{ab}{}^d B^a{}_\mu B^b{}_\nu\big) dx^4 + \\
&+ \Big\{ d\xi_1{}^a{}_\nu(x') - \big[\partial^\mu G_{\mu\nu}^a - i\hat{g}_1 A^b{}_\mu G^{c\mu}{}_\nu C_{bc}^{1a} - \chi_1 \hat{g}_1 \Psi_1{}^+ \gamma_\nu\, M^{1a} \Psi_1\big] dx^4 \Big\} \wedge \\
&\wedge \Big\{ d\xi^{2\nu}{}_a(x') - \big[\partial^\mu F_{a\mu}{}^\nu - i\hat{g}_2 B^b{}_\mu F^{c\mu\nu} C_{abc}^2 - \chi_2 \hat{g}_2 \Psi_2^+ \gamma^\nu M_a^2 \Psi_2\big] dx^4 \Big\} + \\
&+ d\xi_1{}^a{}_\nu(x') \wedge d\rho_a^{1\nu}(x') + d\xi^{2\nu}{}_a(x') \wedge d\rho^{2a}{}_\nu \Big\}
\end{aligned} \tag{25}$$

Here: $\Psi_1^+\Psi_1$ – density operator for the number of particles generating the calibration field $G_{\mu\nu}^a$, $\Psi_2^+\Psi_2$ – density operator for the number of particles generating the calibration field $F_{\mu\nu}^a$, γ^ν – standard γ–matrices included in the Dirac equation, $\pi_{1d}{}^{\mu\nu}, \pi_2{}^{\mu\nu}{}_d, \xi_1^a{}_\nu, \xi^{2\nu}{}_a, \rho^{1\nu}{}_a, \rho^{2a}{}_\nu$ – auxiliary function, χ_1 and χ_2 are coefficients that link currents and the calibration fields they generate, x^4 – auxiliary parameter.

Cartan's equations for the symplectic metric (25) have the form:

$$\begin{aligned}
&0 = \frac{\delta\Omega_1}{\delta dg_{\alpha\beta}(x)} = \frac{\delta\Omega_1}{\delta d\theta_1^{\alpha\beta}(x)} = \frac{\delta\Omega_1}{\delta d\pi_{\alpha\beta}(x)} = \frac{\delta\Omega_1}{\delta \pi_\alpha{}^\beta(x)} = \\
&= \frac{\delta\Omega_1}{\delta d\pi_{1d}^{\mu\nu}(x)} = \frac{\delta\Omega_1}{\delta d\pi_{2d}^{\mu\nu}(x)} = \frac{\delta\Omega_1}{\delta d\xi_1{}^a{}_\nu(x)} = \frac{\delta\Omega_1}{\delta d\xi_a^{2\nu}(x)} = \\
&= \frac{\delta\Omega_1}{\delta d\rho_a^{1\nu}(x)} = \frac{\delta\Omega_1}{\delta d\rho_\nu^{2a}(x)} = d\theta_1^{\alpha\beta} - \left(R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R - \chi T^{\alpha\beta} \right) dx^4 = \\
&= dg_{\alpha\beta} - \omega_{\alpha\beta} - \omega_{\beta\alpha} = d\theta_1^{\alpha\beta} = d\omega^\alpha{}_\beta + \omega^\alpha{}_\rho \wedge \omega^\rho{}_\beta - R^\alpha{}_\beta = \\
&= \big(G_{\mu\nu}^d - \partial_\mu A_\nu^d + \partial_\nu A_\mu^d - \hat{g}_1 C^1{}_{ab}{}^d A_\mu^a A_\nu^b\big) dx^4 = \\
&= \big(F_{\mu\nu}^d - \partial_\mu B_\nu^d + \partial_\nu B_\mu^d - \hat{g}_2 C^2{}_{ab}{}^d B_\mu^a B_\nu^b\big) dx^4 = \\
&= d\xi_a^{2\nu} - \big[\partial^\mu F_{a\mu}{}^\nu - i\hat{g}_2 B_\mu^b F^{c\mu\nu} C_{abc}^2 - \chi_2 \hat{g}_2 \Psi_2^+ \gamma^\nu M^2{}_a \Psi_2\big] dx^4 = \\
&= d\xi_{1\,\nu}^a - \big[\partial^\mu G_{\mu\nu}^a - i\hat{g}_1 A_\mu^b G^{c\mu}{}_\nu C_{1\,bc}^a - \chi_1 \hat{g}_1 \Psi_1^+ \gamma_\nu M^{1a} \Psi_1\big] dx^4 = d\xi_{1\,\nu}^a = d\xi_a^{2\nu}.
\end{aligned} \tag{26}$$

This system of Cartan equations is equivalent to the system of gravitational field equations:

$$\left.\begin{aligned}
&R^{\alpha\beta}-\frac{1}{2}g^{\alpha\beta}R=\chi T^{\alpha\beta}\\
&d\omega^{\alpha}{}_{\beta}+\omega^{\alpha}{}_{\rho}\wedge\omega^{\rho}{}_{\beta}=R^{\alpha}{}_{\beta}\\
&dg_{\alpha\beta}=\omega_{\alpha\beta}+\omega_{\beta\alpha}
\end{aligned}\right\} \tag{27}$$

and a system of equations for two calibration fields [25-31]

$$\left.\begin{aligned}
&G^{d}{}_{\mu\nu}=\partial_{\mu}A^{d}_{\nu}-\partial_{\nu}A^{d}_{\mu}+\hat{g}_{1}C^{1}{}_{ab}{}^{d}A^{a}_{\mu}A^{b}_{\nu}\\
&\partial^{\mu}G^{a}_{\mu\nu}-i\hat{g}_{1}A^{b}_{\mu}G^{c\mu}{}_{\nu}C^{a}_{1\,bc}=\chi_{1}\hat{g}_{1}\Psi^{+}_{1}\gamma_{\nu}M^{1a}\Psi_{1}\\
&F^{d}_{\mu\nu}=\partial_{\mu}B^{d}_{\nu}-\partial_{\nu}B^{d}_{\mu}+\hat{g}_{2}C^{2}{}_{ab}{}^{d}B^{a}_{\mu}B^{b}_{\nu}\\
&\partial^{\mu}F_{a\mu}{}^{\nu}-i\hat{g}_{2}B^{b}{}_{\mu}F^{c\mu\nu}C^{2}_{abc}=\chi_{2}\hat{g}_{2}\Psi^{+}_{2}\gamma^{\nu}M^{2}_{a}\Psi_{2}
\end{aligned}\right\} \tag{28}$$

Two gauge fields $G^{d}_{\mu\nu}$ and $F^{d}_{\mu\nu}$ are taken in the most General form with arbitrary structural constants C^{1}_{abd} and C^{2}_{abd} since if in our local Universe they have certain values and describe electro-weak and gluon gauge fields, then in other local Universes they can have other and arbitrary values. Quarks and leptons can also be reduced to the gauge fields by the transformation of supersymmetry.

3 Conclusion

Thus, the results obtained in this paper are similar to those obtained in [32]. That is, all the interactions included in the Standard model of elementary particle physics (electroweak and strong interactions) and gravity are combined into a single bond in the stratified space, and it is indicated how fermions can be included in the theory. But, unlike the work [32], it is made much easier, since it is based on the famous works of E. Cartan [21,23]. While Lisi used complex manipulations with connections arising within the E8 Lie group (although such connections also go back to the works of E. Cartan).

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