

## Fuzzy Set Theory in Determining Learning Process Effectiveness\*

Oleg V. Boychenko <sup>1</sup>[0000-0003-3326-1015], Andrej I. Gaponov <sup>1</sup>[0000-0002-0127-9643],  
Oksana Yu. Smirnova <sup>1</sup>[0000-0001-9803-3094] and Ilya V. Gavrikov <sup>1</sup>[0000-0002-7047-9059]

<sup>1</sup> V. I. Vernadsky Crimean Federal University, Simferopol, Russia

bolek61@mail.ru

**Abstract.** The paper considers the viability of applying fuzzy set theory to determining learning process effectiveness in a higher education institution. It also investigates issues that concern relationships between learning process effectiveness criteria, quality of expert assessment, as well as concerning the integrated assessment of learning process effectiveness. The goal of this paper is to propose a tool that would determine learning process effectiveness in a higher education institution. To that end, a mathematical model is presented that demonstrates the relationship between learning process effectiveness criteria and fuzzy set theory. Additionally, a formalized mathematical model is presented using fuzzy set theory and the MATLAB software environment. To implement the technique for learning process effectiveness assessment, an intelligent system has been developed by the authors, based on the MATLAB mathematical package. The value of the integrated assessment is determined using defuzzification of the output variable using the “centre of gravity”, and the principal stages of calculating the assessment are presented on figures within. The paper provides tables of membership function values, active fuzzy inference rules, and a triangular membership function with “non-overlapping” neighbouring terms.

**Keywords:** integrated assessment, fuzzy set theory, effectiveness, learning process, higher education institution, computer modelling, fuzzy logic.

### 1 Introduction

Today, fuzzy modelling is a promising avenue of research and development. The technology is relevant and in-demand because interest in various aspects of intelligent management has risen, and formal and mathematical models of real systems and management processes are growing ever more complex. This tendency means models must more adequately describe their subject area and consider a multitude of factors that influence decision making processes. The fuzzy systems toolset includes fuzzy sets, fuzzy logic, and fuzzy modelling, and is especially useful for solving these emerging

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problems. Applying the fuzzy systems toolset makes it possible to build fuzzy control systems of various classes, which would allow solving management problems in circumstances where traditional methods are ineffective or unable to be applied due to insufficient data about the object [1].

The learning process in pedagogy is the gradual shift in educational acts necessary to facilitate the development of the student's personality. Effectiveness of learning is an important category that contains both process unity and learning results. Assessment of the effectiveness of the learning process in higher education institutions is relevant because ratings do not reflect the reality of the learning process. Today, significant attention is devoted to learning process assessment by higher education institutions, and a multitude of methods and models have been developed to that end. However, the majority of them do not yield accurate assessments, so the task of finding and developing methods for accurate assessment of learning process effectiveness and comparison of higher education institutions is particularly relevant [2, 3, 4].

Many researchers believe that increasing the effectiveness of the learning process is one of the most important pedagogical tasks. Effectiveness, along with performance, is one of the indicators of process quality. The combination of multiple interrelated elements that determine the effectiveness of the learning process makes it difficult to determine its integral (integrated) assessment. The history of this problem is determined by the dynamism expressed in particular by changes during generations, as well as the progressive development of human society. Passing on experience in solving this problem results in the improvement of techniques and methods of teaching. Experience shows that a constructive solution cannot be found when using an approach that considers the learning process as a set of components interconnected in different ways [5].

## **2 Principal Assumptions and Methodology**

Modern pedagogical learning process theory is directed at efficiently forming competencies in students and describes techniques and methods for organizing learning activities. The learning process, being bilateral, consists of two components: teaching – the activity of the teacher – and studying – the activity of the student. Ultimately, learning itself is a tool and method for organizing the educational process, the path towards obtaining a comprehensive education. The efficiency of learning is determined by many criteria. The criteria for learning effectiveness are objective, comparable indicators of the learning process, which are stable for a certain period. According to modern didactical concepts, effectiveness criteria should reflect competency completeness and the development of personal qualities. Effectiveness criteria for the learning process may be considered from various points of view: from one of a competence approach, a personal approach, a systematic approach, or a differentiated approach. V. M. Blinov, a Russian pedagogue, was the first to describe the problem of learning process effectiveness as a didactical category, which is based upon the tenets of activity systems. Blinov determined the weak areas in the theory of learning effectiveness increase and proposed ways to rectify them, as well as didactically characterized learning process effectiveness [6].

Let us consider the criteria for learning process effectiveness, which may be grouped into internal and external criteria. Internal criteria include learning success, academic performance, quality of acquired knowledge and competencies, level of student development, level of education, and educability [7]. External criteria include adaptability of graduates to professional activities and social functioning, the perfection of self-education, professional mastery. Some researchers propose different classifications of learning process effectiveness criteria, for example, cognitive, affective, and psychomotor goals of education. Limiting the scope of this paper, only the following criteria for the effectiveness of the learning process will be considered, in the following order:

Level of teacher professionalism – LP.

Level of learning, or level of educability (i.e. the ability to assimilate knowledge) – LE.

The efficiency of learning process management, delivering good results at minimal cost – EM.

Formation of competencies, i.e. a system of knowledge, skills and competencies that correspond to the Federal State Educational Standard (CF) [8].

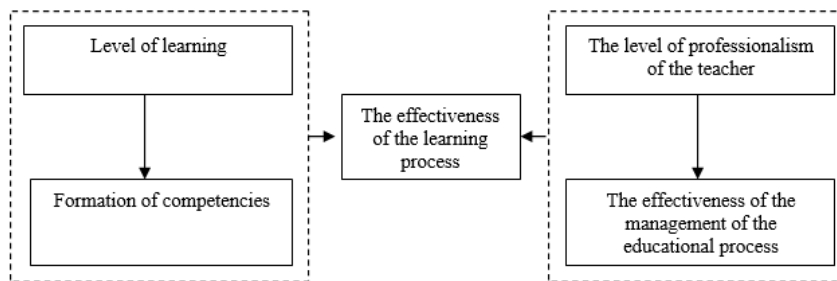


Fig. 1. Relationships between criteria for learning process effectiveness

### 3 Modelling an Integral Assessment of Learning Process Effectiveness

The MATLAB software environment is an application package for various calculations, computer modelling, and solving a wide range of practical problems. The MATLAB software environment includes the base software and some extension packages, which encompass a wide range of problems and subject areas. The development of mathematical models based on fuzzy logic is facilitated by the MATLAB Fuzzy Logic Toolbox extension package. The package facilitates the development and utilization of fuzzy logic models.

To develop and later utilize fuzzy inference systems, the following software tools in the Fuzzy Logic Toolbox have been used: FIS Editor (editing graphical fuzzy inference systems); Membership Function Editor (editing graphical membership functions of the generated fuzzy inference); Rule Editor (editing logical rules of the fuzzy inference system); Rule Viewer (viewing logical rule tables for the fuzzy inference system being analysed); Surface Viewer (visualizing the surface of the fuzzy inference result). The

FIS Editor may be opened using the word *fuzzy* in the command line. This function supports adding and editing the number of input and output variables, the corresponding membership functions, fuzzy inference system type, assumed defuzzification method, etc.

The Membership Function Editor serves to input and edit functions of linguistic term membership in a fuzzy inference system. The Rule Editor is used to set and modify logical rules of the fuzzy inference system using a table view. Complete sentences are created using the words *if*, *then*, *is*, *and*, *or*, when writing rules down in text form. Result visualization and analysis of result changes depending on inputs are performed using the Rule Viewer; the bottom right rectangle marks the defuzzified value of the output variable received as a result of accumulating inferences made using the specified rules. Changing specific input variable values is done by either moving the red vertical line intersecting the input variable rectangles, or specifying values manually in the respective input fields. The Surface Viewer allows the user to analyse the fuzzy inference system surface and visualize how output variables relate to all input variables.

Let us review the applications of MATLAB to performing calculations, computer modelling, and practical problem solutions in the field of education.

The integrated assessment (KO) of learning process effectiveness shall be calculated using fuzzy set theory [9]. Levels of criteria that form the integrated assessment will be determined according to 10 expert assessments. The experts using a qualitative descriptor assess each of these criteria: H for “unsatisfactory”, Y for “satisfactory”, X for “good” and O for “excellent”. Taking into account the high consistency of expert opinions, it can be assumed that the assessments of one criterion will be limited to a pair of adjacent qualitative descriptors, i.e. unsatisfactory-satisfactory, satisfactory-good, good-excellent. To avoid unnecessary complication of the fuzzy inference ruleset in the model, let us use a triangular membership function with “non-overlapping” neighbouring terms.

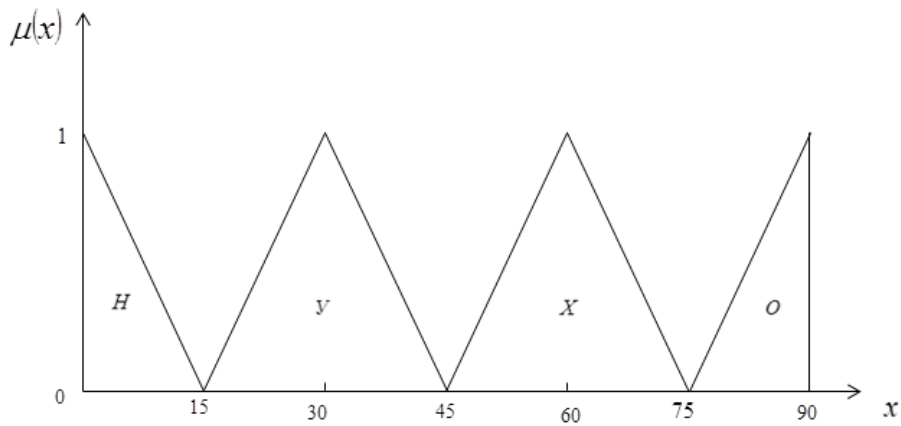
$$\mu_{trimf}(x, a, b, c) = \begin{cases} 0, & x < a; \\ \frac{x-a}{b-a}, & a \leq x \leq b; \\ \frac{c-x}{c-b}, & b \leq x \leq c; \\ 0, & x > c \end{cases}, \quad (1)$$

where  $a$ ,  $c$  are the X coordinates of the points defining the triangle base,  $b$  is the X coordinate of its vertex. This function is plotted in Fig. 2.

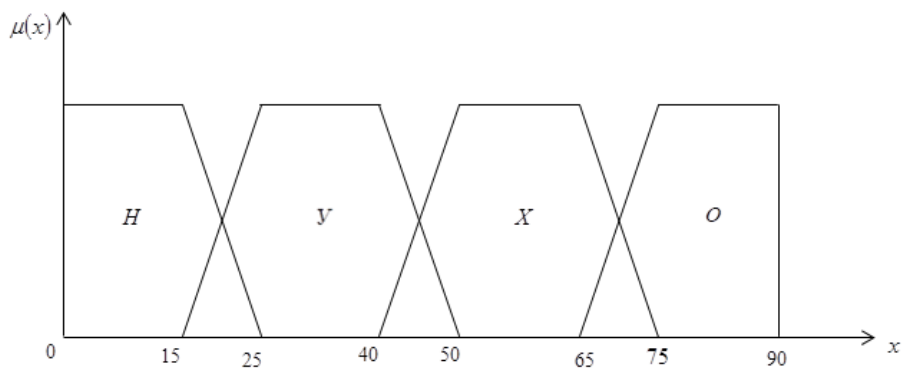
The output linguistic variable, “integrated assessment” (KO) contains four linguistic terms (qualitative descriptors) H, Y, X, O with trapezoid membership functions (see Fig. 3).

$$\mu_{trapezf}(x;a;b;c;d) = \left\{ \begin{array}{l} 0, x \leq a \\ \frac{x-a}{b-a}, a \leq x \leq b \\ 1, b \leq x \leq c \\ \frac{d-x}{d-c}, c \leq x \leq d \\ 0, x \geq d \end{array} \right\}, \quad (2)$$

where  $a, d$  are the X coordinates of the trapezoid lower base,  $b, c$  – the upper.



**Fig. 2.** Membership function plot for input variables LP, LE, EM, CF.



**Fig. 3.** Membership function plot for KO.

The values of the membership functions for the input linguistic variables are defined as

the ratio of the number of each of the H, Y, X, O answers to the total number of respondents [10]. The obtained values are presented in the table below:

**Table 1.** Membership function values

Term membership function value	Criterion			
	LP	LE	EM	CF
$\mu(H)$				
$\mu(Y)$		0,5	0,4	
$\mu(X)$	0,2	0,5	0,6	0,8
$\mu(O)$	0,8			0,2

Fuzzy inference requires a ruleset of the form IF LP IS X AND LE IS Y AND EM IS Y AND CF IS X THEN KO IS Y, formally containing a total of  $4^4 = 256$  inferences. When considering only the “active” rules, 16 inferences remain, which are listed in Table 2.

**Table 2.** Active fuzzy inference rules

№ π/π	Criteria evaluations / Membership function values				Integrated assessment KO/membership function value
	LP	LE	EM	CF	
1.	X/0,2	Y/0,5	Y/0,4	X/0,8	Y/0,2
2.	X/0,2	Y/0,5	Y/0,4	O/0,2	X/0,2
3.	X/0,2	Y/0,5	X/0,6	X/0,8	Y/0,2
4.	X/0,2	Y/0,5	X/0,6	O/0,2	X/0,2
5.	X/0,2	X/0,5	Y/0,4	X/0,8	X/0,2
6.	X/0,2	X/0,5	Y/0,4	O/0,2	X/0,2
7.	X/0,2	X/0,5	X/0,6	X/0,8	X/0,2
8.	X/0,2	X/0,5	X/0,6	O/0,2	X/0,2
9.	O/0,8	Y/0,5	Y/0,4	X/0,8	Y/0,4
10.	O/0,8	Y/0,5	Y/0,4	O/0,2	X/0,2
11.	O/0,8	Y/0,5	X/0,6	X/0,8	X/0,5
12.	O/0,8	Y/0,5	X/0,6	O/0,2	X/0,2
13.	O/0,8	X/0,5	Y/0,4	X/0,8	X/0,4
14.	O/0,8	X/0,5	Y/0,4	O/0,2	X/0,2
15.	O/0,8	X/0,5	X/0,6	X/0,8	X/0,5
16.	O/0,8	X/0,5	X/0,6	O/0,2	O/0,2

#### 4 Results and Future Work

Because the resulting membership function for each subinference corresponds to the minimum of its component membership functions, and the output variable membership

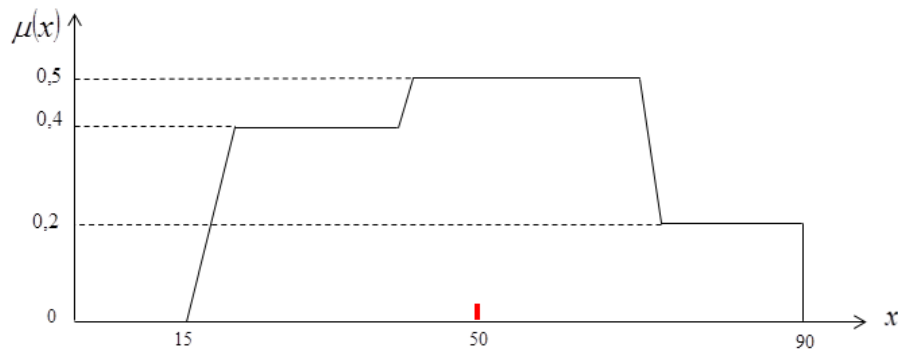
function corresponds to the maximum of the membership functions defined by the sub-inferences being analysed, then it would suffice to leave only those “active” rules which define the maximum value of the membership functions of each of the output variable’s terms.

$$\mu_{KO}(Y) = 0,4; \quad (3)$$

$$\mu_{KO}(X) = 0,5; \quad (4)$$

$$\mu_{KO}(O) = 0,2 \quad (5)$$

The union of the resulting sets corresponds to the resulting output variable membership function. The final value of KO may be calculated using defuzzification on the output fuzzy variable using the “centre of gravity” method [11] (see Fig. 4).



**Fig. 4.** Defuzzification of the “integrated assessment” output linguistic variable

The final result may then be calculated [12]:

$$KO = \frac{\int_{15}^{90} x\mu(x)dx}{\int_{15}^{90} \mu(x)dx} = 50,35 \quad (6)$$

## 5 Conclusion

The resulting value is equivalent to a “good” qualitative assessment. The paper’s results may therefore be formulated thus:

1. Based on fuzzy set theory, a method is presented for determining a quantitative assessment of learning process effectiveness based on the results of qualitative expert assessments of the criteria that form the “integrated assessment”;
2. A fuzzy inference technique is presented that uses an “abbreviated” ruleset, i.e. taking into account only active rules defined by the expert assessments under consideration. The numerical value (defuzzification) of the “integrated assessment” output linguistic variable was obtained using a fuzzy inference algorithm, taking into account disagreements in expert estimates;
3. An example is provided for calculating the integrated assessment of learning process effectiveness criteria.

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