

# Influence of Costs Inflation on the Development of Industries and Spheres of the National Economy\*

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**Abstract.** In the classical version of the model the relationship dynamics of prices, sales, and stocks of goods based on differential equations accounted for only one type of inflation - inflation demand. In conditions of the domestic economy in recent decades up to the present time had to deal with the opposite kind of inflation - inflation costs. This fact determined the directions of the modernization model.

Interpretation of the results of the modernized system of differential equations allows us to draw the following conclusions. After a certain transition process, all dynamic values approach their asymptotic values. The zero solution of the homogeneous system is asymptotically stable. This leads to the fact that at low levels of cost inflation homogeneous part of the solution over time, «dies», and the function of prices, sales volume, and inventory of goods will tend to asymptotic values regardless of the initial conditions. Therefore, within this model it is possible to maintain the inventory level of an item at a certain beforehand specified level, using a flexible mechanism of price changes. If inflation is large, the mathematical model becomes unacceptable.

**Keywords:** dynamics of prices, sales volume, and inventory of goods, inflation costs.

## 1 Introduction

The functioning of a market economy is necessarily accompanied by such an important and mandatory attribute as inflation, which, is an important indicator of economic development. Inflation is a very complex socio-economic phenomenon, which, as a rule, manifests itself in the growth of commodity prices and the relative depreciation of the national currency, and therefore affects the interests of almost every member of our society. Inflation is present to one degree or another in any modern economy. At the same time, inflation rates in different periods in different countries made it one of the

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most acute and critically experienced national problems requiring solutions at the state level. At one time, inflation was declared in the US by President J. Ford was the "enemy of society number one," and President Reagan dubbed it the "most severe tax." During particularly strong inflation, such as in Russia during the Civil War, or Germany in the 1920s, monetary circulation can generally give way to in-kind exchange. There are examples when government policies led to a long period of lower retail prices with higher wages (for example, in the USSR in the last years of JV Stalin's life and under the government of L. Erhard in West Germany starting in 1948).

The inflation problem in the Russian Federation attracts particular attention after the financial crisis of August 17, 1998, when the price of a consumer basket increased 2910 times. Such a colossal increase in inflation significantly affected the economic relations that had developed at that time, complicating them most severely. It was during this period that Russia first encountered the need to pay constant attention to this important factor in the market economy and implement special measures to keep inflation at an acceptable (normally safe) level, which was adopted 3-5% per year.

In Russia, inflation is traditionally higher than in the United States or the European Union. But in recent years, inflation has fallen below the psychological barrier of 10% per year. Any inflation management measures should be based on scientifically-based methods for forecasting and controlling inflation processes occurring in a particular economic system. These aspects determined the relevance of the research.

The analysis of many literary sources showed that today science knows two areas of inflation research - classical and Keynesian. Representatives of the classical school, among which the most striking were foreign authors (A. Smith, J. Boden, J. Vanderlint, D. Hume, F. Galiani, J. Stuart, W. Jevons, K. Marx, A. Wagner, A. Marshall) and domestic (A. Antonovich, V. Bez-obrazov, N. Bunge, I. Gorelov, P. Migulin, I. Vyshnegradsky, S. Witte) argued that price increases are always the result of an increase in the money supply. Their opponents are Keynesian representatives (foreign: J. Ricardo, G. Thornton, Lord King and J. Mill, G. Knaap, F. Bendixen, O. Gein, K. Pixel; domestic: A. Kra-silnikov, S. Sharanov, N. Shavrov, A. Shipov, N. Danilevsky), without denying the general correlation between these two concepts, they proved the presence of other reasons for inflation [1].

And here it should be noted the work of J. Keynes, in which the special role of the state in the economic process was clearly defined — the need to use state power in the field of taxation, expenditures, and monetary policy aimed at ensuring economic stability. [1, 2]

At the same time, Keynes drew attention to the limits of state intervention in the activities of business entities. One of his indisputable quotes should be recognized as follows: "economic ideas govern the actions of political leaders, but these are not always the best of these ideas." Keynes's answer was the doctrine of monetarism, which preaches free competition with minimal government intervention in the activities of business entities. And here one cannot but ignore A. Smith's catchphrase that "the invisible hand of the market" acts as the coordinator of the actions of "independent economic players" [3,4].

The authors of this direction following A. Smith's theory of monetarism criticized Keynesianism as a worldview that considers the economy as a system that does not

have sufficient resources for self-regulation, which requires targeted correction by the state.

The greatest activity of monetarism occurred in the 70s of the twentieth century when a sharp increase in oil prices caused stagflation in most industrialized countries of the world. At the same time, attempts by the governments of these countries to even out fluctuations in market conditions by reducing the budget deficit led only to an increase in the amplitude of cyclical fluctuations in macro parameters. It was during this period that the attention of the central banks of many countries was drawn to the proposals of monetarists to maintain the established rate of increase in money supply following the average GDP growth rates, apply the so-called targeting policy for the growth of monetary indicators [3,4,5].

The experience of applying the US targeting policy was sad when in 1979 its implementation led to such negative consequences as:

- - increase in interest rate volatility;
- - violation of the stability of the speed of money circulation;
- - inflation growth; decline in production;
- - destabilization of the growth rate of the money supply.

Neoclassical and Keynesian approaches became the basis for the formation and development of the theory of demand for money (Keynesian-neoclassical synthesis), which confirmed empirical data on the neutrality of money. Moreover, it was proved that money is not neutral in the relatively short term, but in the long term - it is close to neutrality [Miller, 2000].

The following causes of inflation are distinguished in economic science [6,7]:

1. The growth of government spending, for the financing of which the state is resorting to money emission, increasing the money supply above the needs of commodity circulation. Most pronounced during the war and crisis periods.
2. Excessive expansion of the money supply due to mass lending, and the financial resource for lending is taken not from savings, but the issue of unsecured currency.
3. The monopoly of large firms to determine the price and their costs of production, especially in the commodity sectors.
4. The monopoly of trade unions, which limits the ability of the market mechanism to determine an acceptable level of wages for the economy.
5. A reduction in the real volume of national production, which, given a stable level of money supply, leads to an increase in prices, since the same amount of money corresponds to a smaller volume of goods and services.
6. The depreciation of the national currency with a stable level of money supply and a large volume of imports of goods.
7. An increase in state taxes and duties, excise taxes, etc., with a stable level of the money supply.

The two most important sources of inflation due to rising costs are the increase in nominal wages and prices for raw materials and energy.

One of the leading places in the world economy is occupied by the Russian economic school of the first quarter of the twentieth century, represented by such outstanding scientists as M. Tugan-Baranovsky, E. Slutsky, S. Strumilin, A. Sokolov, and others, who in their works are based on statistical data built economic and mathematical models of economic processes, including models of inflationary processes.

Subsequent studies of inflationary processes led to the conclusion about the non-monetary nature of inflation. We fully support the opinion of many domestic and foreign scientists who state that inflation is a multifactorial process, which is a consequence of the impact on the economic system of a combination of factors that differ both in nature and in terms of the degree of intensity of influence on the system [8].

To assess the impact of inflationary processes on the main economic indicators characterizing macroeconomic dynamics, it is necessary to have models that adequately reflect both the inflationary processes themselves and their relationship with economic dynamics. Therefore, this work aims to develop a mathematical model that satisfies the above requirements.

## **2 Analysis of Existing Approaches to Modeling the Inflation Process**

As for inflation models, they are represented by several options, among which the following are considered classical:

1. Friedman's model, which allows evaluating the "optimal" rate of inflation in terms of the maximum value of real income received from issuing money. The model assumes that the inflation rate does not affect economic growth, and inflation expectations coincide with actual inflation. The model also proceeds from the constancy of the real interest rate, that is, changes in the nominal interest rate are associated only with changes in inflation expectations;
2. The Cagan hyperinflation model is a mathematical model that simplifies the dynamics of inflation when money demand depends only on inflation expectations and in the absence of economic growth. This model describes hyperinflation situations in which inflationary expectations begin to play a decisive role in the economy;
3. Bruno-Fischer's model showing the dependence of inflation, budget deficits, and methods of financing. It is based on a certain dependence of the specific real demand for money on one factor - expected inflation, on the so-called adaptive inflationary expectations. There is a simplified form of this model, in which it is assumed that the entire budget deficit is financed by emission, and complicated, where emission financing of the deficit and financing through borrowing is allowed;
4. Sargent-Wallace model, which is based on rational expectations, where current inflation depends not only on the current (current) but also on the future monetary policy. From the model, in particular, it follows that with a restraining monetary policy, inflation in the future may be greater than with a less rigid policy and current inflation may be higher than with a less restrictive policy;

5. The accelerationist inflation model is a theory that considers the effect of the movement of the inflation rate on the real national product and unemployment. In contrast to the model of aggregate supply and demand, the acceleration model considers not only the change in the general price level but also the speed of this change. The underlying assumption in the model is that the economy does not behave identically during periods when inflation accelerates or slows down and when price increases are stable.

The dynamics of inflationary processes largely determine the perception of the economic and political situation in the country by Russian and foreign expert organizations, including international rating agencies. Inflationary processes are a complex economic phenomenon and an urgent topic for scientific debate. There are many modern theories and models designed to explain the nature and causes of inflation.

So, in [9], the inflation model is built based on the formation of a dynamic function of aggregate demand and a dynamic function of aggregate supply, from the conditions of the interaction of which one can express the inflation rate and obtain a dynamic model for determining the level of inflation. Also, it was taken into account that the inflationary process is cyclical. Therefore, the cyclic component, which can be modeled using the Fourier series for one harmonic, should be included in the equation for inflation. As a result, the mathematical model of the inflation process in the Russian economy can be represented as:

$$\pi_t = a_0 + a_1 M_t + a_2 R_t + a_3 \varepsilon_t + b \cos t + b \sin t + \eta_t$$

where  $M_t$  is the money supply growth rate,  $R_t$  is the interest rate level,  $\varepsilon_t$  is the exchange rate,  $t$  is the period,  $a$  and  $b$  are the Fourier series coefficients,  $\eta_t \sim N(0.0007; 0.97)$ .

It should be noted that the approaches considered do not fully reflect the inflationary processes taking place in the Russian economy, therefore, the authors attempted to develop a more adequate model that will allow for more significant inflationary factors to be taken into account and more significant forecasts and conclusions to be drawn.

The advantage of the approach to building the inflation model described in the work is the accounting of both monetary (growth rate of the money supply) and non-monetary factors (change in autonomous demand) affecting the inflation rate. Also, the model characterizes the development of the inflationary process in time and at the same time, the state of the system in the past and expectations of the future are taken into account.

In the classic version of the model of the relationship between price dynamics, sales, and stocks of goods based on differential equations (1), only one type of inflation was taken into account - demand inflation.

$$\begin{cases} \frac{dI}{dt} = Q - S \\ \frac{dP}{dt} = \alpha(I - I^*) \\ \frac{dS}{dt} = \beta(P - P^*), \end{cases}$$

where  $S$  is the sales volume per unit time,  $P$  is the current price,  $P^*$  is some equilibrium price close to the average market price,  $0 < \alpha$  is the proportionality coefficient between the shortage of goods in the warehouse and the price increase,  $I$  is the current quantity of goods in the warehouse,  $I^*$  is the standard quantity of goods in the warehouse,  $Q$  is the rate of receipt of goods from the manufacturer or supplier,  $0 < \beta$  is the proportionality coefficient between the deviation of the current price  $P$  from the equilibrium value of  $P^*$  and the change in the rate of sales.

The first of equations (1) describes the balance of goods in a warehouse: a change in the number of goods in a warehouse  $I'$  is related to the rate of goods received from a manufacturer or supplier  $Q$  and the speed of sale of goods  $S$ . The second of equations (1) describes the relationship between the price of goods and its quantity in a warehouse: with its deficit  $I < I^*$ ,  $P' > 0$  - the price increases, with an excess of  $I > I^*$ ,  $P' < 0$  - the price decreases. The last of equations (1) describes the relationship between the sales rate  $S'$  and the deviation of the current price  $P$  from the equilibrium value  $P^*$ . For  $P < P^*$ ,  $S' > 0$  - the sales rate increases at a price lower than the equilibrium one and decreases at a price higher than the equilibrium one, since the coefficient  $\beta$  is negative. It models an aggressive marketing strategy that is often used, for example, in the sales season [10, 11].

In the conditions of the domestic economy of recent decades, up to the present time, one has to deal with the opposite type of inflation - cost inflation.

The theory of cost inflation is an economic theory, suggesting that the main reason for the increase in prices is the increase in production costs.

In conditions of cost inflation, the growth of the money supply is not the cause of price increases, but a consequence of inflation. Initially, an increase in prices is carried out based on increased average production costs due to an increase, for example, in prices and tariffs for goods and services of natural monopolies. Then nominal GDP increases and demand for money increases, which leads to an increase in the money supply. These economic processes can be observed over a certain period and affect the quality and reliability of the obtained estimates of the influence of individual factors on the value of inflation. It should be borne in mind that the influence of certain factors on the rate of inflation can occur with some lag. Various works devoted to the analysis of the influence of various factors on inflation in the domestic economy contain data on lags from 3 to 12 months. On the other hand, inflation expectations also determine the exact opposite trend - the anticipation of future inflation factors on the dynamics of current prices. This, of course, does not mean that the influence of various factors does not appear in a longer period. The process of reproduction is continuous, and changes in the present period are inextricably linked with those processes that took place in the distant past.

In foreign theories of cost inflation, based on the tenets of the theory of A. Smith, the most significant are three factors (their dynamics): wages, profits, prices of imported labor items (raw materials, energy, etc.).

One of the directions of the theory of cost inflation was a concept that linked the growth of wages (labor costs) and price increases. An attempt to prove the effect of wages on price increases was made back in the 19th century. The idea of this concept is that wage growth if it occurs during periods of cyclical decline in production, can

increase costs and prices. This idea is as follows: unions are seeking an “excessive” increase in wages, which drastically reduces profits and provokes a rise in prices as a response of entrepreneurs to rising labor costs. Rising prices reduce real wages, as a result of which trade unions again put forward demands for higher wages, etc. This doctrine is called the spiral of “wages - prices.” In the words of the English economist J. Trevitik, “this process resembles a cat chasing its tail with an ever-increasing speed.” Modern inflation, according to the American economist J. Tobin, is explained by the “concentration of economic power in the hands of large companies and trade unions.” These powerful monopolies do not obey the laws of competition in setting wages and prices. Unions raise wages beyond what would be competitive. Moreover, they are trying to get most of the profits of those monopolies and oligopolies with which they are negotiating. But in this regard, they do not succeed, because companies simply shift the increased costs of their wages with the help of higher prices for defenseless customers [12,13].

This interpretation of the causes of inflation does not take into account the influence of monopolies on the general dynamics of prices under the influence of the desire to receive super-profits. Within the framework of this concept, it is proved that monopolies are interested in achieving a minimum rate of profit, and price increases are dictated solely by increased costs. The most important role in the spiral mechanism “wages - prices” is played by labor productivity:

- with its rather high growth, the pressure of wages on prices softens;
- with a small increase in labor productivity, not to mention stability or a decrease in this indicator, the pressure of rising wages on prices is noticeably increased.

A certain role in increasing price changes may play an increase in profits. In the West, there was even such a thing as “administered inflation”, understood as a price increase, caused mainly by the policy of company management (their administrations) to increase the mass and rate of return. An important condition for “administered inflation” is the presence of serious market power in the group of the largest producers in most sectors of the modern economy (in the so-called oligopolistic structures of these sectors) [14, 15].

The third area of the theory of cost inflation-linked inflationary processes with rising prices for imported goods. This factor of cost inflation often plays a rather important role under the influence of two decisive reasons: with a marked increase in prices for energy and raw materials on the world market (as was observed in the 70s of the 20th century) and as a result of changes in the exchange rate of the national currency for the dollar and others hard currencies. When the dollar exchange rate falls, all imported goods rise in price, which undoubtedly leads to an increase in domestic inflation (the more goods a country imports; moreover, the increase in the price of imported objects of labor through the impact on rising costs of goods produced from them cumulatively increases inflation effect of more expensive imports). With an increase in the exchange rate through cheaper imports, the reverse anti-inflation effect is manifested.

In the 60s - early 70s of XX century the theory of cost inflation experienced a rebirth called structuralism. His key idea was to explain inflation with specific structural country factors, such as the ratio of the “progressive” (industrial) sector to the “traditional” (export and agricultural) sectors of the economy [16, 17].

In this regard, more attention should be paid both theoretically and in practical terms to the role of cost inflation in the modern economy.

One of the most famous representatives of the monetarist theory M. Friedman believes that the impact of cost inflation is completely negligible. However, the experience of the domestic economy in the 90s indicates that the increase in costs caused by the increase in prices for materials and services does not boil down to a temporary increase in the general price level. A drop in production, a decrease in the share of capacities used, their obsolescence, and retirement leads to a permanent increase in total average costs, which causes a new round of price growth.

The significance of the impact of cost inflation, the price level of goods and services, is confirmed by the analysis of statistical data. The dynamics of the inflation rate (calculated by the consumer price index) and the producer price index of the most important raw materials, goods, and services in 1995–2014 are closely correlated over this period.

So, in 1995, the highest inflation rates compared to the previous period were observed in the 1st quarter of the year, when the producer price indices in the fuel industry and freight transportation tariffs also had the maximum value during the year, and the producer price index in the electric power industry was the second the largest during the year. In 1997, the highest inflation rates were also in the 1st quarter, when producer price indices in the electric power industry and the fuel industry had the maximum value. Since 1999, the importance of monetary factors has increased in connection with the favorable dynamics of world prices for oil and other export goods from Russia. Nevertheless, the cost factors have retained their significance, although under the conditions of economic growth their influence is objectively reduced. However, in 2001 and 2003, the highest inflation rates took place in the 1st quarter, when the index of producers in the electric power industry showed the highest rates of price growth. In 2002, the maximum value at the highest inflation rates was the freight tariff index. In 2004, a similar synchronism of indicators was observed for consumer price and producer indices in the electric power industry and the fuel industry. Also, in the 2000s, the inflation rate was significantly affected by a significant increase in utility tariffs.

The neglect or even complete denial of the impact of cost inflation, by many scientists and decision-makers, stems from the postulates of the monetarist theory that relates the dynamics of inflation to the characteristics of the money circulation system. It is based on the classical quantitative theory of money. The relationship of money supply and the price level is represented by the I. Fisher exchange equation:

$$MV = PY,$$

where M is the amount of money in circulation, V is the speed of their circulation, P is the price level,

Y is the amount of output (real GDP).

Irving Fisher based on this equation assumes that price dynamics show an increase or decrease both in direct proportion to the amount of money and their velocity and

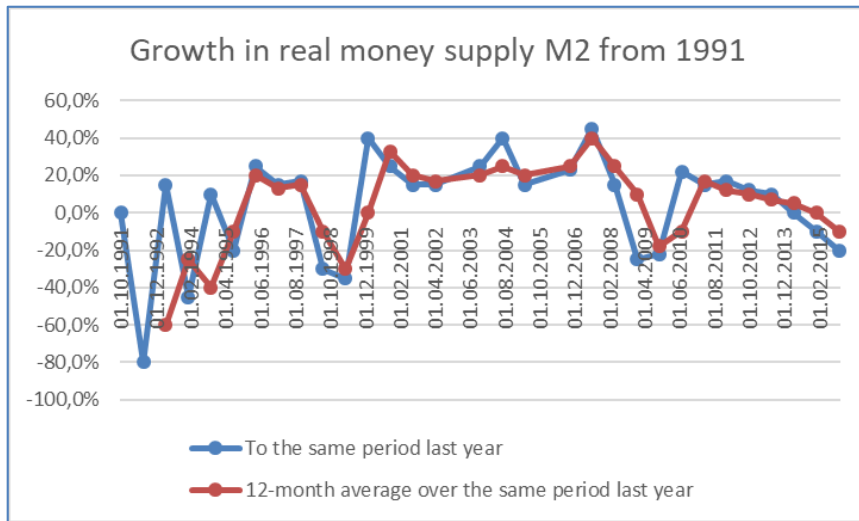


inversely to the volume of trading performed with money. However, data on the development of the real Russian economy do not indicate a proportional change in money supply growth and price levels. In 1994-1997, there was a direct correlation between the growth rate of the M2 monetary aggregate and inflation. But this can be considered proportional only during the jumping inflation of 1994-1995. So, in 1994, the inflation rate in the consumer price index (CPI) was 215% with an increase of M2 by 194.5%, in 1995 by 131% and 135.7%, respectively. In 1996-1997, these changes were not proportional. In 1996, with an increase in the M2 monetary aggregate by 30.8%, the inflation rate was 21.8%, in 1997 - 29.8% and 11.0%, respectively. In 1998, despite a significant decrease in the growth rate of M2 compared to the previous period to 19.8%, the inflation rate increased significantly - up to 84.4%. In 1999, 2000, and 2003, the money supply compared to the previous year increased by 57.2%, 61.5%, and 50.5%, respectively, and the inflation rate decreased - by 36.5%, 20.2%, and 12, 0%.

In 2001, 2002, 2004, 2009, 2013, 2014, 2014-2017 the analyzed indicators changed in comparison with the previous year in one direction: the growth rate of M2 decreased (39.7; 32.4 and 35.8%), and the inflation rate decreased (18.6; 15.1 and 11.7%). Thus, in 1997-2017. There was not only a proportional, but also, in some years, the direct correlation between the growth rate of the M2 money aggregate and the dynamics of inflationary processes, which casts doubt on the purely monetary nature of Russian inflation.

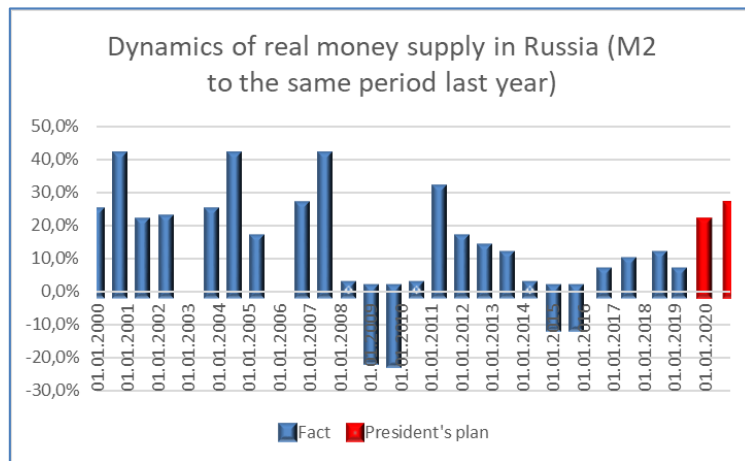
In theoretical terms, this is because the equation of exchange itself is not a fully adequate model for the balance of money supply at present. The relationship between the volume and speed of the money supply and the price level, of course, exists, but this does not mean their inevitable proportional change. Firstly, only part of the increase in the money supply goes to the purchase of goods and services, and part - to savings. Secondly, the dynamics of money circulation changes over time. Thirdly, it is necessary to take into account the use of foreign currency in the national economy.

Under the pretext of fighting inflation, monetary authorities slowed down the growth rate or reduced the real money supply, which always led to a reduction in production and crisis phenomena in the Russian economy (Fig. 1, 2). If the growth rate is less than 20 %, then within six months there will be a reduction in production in sectors producing investment goods (for example, machine tools, locomotives, trucks) [18, 19].



**Fig. 1.** The relationship of the dynamics of changes in real money supply and crisis phenomena in the Russian economy

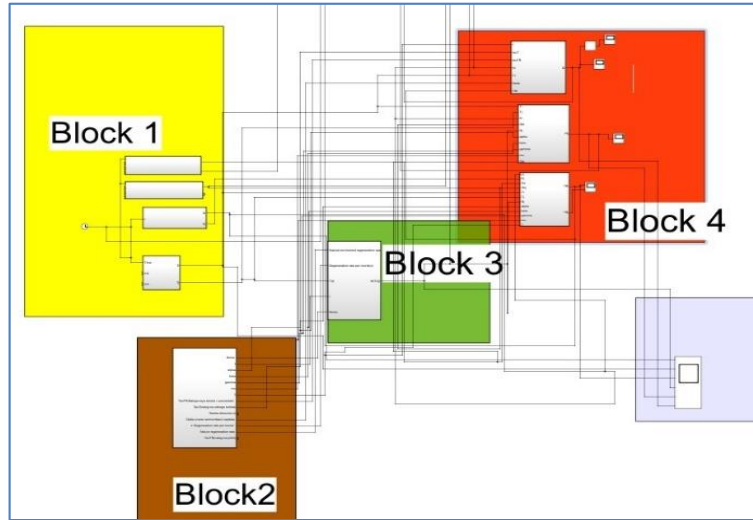
All of the above has determined the importance of taking into account the factor of cost inflation in the model of the relationship between price dynamics, sales, and stocks of goods.



**Fig. 2.** Dynamics of real money supply in Russia (M2 to the same period last year)

Data explain well both the ups and downs of the Russian economy.

Fig. 3 shows a block diagram characterizing the influence of the money supply and other basic parameters on the main indicators of economic development. To create the model, the Simulink environment was used, included in the Matlab package - version 2016b.



**Fig. 3.** Block diagram characterizing the influence of the money supply and other basic parameters on the main indicators of economic development

Part of the program code of the model is presented in Table 1.

**Table 1.** Part of the program code of the model

Parameter	Value
Plot type	Stair
X value from	Event time
Open scope at the start of the simulation	on
Initial X-axis lower limit	0
Initial X-axis upper limit	10
If X value is beyond the limit	Stretch axis limits
Initial Y-axis lower limit	0
Initial Y-axis upper limit	10
If Y value is beyond the limit	Stretch axis limits
Show grid	on
Figure position	se_figureposition([5 60 25 25])
Show number of points	off
Store data when the scope is closed	Limited
Limit data points to	10000
Number of points plotted, #c	off
Line colors	b
Line styles	-
Line markers	.
Show toolbar	on

To solve this problem, instead of the system of differential equations (1), the following system of equations is proposed:

$$\begin{cases} \frac{dI}{dt} = Q - S \\ \frac{dP}{dt} = -\alpha(I - I^*) + \gamma P \\ \frac{dS}{dt} = -\beta(P - P^*) \end{cases} \quad (1)$$

here  $\gamma$  is the cost inflation coefficient,  $\alpha$  and  $\beta$  are positive values, the remaining notation is the same as in system (1). An additional term in the second equation of system (2):  $\gamma P$  gives an increase in price due to inflation  $e^{\wedge} \gamma t$ .

The system of equations (2) can be represented as follows:

$$\dot{\vec{X}} = A\vec{x} + \vec{b}, \text{ где } \vec{X}(t) = \begin{pmatrix} I(t) \\ P(t) \\ S(t) \end{pmatrix}, \vec{b} = \begin{pmatrix} Q \\ \alpha I^* \\ \beta P^* \end{pmatrix}, A = \begin{pmatrix} 0 & 0 & -1 \\ -\alpha & \gamma & 0 \\ 0 & -\beta & 0 \end{pmatrix} \quad (2)$$

The solution to equation (3) will be sought in the form

$$\vec{X}(t) = \vec{X}_0(t) + \vec{y}, \quad (3)$$

where  $\vec{X}_0(t)$  is the solution of the equation:

$$\begin{aligned} \dot{\vec{X}}_0(t) &= A\vec{X}_0(t), \\ \text{at } \vec{X}_0(0) &= \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix}. \end{aligned} \quad (4)$$

Substituting equality (4) into equation (3), we obtain the condition for determining  $\vec{y}$ :

$$\dot{\vec{X}}(t) = \dot{\vec{X}}_0(t) = A\vec{X}_0(t) + A\vec{y} + \vec{b} = A\vec{X}_0(t), \quad (5)$$

which gives the equation for finding  $\vec{y}$ :

$$A\vec{y} = -\vec{b}. \quad (6)$$

We start by solving equation (5), which can be represented as:

$$\vec{X}_0(t) = \exp\{At\} \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix}. \quad (7)$$

Represent the exponential form of the formula (7) as a series:

$$\exp\{At\} = a(t)\mathbf{1} + b(t)A + c(t)A^2, \quad (8)$$

here  $\mathbf{1}$  is the block operator. The eigenvalues of the matrix  $A$  are found from the equation:

$$0 = |A - \lambda \mathbf{1}| = \begin{vmatrix} -\lambda & 0 & -1 \\ -\alpha & \gamma - \lambda & 0 \\ 0 & -\beta & -\lambda \end{vmatrix} = \begin{vmatrix} \alpha & \gamma - \lambda \\ 0 & -\beta \end{vmatrix} + \lambda \begin{vmatrix} \lambda & 0 \\ \alpha & \gamma - \lambda \end{vmatrix} =$$

$$= -\lambda^3 + \gamma\lambda^2 - \alpha\beta \quad (9)$$

The root of this equation can be represented as an expansion in the parameter  $\gamma$ :

$$\lambda = -\sqrt[3]{\alpha\beta} + \mu\gamma \quad (10)$$

and substitute (10) in equation (9):

$$(-\sqrt[3]{\alpha\beta} + \mu\gamma)^3 - (-\sqrt[3]{\alpha\beta} + \mu\gamma)^2 + \alpha\beta = 0 \quad (11)$$

Solving this equation for  $\mu$  (taking into account the fact that  $\gamma$  has the first order of smallness), we obtain  $\mu = \frac{1}{2}$  and, therefore, from formula (10) we obtain the expression:

$$\lambda = -\sqrt[3]{\alpha\beta} + \frac{1}{2}\gamma. \quad (12)$$

Substituting expressions (11) and (12) into equation (8), differentiating (8), and presenting the result of this differentiation concerning the operator  $A$ , we obtain:

$$\begin{cases} \exp\{At\} = a\mathbf{1} + bA + cA^2 \\ t\exp\{At\} = b + 2cA \\ t^2\exp\{At\} = 2c. \end{cases} \quad (13)$$

Acting on the right and left sides of formulas (13) on any of the eigenvectors of the operator  $A$ , we obtain a system of equations for determining the quantities  $a$ ,  $b$ , and  $c$  in the system of equations (8):

$$\begin{cases} \exp\{\lambda t\} = a + b\lambda + c\lambda^2 \\ t\exp\{\lambda t\} = b + 2c\lambda \\ t^2\exp\{\lambda t\} = 2c. \end{cases} \quad (14)$$

The system of equations (14) is easily solved:

$$c = \frac{t^2}{2}\exp\{\lambda t\}, b = (t - \lambda t^2)\exp\{\lambda t\}, a = \left(1 - \lambda t + \frac{\lambda^2 t^2}{2}\right)\exp\{\lambda t\},$$

which gives an expression for the exponent in the formula (8) of the expression:

$$\exp\{At\} = \exp\{\lambda t\} \left\{ \left(1 - \lambda t + \frac{\lambda^2 t^2}{2}\right) \mathbf{1} + (t - \lambda t^2)A + \frac{t^2}{2}A^2 \right\}, \quad (15)$$

which allows us to give the formula (7) of the form:

$$\vec{X}_0(t) = \exp\{At\} \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} =$$

$$= \exp\{\lambda t\} \left\{ \left(1 - \lambda t + \frac{\lambda^2 t^2}{2}\right) \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} + (t - \lambda t^2) A \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} + \frac{t^2}{2} A^2 \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} \right\}, \quad (16)$$

Find a solution to equation (6).

Let the vector  $\vec{y}$  have the form

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}, \text{ then}$$

$$A \vec{y} = \begin{pmatrix} 0 & 0 & -1 \\ \alpha & \gamma & 0 \\ 0 & -\beta & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = -\vec{b} = \begin{pmatrix} -Q \\ -\alpha I^* \\ -\beta P^* \end{pmatrix}.$$

From here we get  $-y_3 = -Q$ ,  $-\beta y_2 = -\beta P^*$ ,  $-\alpha y_1 + \gamma y_2 = -\alpha I^*$  which gives expressions for the components of the vector  $\vec{y}$ :

$$y_1 = I^* + \frac{\gamma}{\alpha} P^*, \quad y_2 = P^*, \quad y_3 = Q.$$

So, the vector  $\vec{y}$  has the form:  $\vec{y} = \begin{pmatrix} I^* + \frac{\gamma}{\alpha} P^* \\ P^* \\ Q \end{pmatrix}$ .

Thus, the final solution of equation (3) can be represented:

$$\begin{aligned} \vec{X}(t) &= \begin{pmatrix} I(t) \\ P(t) \\ S(t) \end{pmatrix} = \vec{x}_0(t) + \vec{y} = \\ &= \exp\{\lambda t\} \left\{ \left(1 - \lambda t + \frac{\lambda^2 t^2}{2}\right) \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} + (t - \lambda t^2) A \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} + \frac{t^2}{2} A^2 \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} \right\} + \begin{pmatrix} I^* + \frac{\gamma}{\alpha} P^* \\ P^* \\ Q \end{pmatrix} = \\ &\exp\{\lambda t\} \left\{ \left(1 - \lambda t + \frac{\lambda^2 t^2}{2}\right) \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} + (t - \lambda t^2) \begin{pmatrix} -S_0 \\ \gamma P_0 - \alpha I_0 \\ -\beta P_0 \end{pmatrix} + \frac{t^2}{2} \begin{pmatrix} \beta P_0 \\ \alpha S_0 + \gamma^2 P_0 - \\ \alpha \beta I_0 - \gamma \beta P_0 \end{pmatrix} \right\} + \begin{pmatrix} I^* + \frac{\gamma}{\alpha} P^* \\ P^* \\ Q \end{pmatrix}. \end{aligned} \quad (17)$$

The following expressions are used here:

$$A \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} = \begin{pmatrix} -S_0 \\ \gamma P_0 - \alpha I_0 \\ -\beta P_0 \end{pmatrix} \quad \text{и} \quad A^2 \begin{pmatrix} I_0 \\ P_0 \\ S_0 \end{pmatrix} = \begin{pmatrix} \beta P_0 \\ \alpha S_0 + \gamma^2 P_0 - \alpha \gamma I_0 \\ \alpha \beta I_0 - \gamma \beta P_0 \end{pmatrix}.$$

The components of a vector (17) give a solution to the problem:

$$I(t) = \exp\{\lambda t\} \left\{ \left(1 - \lambda t + \frac{\lambda^2 t^2}{2}\right) I_0 - (t - \lambda t^2) S_0 + \beta \frac{t^2}{2} P_0 \right\} + I^* + \frac{\gamma}{\alpha} p^* ; \quad (18)$$

$$P(t) = \exp\{\lambda t\} \left\{ \left(1 - \lambda t + \frac{\lambda^2 t^2}{2}\right) P_0 + (t - \lambda t^2)(\gamma P_0 - \alpha I_0) + \frac{t^2}{2} (\alpha S_0 + \gamma^2 P_0 - \alpha \gamma I_0) \right\} + p^* ; \quad (19)$$

$$S(t) = \exp\{\lambda t\} \left\{ \left(1 - \lambda t + \frac{\lambda^2 t^2}{2}\right) S_0 - \beta (t - \lambda t^2) P_0 + \frac{t^2}{2} (\alpha \beta I_0 - \gamma \beta P_0) \right\} + Q . \quad (20)$$

Formulas (18), (19), and (20) describe the behavior of the functions  $I(t)$ ,  $P(t)$ , and  $S(t)$  depending on the parameters of the problem.

The model contains six parameters:  $\alpha, \beta, \gamma, I^*, p^*, Q$  and three initial values of the variables:  $I_0, P_0, S_0$ . Since the exponent in formulas (18), (19), and (20)  $\lambda$  is equal to:

$$\lambda = -\sqrt[3]{\alpha\beta} + \frac{1}{2}\gamma, \quad (21)$$

moreover,  $\alpha, \beta$  and  $\gamma$  are greater than zero, then if cost inflation is small, namely, the inflation coefficient  $\gamma$  satisfies the condition:

$$\gamma < 2\sqrt[3]{\alpha\beta}, \quad (22)$$

then  $\lambda < 0$  and the dynamic quantities  $I(t)$ ,  $P(t)$  and  $S(t)$  tend to their asymptotic values regardless of the initial conditions:

$$\lim_{t \rightarrow \infty} I(t) = I^* + \frac{\gamma}{\alpha} p^*, \quad \lim_{t \rightarrow \infty} P(t) = p^* \quad \text{и} \quad \lim_{t \rightarrow \infty} S(t) = Q. \quad (23)$$

### 3 Conclusions

The interpretation of the results of solving the modernized system of differential equations allows us to draw the following conclusions. After a certain transition process, all dynamic quantities approach their asymptotic values. The zero solution of a homogeneous system is asymptotically stable.

This leads to the fact that with a low level of cost inflation, the homogeneous part of the solution fades with time, and the functions of prices, sales volumes, and stocks of goods will tend to asymptotic values regardless of the initial conditions. Therefore, within the framework of this model, it is possible to maintain the level of stocks of goods at a predetermined level using a flexible mechanism for changing prices.

If inflation is high, that is,  $\gamma \geq 2\sqrt[3]{\alpha\beta}$ , then this mathematical model becomes unacceptable, and we need to move from a system of linear equations (2) to a system of nonlinear equations that refine the mathematical model (2).

Foreign and domestic experience shows that many proven recommendations take into account the cost inflation factor, how to use money supply growth to ensure effective growth of the national economy. In particular, this increase may be aimed at increasing government spending.

Of course, any increase in budget expenditures leads to a multiplicative increase in nominal GDP and the aggregate demand of the population and companies. But there are certain anti-inflation rules:

government spending should grow smoothly, given the possibility of using unused production not any budget expenditures are subject to growth, but mainly those that can cause a multiplicative development of production. This includes a state order for products, for example, material-intensive and unloaded production.

Such a state order directly leads to an increase in production by an amount equal to the volume of the state order, an increase in demand for related products, as well as consumer demand, a revival of investment activity, and an expansion of the tax base.

In foreign practice, these rules are applied as “stimulating low inflation” and are known, first of all, thanks to John Maynard Keynes and the experience of overcoming the Great Depression in the USA (1929-1933). Typically, depression was overcome due to an increase in the amount of money in circulation, but it began, according to M. Friedman, due to a decrease in the amount of cash in the United States by a third.

The most acceptable recipes for anti-crisis policies are combinations of projects to expand housing construction and infrastructure development (transport, information, etc.). The implementation of such projects can bring the greatest multiplier effect, reduce overall production costs. These include high-tech projects, primarily in the defense sector, followed by the transfer of dual-use technologies to the civilian sector.

It is also necessary to ensure future investment growth mainly due to domestic equipment and technological lines to protect the national investment sector of machine tools. We emphasize that the growth of investment in 2000-2008. It was accompanied by the accelerated import of foreign equipment. Despite the importance of the credit and financial sector, services, material, industrial, agricultural production are the basis for the development of economic service sectors (both at the regional, national, and global levels). Therefore, crisis phenomena arise precisely on this basis, for example, due to a decrease in the efficiency of the material sector. This means that anti-crisis policy should be aimed, first of all, at transforming real production.

According to R. Harrod, a classic of growth theory, there is another important reason for the anti-inflationary impact of government orders during the crisis period. “If the aggregate demand in the economy is less than the supply potential, a demand reduction will increase unit costs and contribute to price inflation. Conversely, increased demand helps lower unit costs, as the curve drops as you approach the optimal output point on the left.”

Thus, investments in high-tech projects with long payback periods bring a long-term effect, increase aggregate demand, but do not provide a sharp development of production and expansion of the tax base. Projects with extremely quick returns include investments in projects that provide, in a short time, an increase in export deliveries and the production of consumer goods (for example, food, clothing) with a quick return on cash. The task of a specific investment state and regional investment policy is to determine the proportions between these types of projects.



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