Cordial Labeling of Some Pan Graphs

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Abstract
We introduce cordial labeling of path joining two copies m-pan graph, when (m = 3, 4). We also investigated cordial labeling of path union of r-copies of 3-pan graph is cordial only for r ≡ 0, 1,2(mod4).

Keywords 1
Pan graph, path union of 3-pan graph, cordial graph

1. Introduction

Graph theory plays an important role in the field of mathematics, computer science, operation research, physics, chemistry, biology in general and widely in communication. In the field of graph theory labeling has multiple application like networking, data base management, circuit designing, coding, communication networking. Graph labeling is a function that carries set of vertices or edges to non-negative integers. Throughout the paper K is finite, simple and undirected. We follow Harary [5] for graph theory related basic terminology and notations. The idea of cordial labeling was first invented by Cahit [1] 1987.

2. Terminology and Notation

2.1. Definition

Let K = (V, E) be a graph and f : V (K) → {0, 1} is said to be a (0−1) or a binary vertex labeling of K and the vertex label of K under f is represented by f(l). Consider the induced mapping f*(kl) : E(K) → {0, 1} and derived as f*(kl) = |f(k) − f(l)| where e = kl. We represent, l_f (h) as for any vertex v ∈ V (K) such as f(v) = h and e_f (h) as for any edge e ∈ E(K) such as f*(e) = h, where h ∈ {0, 1}.

A labeling (0 − 1) of a graph K is a cordial labeling if following criteria satisfies

a) $|l_f(0) - l_f(1)| \leq 1$

b) $|e_f(0) - e_f(1)| \leq 1$. A graph K which preserves cordiality known as a cordial graph. [6]

2.2 Definition

A resultant graph K is called m-pan graph if it is constructed with joining a cycle graph C_m to a pendent vertex.

WCNC-2021: Workshop on Computer Networks & Communications, May 01, 2021, Chennai, India.
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2.3 Definition

A path union of K is constructed with attaching r copies of 3-pan graph with the path of size $r - 1$.

3. Results

Theorem 3.1. A graph constructed with attaching 2 copies of 3-pan graph by a path of distinct size preserves cordial labeling.

Proof. We shall denote K as a resultant graph constructed with attaching 2 copies of 3-pan graph by a path of distinct size. Let distinct vertices of 1st and 2nd copies of 3-pan graph be denoted by l_1, l_2, and l_3 and k_1, k_2 and k_3 respectively. Let vertices 1st, 2nd and rth of path P_r be represented by $x_1, x_2, \ldots, x_r$ which carries condition as first vertex x_1 coincides with l_1 and rth vertex, x_r coincides with k_1. Define a vertex labeling mapping $f : V(K) \rightarrow \{0, 1\}$ as given below

Case I for $r \equiv 1(\text{mod}4)$

$$f(l_h) = \begin{cases} 0\,;\ \mathrm{h} = 1(\mathrm{mod}4) \\ 1\,;\ \mathrm{h} = 2,3(\mathrm{mod}4) \end{cases}$$

$$f(k_h) = \begin{cases} 0\,;\ \mathrm{h} = 1(\mathrm{mod}4) \\ 1\,;\ \mathrm{h} = 2,3\ (\mathrm{mod}4) \end{cases}$$

$$f(x_h) = \begin{cases} 0\,;\ \mathrm{h} = 0,1(\mathrm{mod}4) \\ 1\,;\ \mathrm{h} = 2,3(\mathrm{mod}4) \end{cases}$$

Case II For $r \equiv 2(\text{mod}4)$

$$f(l_h) = \begin{cases} 0\,;\ \mathrm{h} = 1(\mathrm{mod}4) \\ 1\,;\ h = 2,3(mod4) \end{cases}$$

$$f(k_h) = \begin{cases} 0\,;\ h = 1(mod4) \\ 1\,;\ \mathrm{h} = 2,3(\mathrm{mod}4) \end{cases}$$

$$f(x_h) = \begin{cases} 0\,;\ \mathrm{h} = 1,2(\mathrm{mod}4) \\ 1\,;\ h = 0,3(mod4) \end{cases}$$

Table 1: Edges Labeling Pattern

Observations of r	Labeling of Edges
$r \equiv 1(mod4)$	$e_f(0) = e_f(1)$
$r \equiv 2(mod4)$	$e_f(0) = e_f(1) + 1$

Above table elaborate labeling pattern of edges. Clearly the condition of cordiality $|e_f(0) - e_f(1)| \leq 1$ is satisfied. Hence, the resultant graph K is cordial.

Example 3.1. The cordial labeling resultant graph K under certain condition r ≡ 1(mod4) is constructed with Figure 1.

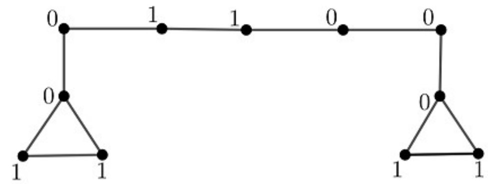

Figure 1: Path joining of 2 copies of 3-pan graph

Theorem 3.2. A graph constructed with attaching 2 copies of 4-pan graph by a path of distinct size preserves cordial labeling.

Proof. We shall denote K as a resultant graph constructed with attaching 2- copies of 4-pan graph by a path of distinct size. Let distinct vertices of 1^{st} and 2^{nd} copies of 4-pan graph be denoted by l_1, l_2, l_3 and l_4 and k_1, k_2, k_3 and k_4 respectively. Let the vertices 1^{st}, 2^{nd} and r^{th} of path P_r be represented by $x_1, x_2, \ldots, x_r$ which carries condition as first vertex x_1 coincides with l_1 and r^{th} vertex, x_r coincides with k_1.

Define a vertex labeling mapping $f : V(K) \to \{0, 1\}$ as given below:

For $r \equiv 0(\mathrm{mod}4)$

$$f(l_h) = \begin{cases} 0\,;\; h = 1, 2(\mathrm{mod}4) \\ 1\,;\; h = 0, 3(\mathrm{mod}4) \end{cases}$$

$$f(k_h) = \begin{cases} 0\,;\; h = 1, 2(\mathrm{mod}4) \\ 1\,;\; h = 0, 3(\mathrm{mod}4) \end{cases}$$

$$f(x_h) = \begin{cases} 0\,;\; h = 0, 1(\mathrm{mod}4) \\ 1\,;\; h = 2, 3(\mathrm{mod}4) \end{cases}$$

Table 2: Edges Labeling Pattern

Observations of r	Labeling of Edges
$r \equiv 0(mod4)$	$e_f(0) = e_f(1) + 1$

Above table elaborate labeling pattern of edges. Clearly the condition of cordiality $|e_f(0) - e_f(1)| \leq 1$ is satisfied. Hence, the resultant graph K is cordial.

Example 3.2. The cordial labeling of the resultant graph K under certain condition r ≡ 0(mod4) is elaborated by **Figure 2.**

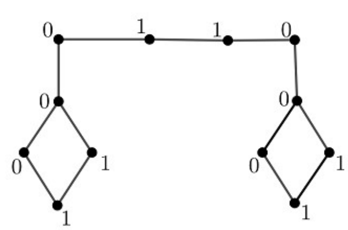

Figure 2: Path joining of 2 copies of 4-pan graph

Theorem 3.3. Any path-union of r-copies of 3-pan graph admits cordiality.

Proof. Consider K as a resultant graph constructed with attaching r copies of 3-pan graph by a path of distinct r − 1 size. Let the vertices of 1st, 2nd , . . . , rth copy graph be $l_{h1}, l_{h2}, \ldots, l_{hr}$. Let $x_1, x_2, \ldots, x_r$ be the vertices of path P_r with condition $x_1 = l_{h1}, x_2 = l_{h2}, \ldots, x_r = l_{hr}$.

Define a vertex labeling mapping $f : V(K) \rightarrow \{0, 1\}$ is stated below

Case I: For $r \equiv 0(mod4)$

$$f(l_{hq}) = \begin{cases} 0 ; \ h = 1(\mathrm{mod}4) \\ 1 ; \ h = 2,3(mod4) \end{cases}$$

$$f(l_{hq-2}) = \begin{cases} 0 ; \ h = 2,3(\mathrm{mod}4) \\ 1 ; \ h = 1(\mathrm{mod}4) \end{cases}$$

Case II: For $r \equiv 1(mod4)$

$$f(l_{hq}) = \begin{cases} 0 ; \ h = 1(\mathrm{mod}4) \\ 1 ; \ h = 2,3(mod4) \end{cases}$$

$$f(l_{hq-2}) = f(l_{hq-3}) = \begin{cases} 0 ; \ h = 2,3(\mathrm{mod}4) \\ 1 ; \ h = 1(\mathrm{mod}4) \end{cases}$$

Case III: For $r \equiv 2(mod4)$

$$f(l_{hq}) = \begin{cases} 0 ; \ h = 1(mod4) \\ 1 ; \ h = 2,3(mod4) \end{cases}$$

$$f(l_{hq-2}) = f(l_{hq-3}) = \begin{cases} 0 ; \ h = 2,3(\mathrm{mod}4) \\ 1 ; \ h = 1(mod4) \end{cases}$$

Table 3: Edges and Vertices Labeling

Observations of r	Labeling of Vertices	Labeling of Edges
$0\ (mod\ 4)$	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
$1\ (mod\ 4)$	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
$2\ (mod\ 4)$	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$

Above table elaborate labeling pattern of edges. Clearly the condition of cordiality $|e_f(0) - e_f(1)| \leq 1$ is satisfied. Hence, any path union of r-copies of 3-pan graph admits cordial labeling.

Example 3.3 Cordial labeling of path-union of r-copies of 3-pan graph under certain condition $r \equiv 2(mod4)$ is shown in Figure 3.

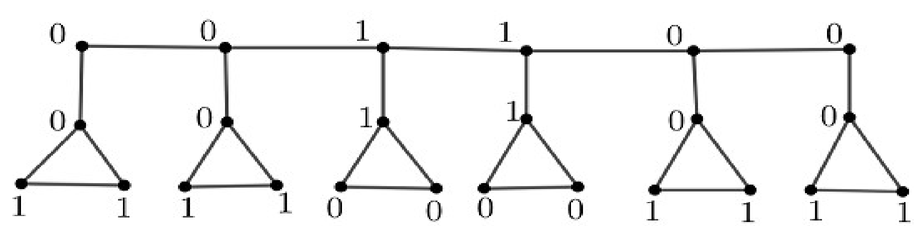

Figure 3: Path union of 6-copies of 3-pan graph

4. Conclusion

In this paper we emphasized path joining two copies m-pan graph and path-union of r copies of 3-pan graph is cordial. To elaborate several families of graph which satisfies corresponding results is an open problem for the further research. In the field of graph theory labeling has multiple application like networking, data base management, circuit designing, coding, communication networking. Graph theory plays an important role in the field of mathematics, computer science, operation research, physics, chemistry, biology in general and widely in communication.

5. Acknowledgement

Author and co Author would like to thank Cahit. I. for introducing the concept of cordial labeling, using that we have generated cordial labeling of some pan graphs.

6. References

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