Enhancing the spatial resolution of digital images using circular scanning

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Abstract

The article describes original algorithms developed to improve the spatial resolution of digital images using a circular scan procedure. Examples of image reconstruction in the presence and absence of noise are given, and the error in reconstructing the original signal is calculated.

Keywords

Image processing, circular scanning, enhancing of spatial resolution.

1. Introduction

In many scientific and technical fields and practical applications of computer science, it becomes necessary to obtain an ideal initial signal (image) from the results of measurements carried out using low-resolution photo matrices. The use of photo matrixes with high spatial resolution is often hampered by high cost, and sometimes (for example, for infrared matrices) difficulties are associated with technological problems in the manufacture of such receivers.

The complexity of increasing the resolution of the observed original image is determined by the required reconstruction accuracy and depends on the applied registration method, which in the general case is described by two characteristics - the trajectory of movement of the recording photo matrix and the shooting frequency. To date, many authors have proposed [1-7] a large number of high-performance schemes and algorithms for increasing the resolution of digital images using various computational methods and optimization procedures. In a number of publications on this topic [8-10], it is assumed that the set of observed data, which should be used to reconstruct the original high-resolution image, was obtained by sequentially moving a low-resolution photo matrix in two mutually perpendicular directions by an amount less than the linear dimensions of the integrating element of the photo matrix.

But this approach is not effective enough in the operation of technological control systems, where the recording photo matrix cannot move along the axes of a rectangular coordinate system, but a circular scanning trajectory is required. This article is devoted to this variant of reconstruction (estimation) of the initial noisy field. Below is a technique for solving the entire chain of problems associated with circular scanning of images, starting from the development of the necessary mathematical model and ending with the construction of final programs and algorithms for optimal subpixel reconstruction of original digital images with improved spatial resolution.

2. Enhancing spatial resolution using circular scanning

Within the framework of the model under consideration, it is assumed that the original digital image to be reconstructed (see Figure 1) is a two-dimensional numeric array F_{ij} corresponding to the zero-order approximation $f^{(0)}(x, y)$ of a function of two variables $f(x, y)$. In other words, there is a set

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of samples regularly specified on a square lattice, which corresponds to a piecewise discontinuous function $f^{(0)}(x, y)$, which is a constant inside each elementary pixel (i, j) with area $S_\Delta=\Delta^2$:

$$f^{(0)}(x,y)=F_{ij}, \quad (i\text{-}1)\Delta<x<i\Delta, \quad (j\text{-}1)\Delta<x<j\Delta, \quad (i,j=1,..,N), \text{ where}$$

$$F_{ij}=\frac{1}{S_\Delta}\int_{(i-1)\Delta}^{i\Delta}\int_{(j-1)\Delta}^{j\Delta} f(x,y)dxdy.$$

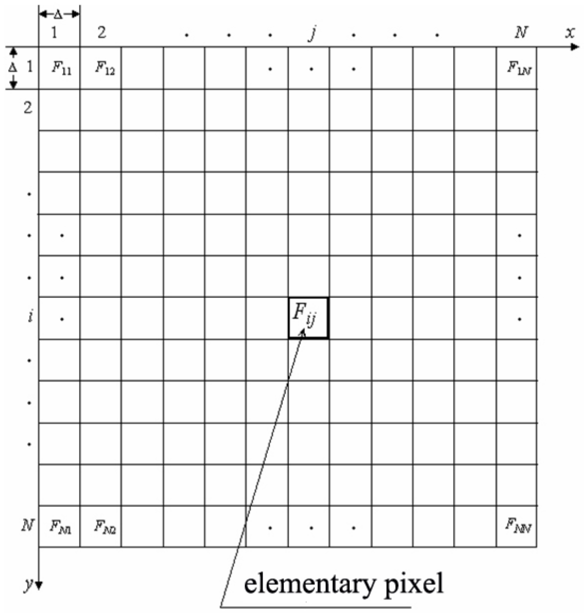

Figure 1: Representation of the original (to be reconstructed) digital image.

The process of circular scanning of such an image is a multiple reading of the original image by a moving photo matrix having an integration aperture $(\Omega\times\Omega)$, $\Omega>\Delta$, i.e. an aperture, the linear dimensions of which are (Ω/Δ) times greater than the linear dimensions of the pixel of the reconstructed image. In this case, the trajectory of movement of the photo matrix is a circle of radius R (Fig. 2).

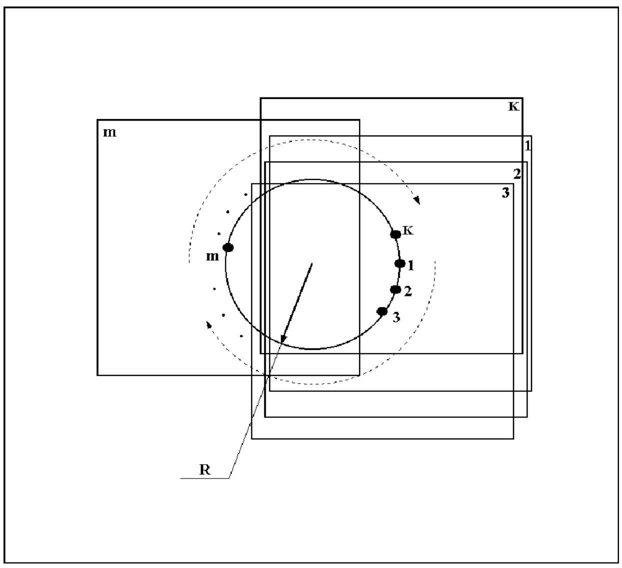

Figure 2: The trajectory of element's movement of the low-resolution photo matrix.

Below, for simplicity, we will assume that both the photo matrix as a whole and each of its elements sequentially "run through" K positions uniformly distributed around a circle of radius R (each element of the photo matrix has its own circle and its own discrete set of K positions on these circles). For simplicity, it is assumed that immediately during registration, the circular motion of the photo matrix is suspended, so that at the moment of "exposure" it remains motionless, and the transition from one point of the circle to the next occurs instantly. Thus, the possible "blur" of the image associated with the circular motion of the photo matrix is not taken into account at the first stage. The result of such registration is K digital images of low resolution (the resolution for each coordinate is (Ω/Δ) times lower than the original image), which we need to reconstructed the original digital image F_{ij}.

The developed approach and the software system implemented on its basis allows using simulation tools to test the algorithms for the synthesis and reconstruction of digital images from a set of images of lower resolution obtained in the process of circular scanning of the original two-dimensional field on a computer. The input data for programs that reconstruct images with the original (i.e. increased) spatial resolution are:

1. A set of K observed images (parameter K is controlled by software).
2. $L = \Omega/\Delta$ - the ratio of the linear size of the element of the low-resolution photo matrix to the linear size of the pixel of the reconstructed image (adjustable in the program).
3. R/Δ - radius of circular scanning, specified in relative values. In this case, the optimal value of the parameter R/Δ should be determined based on the results of calculations.

So, schematically, the process of software simulation of the circular scanning process with subsequent reconstruction of the original digital image can be described as follows. For each fixed k, a separate sample is, as already noted, the result of "integrating" the original digital image F_{ij} over the aperture $(\Omega\times\Omega)$ centered at the point

$$(x_{st}^{(k)}, y_{st}^{(k)}) = (x_{st}^{(0)} + R\cos\frac{2\pi(k-1)}{K};\ y_{st}^{(0)} + R\sin\frac{2\pi(k-1)}{K}).$$

Figure 3 shows an example of the original image F_{ij} (a fragment of the chessboard).

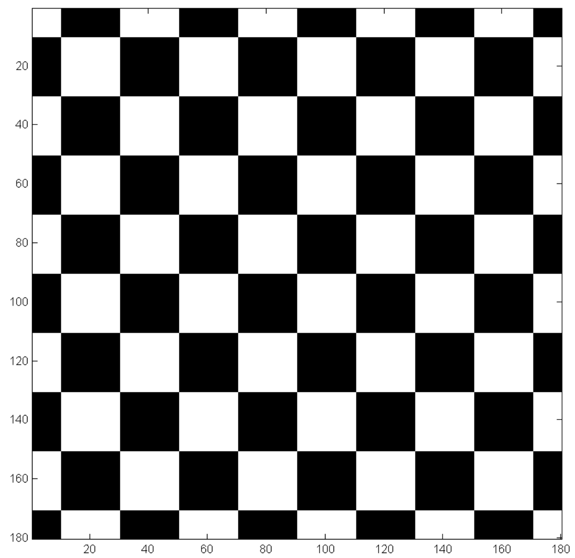

Figure 3: The original image (a fragment of a chessboard).

Figure 4 shows two of 16 images with low spatial resolution, obtained by emulating the circular scanning process of the original image 3 with the following parameters: $K = 16$; $L = 89/29$ ($\approx$3.069); R = 1.1.

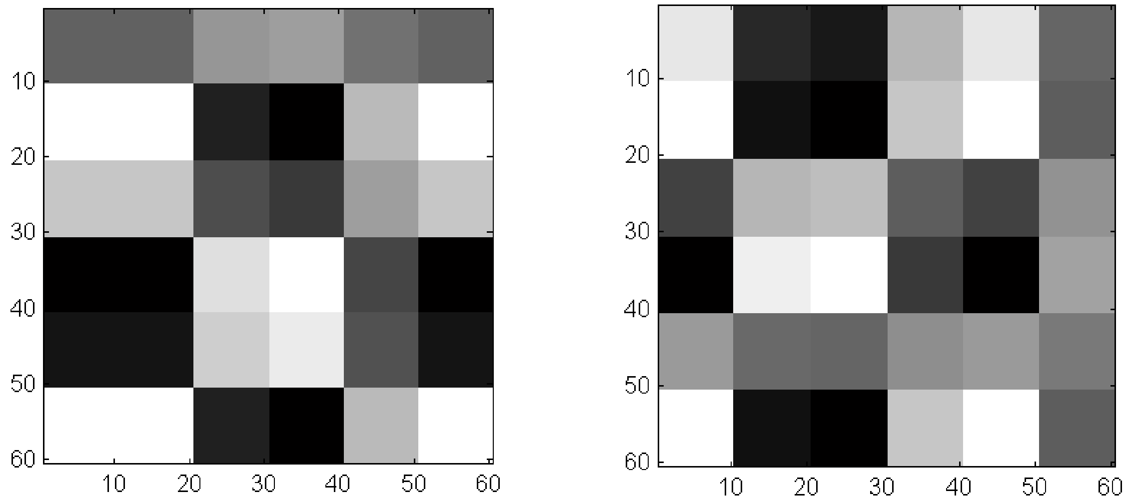

Figure 4: Two out of 16 low-resolution simulated images; the original high-resolution image was reconstructed from them.

Physically, the process of circular scanning of images, i.e. the process of analog photoreading of a real three-dimensional spatially distributed signal using a low-resolution photo matrix is inevitably accompanied by interference (they are associated, on the one hand, with dynamic spatially distributed random noise, and, on the other hand, with the noise of the recording path itself). Therefore, in our case, computer simulation of the process of synthesis of recorded images included, as a component stage, a software simulation of this kind of pseudo-noise. The presented program model provides for the generation of random noise of two types: multiplicative, the level of which is correlated with the level of the integrated signal, and additive random noise, the level of which does not depend on the recorded signal, but only characterizes the recording path itself.

As a result of modeling the circular scanning process, a system of linear equations arises that connects each of the expressions $U^{(k)}_{st}$ with the samples F_{ij} of the original image to be estimated, that fall into the integration aperture $(\Omega\times\Omega)$ centered at a point $(x^{(k)}_{st}, y^{(k)}_{st})$.

In the general case, the number of linear equations connecting the multidimensional array of registered observations $U^{(k)}_{st}$ with the two-dimensional reconstructed array of unknowns F_{ij} exceeds the number of unknowns N^2, i.e. the resulting system of linear equations is overdetermined. It is solved by the standard least squares method, so that the final expression for each of the components F_{ij} of the solution vector $\boldsymbol{F}$ is a linearly weighted combination of the observed values with a matrix of weight coefficients $W(1{:}N^2; 1{:}(K\times[\boldsymbol{entier}(N/(\Omega/\Delta)]^2))$.

This means that the matrix W for the given parameters of circular scanning can be calculated once, and then used to reconstruct the unknown original high-resolution image by multiplying by the observation vector U.

3. Results

As an illustration, Fig. 5 shows one of the examples of reconstruction of a noisy image by the circular scanning method.

Circular scan options:
radius R = 1.2;
relative integration aperture Ω/Δ = 3.069;
number of low-resolution images = 16;
additive noise level = 0.2;
multiplicative noise level = 0.01.
reconstruction quality obtained = 0.07851.

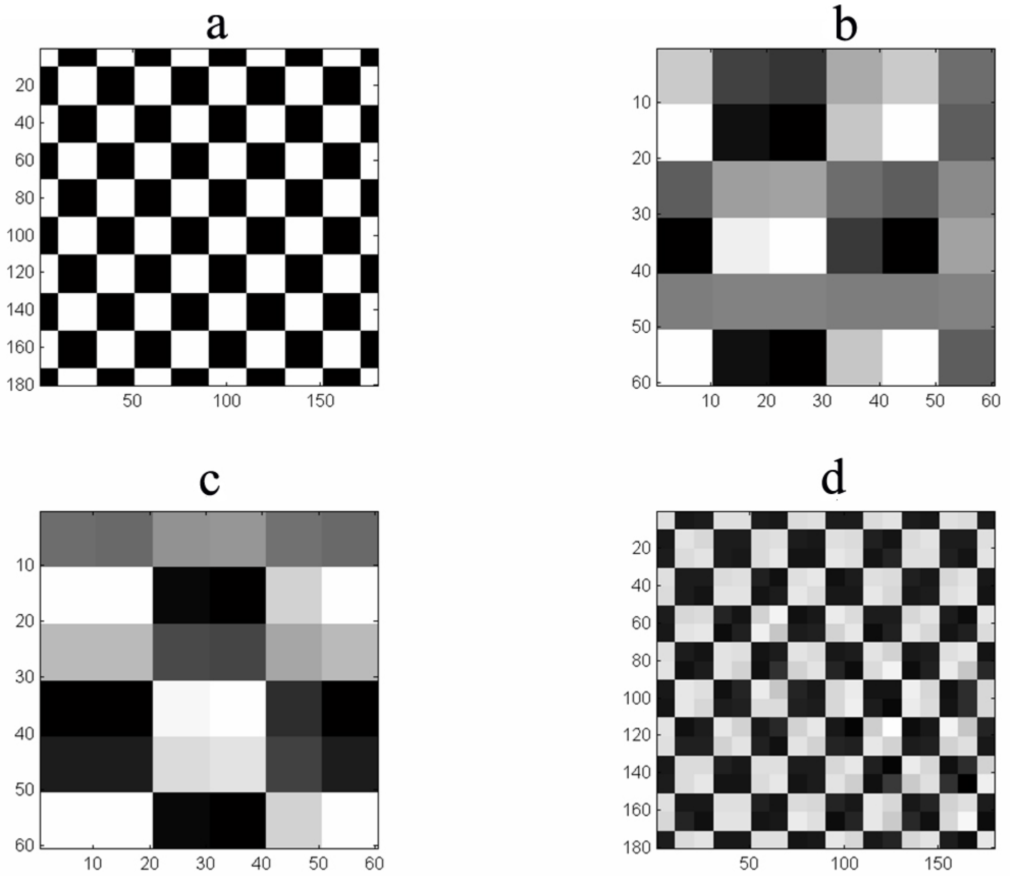

Figure 5: An example of reconstructing the original image in the presence of noise: a) the original image; b), c) two out of 16 low-resolution images, which were used for reconstruction; d) the reconstructed image

The developed software system allows us not only to carry out the image reconstruction process with increased resolution, but also to calculate the optimal parameters of the circular scanning process itself. The data from Table 1 can be an example of such optimization.

Table 1. Calculation of the error in reconstructing the original signal (noise/signal ratio) depending on the scanning radius.

Scanning radius Realization №	0,9	1	1,1	1,2	1,25	1,3	1,4	1,5
Realization 1	0,105	0,074	0,077	0,073	0,073	0,071	0,238	3,792
Realization 2	0,276	0,076	0,112	0,137	0,087	0,076	0,170	3,149
Realization 3	0,142	0,136	0,114	0,160	0,073	0,112	0,185	2,794
Realization 4	0,190	0,260	0,115	0,081	0,073	0,115	0,126	5,773
Realization 5	0,447	0,133	0,118	0,106	0,086	0,132	0,103	4,608
Realization 6	0,317	0,086	0,188	0,103	0,071	0,103	0,204	2,669
Realization 7	0,493	0,235	0,200	0,110	0,064	0,066	0,218	4,436
Realization 8	0,270	0,274	0,101	0,116	0,068	0,128	0,118	7,441
Realization 9	0,294	0,146	0,106	0,119	0,077	0,131	0,172	3,742
Realization 10	0,620	0,082	0,254	0,102	0,074	0,056	0,100	5,738
Average error	0,315	0,150	0,139	0,111	0,075 (min)	0,099	0,163	5,414

The purpose of the calculations was to determine the optimal scanning radius at a fixed value of the relative integration aperture Ω/Δ. The criterion for the optimality of the reconstruction algorithm was the ratio σ_1/σ_0, where σ_1^2 is the variance of the difference image (difference between the reconstructed and the original images), and σ_0^2 is the variance of the original image.

From the data given in Table 1, it can be seen, for example, that to improve the spatial resolution in both coordinates by a factor of 89/29 ≈ 3.069, the optimal radius of the circular path should be $R = 1.25\Delta$. This is clearly manifested in the form of a minimum on the last line of Table 1. The numerical value of this minimum, equal to 0.075, corresponds to the noise/signal value and indicates a high quality of the original signal reconstruction achieved at $R_{opt} = 1.25\Delta$ (control calculations were carried out in the presence of multiplicative and additive noise. To obtain statistically reliable estimates, all results were averaged over 10 realizations of random noise).

4. Conclusion

New efficient algorithms for increasing the spatial resolution of photo matrix images based on optimal digital processing of circular scanning results have been constructed. Testing has demonstrated the stable operation of the algorithms in the presence of additive and multiplicative noise.

5. Acknowledgements

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