Minimization of the Average Risk in Pattern Recognition for Smart Grid Systems

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Abstract

In problems of object and signal recognition, each of the errors of the first and second kind has its own cost, which takes on non-negative values. If they are equal, then the problem is relatively easy to solve. Since, after some transformations, the equation is transformed so that the Laplace function can be applied to it and the approximate values can be found. However, finding more accurate values, with inequality of errors of the first and second kind, and minimizing the average risk is in demand and necessary. In the course of the study, a method was developed for finding the minimum value of the average risk for two functions that have a normal distribution, as well as an independent mathematical expectation and standard deviation.

The obtained theoretical results are simulated on a computer. In the course of modeling, various combinations of the probabilities of errors of the first and second kind were set, in the course of which the tendency of change in the average risk was determined. The results of computer modeling show the effectiveness of the proposed technique.

A mathematical model is built to estimate the errors of the measure of proximity between objects when solving problems of pattern recognition when recognizing signals, and the conditions for minimizing errors of the measure of proximity between objects are derived from it. The fulfillment of these conditions allows two to four times to reduce the errors in estimating the measure of proximity between objects.

Keywords

Pattern recognition, average risk, first kind error, second kind error, first kind error probability, second kind error probability, risk minimization, proximity between objects measure.

1. Introduction

In the last decade, the Smart Grid concept, which means an intelligent power system, has been actively discussed and developed abroad. Smart Grid is a fully integrated self-regulating and self-renewing electric power system with a network topology that includes all generation sources, trunk and distribution networks and all types of electricity consumers, which are controlled using a single network of information and control devices and real-time systems. In fact, an intelligent electric network unites not one, but two networks – an electric and information control network, which closely interact with each other and function simultaneously [1].

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Without proper coordination may not yield satisfactory results. In order to achieve such coordination and, at the same time, avoid the single points of failure typical of centralized controller architectures and dedicated communication links, advanced smart grids should incorporate distributed and autonomous controllers. Greater numbers of distributed and autonomous controllers also reduce the risk of intentional and unintentional outages due to breaches of cybersecurity. There is an underlying paradox here, however: the more distributed and autonomous the control structure is, the more complex it also tends to be. Since more complex systems may be more prone to operational failures, without proper planning and design, distributed and autonomous control architectures may yield worse reliability performance than expected [2].

The computerized control in the electric power industry objects by SCADA systems involves the problems of checking the consistency of their parameters with the required values and, depending on test results, forming appropriate signals to check and control the processes proceeding in these objects. Such problems can be solved with the help of pattern recognition systems (PRS). But existing systems cannot provide sufficient certainty in pattern recognition, because of the proximity measure between patterns (PMBP) corresponding to the current and required states of power objects [3-5].

The existing methods of checking pattern recognition (PR) certainty involve sophisticated algorithms and structural solutions which allow the reduction of PMBP estimation errors but complicate the structure and decrease the speed of PRS.

In the absence of a mathematical model for the analysis of PMBP estimate errors, its development and the construction of a model for the correction of these errors are real problems [6, 7].

2. Problem statement

In the development of the principles of invariant pattern recognition each object can be represented in the form X_i, where $i = 1,2, \dots, n$ and $X_i = (x_i; b_1, b_2, \dots, b_n)$; x_i is the real value of the *i*-th parameter of the pattern to be recognized; $b_1, b_2, \dots, b_n$ are the parameters of influence of destabilizing factors on value x_i. A standard pattern can be represented in the form y_i, where $y_i = f(y_i; c_1, c_2, \dots, c_n)$; y_i is the real admissible value of the i-th parameter of unhealthy influences; $c_1, c_2, \dots, c_n$ are the parameters of influences of destabilizing factors on values y_i [8,9]. The value of PMBP, evaluated indirectly by the i-th parameter, can be determined as follows:

$$Z_i = F[\, f_i(x_i; b_1, b_2, \dots, b_n);\ f_2(y_i; c_1, c_2, \dots, c_n)] \tag{1}$$

As full invariance of pattern recognition is reached, the desired value of Z_i is specified by the expression:

$$Z_i^* = F\ [\, f_i(x_i);\ f_2(y_i)] \tag{2}$$

To determine condition $Z_i = Z_i^*$, formula (1), on rearrangement, can be expanded into Taylor's series. Taking linear terms, we can derive the conditions of invariance of the PMBP value with respect to destabilizing factors:

$$\sum_{i=1}^{n} \left(\frac{\partial f_1}{\partial b_i} db_i + \frac{\partial f_2}{\partial c_i} dc_i \right) = 0 \tag{3}$$

There are three ways to meet these conditions:

$$\frac{\partial d_1}{\partial b_i} db_i = 0,\ \frac{\partial f_2}{\partial c_i} dc_i = 0; \tag{4}$$

$$\left(\frac{\partial f_1}{\partial b_i} db_i + \frac{\partial s_2}{\partial c_i} dc_i \right) = 0; \tag{5}$$

$$\sum_{i=1}^{n} \left(\frac{\partial f_1}{\partial b_i} db_i + \frac{\partial s_2}{\partial c_i} dc_i \right) = 0. \tag{6}$$

The first solution is that using different algorithmic and structural methods. We can minimize each error of direct parameter measurement of the pattern to be recognized and the standard one. Such a solution can be realized in the case when the number of destabilizing factors is small. But in practice, this number is usually large (ambient temperature, instability of voltage and frequency of a power source, illumination of the vision field of a sensitive device, change in the position of the object to be recognized, ageing of equipment, and others). Therefore, the minimization of separate errors of parameter measurement complicates equipment and reduces its reliability. Moreover, in most cases it leads to the degradation of system dynamic characteristics [3,4].

The second solution implies the invariance of PR for the separate types of errors of parameter value measurement for the object to be recognized and the standard one. But firstly, this solution is not efficient for the above reason, and secondly, the formulation of PMBP reduces to differential measurement with spatial and temporal parameter partitioning, and this requires equivalent spatial and temporal conditions, which are difficult to realize [10, 11].

The third solution implies the PR invariance by the totality of parameter measurement errors for the object to be recognized and the standard one, which seems preferable as compared with other solutions. There is no need in this case to allow for the physical nature and contribution of each destabilizing factor to the total error, which simplifies the solution for all the types of destabilizing factors [12, 13].

Invariance conditions:

$$\sum_{i=1}^{n}\left(\frac{\partial f_1}{\partial b_i}db_i\right)=0;\ \sum_{i=1}^{n}\left(\frac{\partial f_2}{\partial c_i}dc_i\right)=0;\ \sum_{i=1}^{n}\left(\frac{\partial f_1}{\partial b_i}db_i+\sum_{i=1}^{n}\left(\frac{\partial f_2}{\partial c_i}dc_i\right)=0. \quad (7)$$

Show that to minimize the influences of destabilizing factors on the PMBP value, firstly, the influence on each parameter X and Y should be minimized, and secondly, the sum of influences on these parameters should be minimal. Destabilizing factors form systematic and random errors of the PMBP estimate [12, 15]. As a result of the analysis of the Euclidean, Manhattan and Canberra algorithms for steady and alternating components of the PMBP systematic estimate error, we have obtained the following generalized expressions:

$$D_Z=\Delta X_i sign(\Delta X_i)-\Delta Y_i sign(\Delta DY_i), \quad (8)$$

where ΔX_i and ΔY_i are the steady components of systematic errors of parameter measurement for the object to be checked and the standard one; $Sign(\cdot)$ is the sign of error in the estimation of parameter X and Y.

A generalized formula for the determination of the alternating component of the systematic error of PMBP value estimation can be written in the form

$$\begin{aligned}\Delta Z = sign\,(\Delta K_x)f(\gamma_x)\Delta K_x - sign(\Delta K_y)f(\gamma_y)\Delta K_y +\\ +sign(\Delta\gamma_x)f(\Delta K_x)\Delta\gamma_x - sign(\Delta\gamma_y)f(\Delta K_y)\Delta\gamma_y,\end{aligned} \quad (9)$$

where ΔK_x and ΔK_y are the multiplicativity factors of the alternating systematic errors of formulation of parameters γ_x and γ_y; X and Y are the relative values of the last.

As a result of experimental investigations, we could clarify that the random error of PMBP estimation was distributed by the normal law, and this fact is confirmed in literature [16]. The analysis of these data showed that the correlation between the error of parameter measurement of the object to be recognized σ_x was close to zero, since $\delta_{PMBP}=(\sigma_x^2+\sigma_y^2)^{\frac{1}{2}}$. It means that the errors introduced into the measurements of X and Y are reflected on the results of technical vision system (TVS) operation [14,17]. These facts allow some corrections in the determination of the "certainty" concept, by which is meant the fiducial probability of the correct determination of belonging of the object under check to a proper class, reflecting the degree of correspondence of the measured parameters of an input object to the true values of standard object parameter [10,14].

The uncertainty of pattern recognition in this case can be realized as a sum of independent uncorrelated events characterizing the errors of measurement of parameter values of the checked and standard objects. Consequently, the certainty of pattern recognition can be determined as the product of the fiducial probabilities of parameter value measurements of the both objects. This attests that certainty in TVS will be always less than separate fiducial probabilities of measurement of parameters X and Y. To increase the certainty, it is necessary to decrease the error of measurement of this parameters. Thus, the development of effective methods for the correction of measurement errors of image parameters of natural objects is a real problem [12].

Let us represent a mathematical model for the PMBP estimation in the implicit form:

$$Z = F\big(f_1(X, b_1, b_2, \dots, b_n);\ f_2(Y, b_1, b_2, \dots, b_n)\big), \tag{10}$$

where $F\big(f_1(X, b_1, b_2, \dots, b_n)\ and\ f_2(Y, b_1, b_2, \dots, b_n)\big)$ are the output signals of a measuring channel in the measurement of parameters of the object under check and standard one, respectively; $F(\cdot)$ is the function of the PMBP estimation.

We can see from (10) that destabilizing factors affect parameters X and Y identically. Therefore, by minimizing these influences, we can increase the information body of the PMBP value. In order to determine extreme points, we expand (10) into Tailor's series and then only consider linear terms. On some transformations, we can find the conditions of invariance of the PMBP estimation for destabilizing factors:

$$\textstyle\sum_{i=1}^{n}\left(\frac{\partial f_1}{\partial b_i}\right)db_i = 0;\ \sum_{i=1}^{n}\left(\frac{\partial f_2}{\partial b_i}\right)db_i = 0;\ \sum_{i=1}^{n}\left(\frac{\partial f_1}{\partial b_i}\right)db_i + \sum_{i=1}^{n}\left(\frac{\partial f_2}{\partial b_i}\right)db_i = 0. \tag{11}$$

Conditions (11) show that to minimize the influences of destabilizing factors on the PMBP value, it is necessary to minimize these influences on each parameter X and Y and the sum of influences on these parameters should be minimal.

3. Problem solving method

Images available: reference image – RI and another image – AI. Let us assume that the brightness level of the reference image is proportional to some true voltage - y_{true}. The observer measures this voltage with errors - δ_y, as a result, there is an estimate of this voltage – y, so that $y = y_{true} + \delta_y$. The brightness level of another image is proportional to some true voltage x_{true}. In this case, a similar relationship $x = x_{true} + \delta_x$ takes place, where x_{true} – is the true voltage corresponding to the brightness of another image, $\delta_x -$ is the error in measuring this voltage, $x-$ is an estimate of this voltage, or the voltage that is measured by the observer.

Estimates of stresses y *and* x are random values and are distributed according to normal laws with parameters $\{m_y, \sigma_y\}$ and $\{m_x, \sigma_x\}$.

Under normal distribution laws, the true stresses of the reference image (RI) and another image (AI) are equal to the mathematical expectations of $y_{true} = m_y$, $x_{true} = m_x$

In addition, either only the reference image with probability p_1 or only another image with probability p_0 can appear in front of the observer. The probabilities of these mutually exclusive events are related as $p_1 = 1 - p_0$.

The task of the observer is to determine which image he is observing, reference or other, that is, to refer the image he is observing either to the reference image or to another. Error of the first type: It is decided that the observed image is different (or not a reference), while the observed image is in fact a reference. Error of the second type: It is decided that the observed image is a reference (or is not different), while the observed image is actually different.

Since the observer has not yet decided which image is in front of him, then it is not yet known which value is measured by y or x. Therefore, we denote the value of the measured voltage z. For definiteness, let us assume that the mathematical expectation of a voltage proportional to the brightness of the reference image is greater than the mathematical expectation of a voltage proportional to the brightness of another image $m_y > m_x$,

Or the true voltage proportional to the brightness of the reference image is greater than the true voltage proportional to the brightness of the other image $y_{true} > x_{true}$

In that case, a certain threshold - Z_{THLD} is set for the measured voltage z. If the measured voltage z is less than the set threshold (THLD), then a decision is made that another image is observed:

$z < Z_{THLD} \Rightarrow$ observed image = another image.

If the measured voltage z is greater than the set threshold, then a decision is made that a reference image is observed: $z \geq Z_{THLD} \Rightarrow$ observed image = reference image.

In this case, such errors are possible:

$z \geq Z_{THLD} \Rightarrow$ the observer decides that the image is a reference, in reality the image is different, in statistical radio technics this situation is called a false alarm - this is a type 1 error.

$z < Z_{THLD} \Rightarrow$ the observer decides that the image is different, in fact the image is a reference, in statistical radio technics such a situation is called a signal skip - this is a type 2 error. (at the very beginning of the task, these situations are confused and indicated vice versa).

The probability of an error of the first kind will be written as follows:

$$P_\alpha = \int_{Z_{THLD}}^{\infty} f_0(x)dx,$$

where $f_0(x) = \frac{1}{\sqrt{2\pi}\sigma_x} exp\left(\frac{(x-m_x)^2}{2\sigma_x^2}\right) -$ probability density function of random stress - x.

The probability of an error of the second type will be written as follows:

$$P_\beta = \int_{-\infty}^{Z_{THLD}} f_1(y)dy,$$

where $f_1(y) = \frac{1}{\sqrt{2\pi}\sigma_y} exp\left(\frac{(y-m_y)^2}{2\sigma_y^2}\right) -$ probability density function of random stress - y.

The average risk with equal probabilities of the appearance of another and the reference image $p_0 = p_1 = 0.5$ will be written as:

$$C = 0.5 \cdot (\alpha \cdot P_\alpha + \beta \cdot P_\beta), \tag{12}$$

where α and β are the cost of losses from errors of the first and second type, respectively.

And with unequal probabilities, the average risk should be averaged over these probabilities as follows:

$$C = p_0 \cdot \alpha \cdot P_\alpha + (1 - p_0) \cdot \beta \cdot P_\beta = p_0 \cdot \alpha \cdot P_\alpha + \beta \cdot P_\beta - p_0 \cdot \beta \cdot P_\beta. \tag{13}$$

It is required to set such a threshold Z_{THLD} at which the average risk becomes C minimum.

In the above formulation of the problem, the mathematical expectations of random voltages m_x, m_y and, the standard deviations - σ_x and σ_y, the correlation coefficient ρ_{xy} - are fixed and known to the observer. In this case, statistical averaging over these parameters is possible.

To develop a method for minimizing the estimation errors of PMBP, we analyze the composition of the distribution laws for the measurement errors of the parameters of the recognized and reference images.

Suppose the probability densities p (x) and p (y) of the values of the input and reference features x and y have an arbitrary form, and the errors γ_x and γ_y, superimposed on the values x and y, are distributed according to the normal law (m_x and m_y, γ_x and γ_y).

It is assumed that the errors γ_x and γ_y are correlated, but independent of the value of x. If the values of x and y differ by the value z (z = x - y), an error of the first ($x > y$ and $x + \gamma_x \leq y - z + \gamma_y$) and the second ($x \leq y$ and $x + \gamma_x > y - z + \gamma_y$) genera. As is known, in problems of control and recognition, each error has its own cost α and β, which take on non-negative values. In this case, the average risk C will be equal to the mathematical expectation of the cost:

$$C = \alpha P_\alpha + \beta P_\beta, \tag{14}$$

where P_α and P_β are, respectively, the probabilities of errors of the first and second kind.

It is necessary to find the difference between the values of x and y, which corresponds to the value of the PRS output signal. For this, a composition of two normal laws of probability density with respect to the variable $\gamma_x - \gamma_y = \gamma_z$ is compiled:

$$g(\gamma) = \frac{1}{\sigma_0\sqrt{2\pi}} exp(-\frac{(\gamma-m_0)^2}{2\sigma_0^2}), \tag{15}$$

where $m_0 = m_x - m_y$; $\sigma_0 = \sqrt{\sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2}$.

Using the variable γ_z, the conditions for the occurrence of errors of the first and second kind can be represented as:

$x > y \; and \; x - y \leq y - z \quad and \quad x \leq y \; and \; x - y > y - z$

Considering the domains of definition of probabilities P_α and P_β:

$$C = \alpha \int_y^{x_{max}} dx \int_{x-y+z}^{\infty} f(x) g(\gamma) d\gamma + \beta \int_{x_{min}}^{y} dx \int_{\infty}^{x-y+z} f(x) g(z) dz. \tag{16}$$

The value of the output signal should be determined by the minimum average risk. For this, we differentiate the last formula and equate to zero. The solution to this equation gives the desired value of z in general form. Since the exact value of z is determined by a computational operation, we will approximately determine it. By the mean value theorem, we transform formula into the following form:

$$\alpha f(x_\alpha) \int_y^{x_{max}} g(x-y+z)dx + \beta f(x_\beta) \int_{x_{min}}^{y} g(x-y+z)dx = 0. \tag{17}$$

Taking the following notation $a = \alpha * f(x_{\alpha)} / \beta * f(x_\beta)$; $m_1 = y + m_0 - z$ can be written:

$$g(x-y-z) = \frac{1}{\sigma_0\sqrt{2\pi}} exp(-\frac{(x-m_1)^2}{2\sigma_0^2}), \tag{18}$$

We expand the last formula in a Taylor series for $x = m_1$

$$g(x-y+z) = \frac{1}{\sigma_0\sqrt{2\pi}}[1 - \frac{1}{2\sigma_0^2}(x-m_1)^2 + \frac{1}{8\sigma_0^4}(x-m_1)^4]. \tag{19}$$

Substituting this formula into equation (17) we get:

$$\frac{a}{\sigma_0\sqrt{2\pi}}[x - \frac{1}{6\sigma_0^2}\left(x-m_1\right)^3 + \frac{(x-m_1)^5}{40\sigma_0^4}\Big]\Big|_{x_{max}}^{y} \Big| \frac{1}{\sigma_0\sqrt{2\pi}} \frac{(x-m_1)^3}{6\sigma_0^2} \frac{(x-m_1)^5}{40\sigma_0^4} \Big|_{y}^{x_{min}} \Big|. \tag{20}$$

Taking into account the boundary values:

$$a[x_{\max} - \frac{(x_{\max}-m_1)^3}{6\sigma_0^2} + \frac{(x_{\max}-m_1)^5}{40\sigma_0^4} - \frac{(y-m_1)^3}{6\sigma_0^2} - \frac{(y-m_1)^5}{40\sigma_0^4}] = \tag{21}$$
$$y - \frac{(y-m_1)^3}{6\sigma_0^2} + \frac{(y-m_1)^5}{40\sigma_0^4} - x_{\min} + \frac{(x_{\min}-m_1)^3}{6\sigma_0^2} - \frac{(x_{\min}-m_1)^5}{40\sigma_0^4}$$

After some transformations, the last equation will take the form:

$$F(z) = 120\,\sigma_0^4[a(x_{\max} - y) - y + x_{\min}] + 20\,\sigma_0^2[a(z - m_0)^3 - a(x_{\max} - y - m_0 + z)^2$$
$$+ (z - m_0)^3 - (x_{\min} - y - m_0 + z)^3] + 3a\,(x_{\max} - y - m_0 + z)^5 - 3a\,(z - m_0)^5$$
$$- 3(z - m_0)^5 + 3\,(x_{\min} - y - m_0 + z)^5 \quad (22)$$

Let us define the first and second derivatives of this formula:

$$F'(z) = 20\sigma_0^2[3a(z - m_0)^2 - 3a\,(x_{\max} - y - m_0 + z)^2 + 3(z - m_0)^2 - 3(x_{\min} - y - m_0 + z)^2] + \quad (23)$$
$$+ 15a\,(x_{\max} - y - m_0 + z)^4 - 15a\,(z - m_0)^4 - 15(z - m_0)^4 + 15(x_{\min} - y - m_0 + z)^4$$
$$F''(z) = 20\,\sigma_0^2\,[6a\,(z - m_0)^2 - 6a\,(x_{\max} - y - m_0 + z)^2 + 6\,(z - m_0) - 6\,(x_{\min} - y - m_0 + z)] + \quad (24)$$
$$+ 60\,a(x_{\max} - y - m_0 + z)^3 - 60\,a(z - m_0)^3 - 60\,(z - m_0)^3 + 60\,(x_{\min} - y - m_0 + z)^3$$

The exact value of z is determined by the Simpson method using the following algorithm:

$$z_{i+1} = z_i - F(z_i)/F'(z_i). \quad (25)$$

The initial value of z is taken to be the end of the range $[x_{min}, x_{max}]$, at which the following is performed:

$$F(x''_{min})_{min} \; or \; F(x''_{max})_{max}. \quad (26)$$

With a more simplified definition of the value of z, it is required that the condition be met:

$$\frac{\alpha}{\beta} - \frac{\alpha'}{\beta'} \leq |k|\frac{\alpha}{\beta} \quad (27)$$

where k is a coefficient that takes into account variations in the values α and β:

$$\alpha' = \alpha f(x_\alpha) \; and \; \beta' = \beta f(x_\beta) \quad (28)$$

This condition is equivalent to:

$$f(x_\alpha) - f(x_\beta) \leq |k| f(x_\beta), \quad (29)$$

the fulfillment of which is the main constraint imposed on the function *f(x)*. To soften this restriction, it is assumed that γ is limited by the interval nσ (n = 6) and $n\sigma_0 \ll x_{max} - x_{min}$. Thus, the main constraint now leads to the condition usually fulfilled in practice that the relative changes in the function *f(x)* are small when *x* changes in a very narrow interval. The introduction of the condition $n\sigma_0 \ll x_{max} - x_{min}$ limits the class of the considered systems, which have an error of no more than 2.5%.

The accepted restrictions allow α' and β' to be replaced by α and β in formula (17):

$$-\alpha \int_y^{x_{max}} g(x - y + z)dx + \beta \int_{x_{min}}^{y} g(x - y + z)dx = 0. \quad (30)$$

Approximate (30) becomes strict if $f(x) = const$ in the range $n\sigma_0$ or even more so in the entire range of *x*. Then equation (30) is represented as:

$$-\alpha\left[\Phi\left(\frac{x1_{max}}{\sigma_0\sqrt{2}}\right) - \Phi\left(\frac{z-m_0}{\sigma_0\sqrt{2}}\right)\right] + \beta\left[\Phi\left(\frac{z-m_0}{\sigma_0\sqrt{2}}\right) - \Phi\left(\frac{x1_{min}}{\sigma_0\sqrt{2}}\right)\right] = 0. \quad (31)$$

In formula (31), the Laplace functions are tabulated. Due to the fact that $x_{max} - y + z - m_0 > 3\sigma_0$ and $x_{min} - y + z - m_0 > 3\sigma_0$, the first Laplace function is equal to one, and the fourth is equal to zero. Consequently,

$$\Phi\left(\frac{z-m_0}{\sigma_0\sqrt{2}}\right) = \frac{\alpha}{\alpha+\beta}. \tag{32}$$

Expression (32) allows for given α and β to find the number k, which is the tabular value of Φ (*). Hence, the output parameter *z* is estimated by the formula:

$$z = k * \sqrt{2(\sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2)} + m_x - m_y. \tag{33}$$

With the availability of the criterion of an ideal observer (α = β) the z parameter can be determined only by parameters m_x and m_y ($z = m_x - m_y$), because $\phi(k^*)$=0.5 with $k^* = 0$. More sophisticated treatment of this process allows us to propose conditions for providing the invariance of the PMBP value.

For random errors

$$\sigma_x = \sigma_y = min; sign(\sigma_y) or\ \rho = 1. \tag{34}$$

We select $z = F(X,Y)$ provided that $\Delta F(\sigma_x, \sigma_y) = min$.

For systematic errors

$$\Delta x = \Delta y = min; sign(\Delta x) = sign(\Delta y). \tag{35}$$

We select $z = F(X,Y)$ provided that $\Delta F(\Delta X, \Delta Y) = min$.

4. Simulations

To check the correctness of theoretical research, we have performed experiments for greater accuracy of the PMBP estimation in the recognition of electrical power signals by their parameters. The data obtained in the course of parameter measurements of current and desired object states are shown in Table1.

To improve the accuracy of the PMBP estimation in the object parameter measurement channel, conditions (34) and (35) have been met, and the PMBP estimation have been performed by the following algorithm.

At the first stage, using the data of initial arrays, the array of values X_i and Y_i is being formed. At the second stage, mathematical expectations m_z are being determined as well as the mean square deviations of the elements of this array σ_z.

Using the obtained data, new array A_i should be formed to consider the proposed conditions of invariance. Its elements can be determined by the following rules:

$$\begin{aligned} &if\ X_i < m_z, then\ A_i = m_z; \\ &if\ X_i > m_z, then\ A_i = m_i. \end{aligned} \tag{36}$$

The essence of this algorithm is that the PMBP estimation errors by separate parameters for input and standard objects can have positive or negative signs. But their values should be subjected to a proper distribution law. Therefore, when the initial and standard objects coincide, the spread in values of their PMBPs, with the appropriate fiducial probability, should be within a given range. The elements which are within this range and take negative increments are being changed for the PMBP mathematical expectation value.

The errors of formation of parameter X and Y with negative increments are eliminated in the fresh array. The mathematical expectation of new array elements is the refined PMBP value.

Thus, we have discovered in our research that if (34) and (35) are met, then the resulting error of the PMBP estimation can be determined by formulas

$$\delta_{PMBP} = -5.878 \cdot 10^{-3} + 153.86\,\sigma_x - 152.93\,\sigma_y;$$
$$\delta_{PMBP} = -5.4 \cdot 10^{-3} + 130.74\,\sigma_x - 129.67\,\sigma_y;$$
$$\delta_{PMBP} = -18.34 \cdot 10^{-3} + 399.07\,\sigma_x - 398.5\,\sigma_y;$$
$$\delta_{PMBP} = -7.35 \cdot 10^{-3} + 181.46\,\sigma_x - 180.62\,\sigma_y; \quad (37)$$

Table 1

The table of dependences of the rms (root mean square) values of the estimate of proximity between objects measure and the value of proximity between objects measure on the rms values of the parameters measurements of the input and reference objects when the proposed conditions are met.

σ_x	σ_y	δ_{PMBP}	d_{PMBP}
7.6	7.6	8.9	10.4
14.08	14.08	17.39	21.1
20.67	20.67	25.86	31.3
27.3	27.3	34.33	41.6
31.34	31.34	40.45	51.9
33.92	33.92	42.81	52.2
36.98	36.98	48.40	61.2
42.73	42.73	56.37	71.1
44.35	44.35	57.38	79.9
55.16	55.16	72.97	92.4
61.49	61.49	81.37	102.9
67.33	67.33	90.06	115.4
74.31	74.31	98.31	124.1
80.78	80.78	106.83	134.7
87.28	87.28	115.38	145.4
93.79	93.79	123.96	156.1
100.33	100.33	132.57	166.8

Formulas (37) have been constructed by the experimental data obtained with the use the Manhattan, Euclidean and Canberra distances and those proposed in this research for the PMBP estimation. This treatment showed that the resulting PMBP estimation error reduced more than by a factor of two.

Let us discuss the possibility to improve the certainty of TVS operation depending on individual invariance conditions for the measure of object proximity to destabilizing factors. For this purpose, we now prove that the δ error reaches its minimum value if errors σ_x and σ_y are equal. This dependence is symmetric around a minimum point and can be written in the form:

$$\frac{\Delta\delta_{PMBP}}{\delta_{PMBP}} = -0.38 + 1.5\frac{\sigma_x - \sigma_y}{\sigma_y} + 0.004\frac{\sigma_x - \sigma_y^2}{\sigma_y^2}. \quad (38)$$

Formula (38) is true with sign $(\sigma_x) = sign(\sigma_y)$ and shows the necessity of providing symmetry between the processes of parameter measurement for the input and standard objects.

To check the dependence of the fiducial probability of measurement of the proximity measure value between objects on the correlation between errors σ_x and σ_y, we have carried out research to obtain the following formula:

$$P_{PMBP} = 0.572 - 0.6763\rho + 1.6807\rho^2. \quad (39)$$

Dependence (39) has been derived with the condition of change of the ρ correlation within the [0, 0.8] range and the fiducial interval of measurement of the proximity measure between objects which is equal to the mean square deviation (MSD).

Experimental dependencies

$$m_{PMBP} = f(d_{PMBP}, \sigma_x, \sigma_y, \delta_{PMBP}),$$
$$m_{PMBP} = f(d_{PMBP} \sigma_x, \sigma_y),$$
$$\text{and } m_{PMBP} = f(d_{MPBP}, \delta_{PMBP}). \quad (40)$$

have been built to allow quantitative estimation of the results obtained (Table 2).

Table 2

Manhattan, Euclidean and Canberra distances

Experimental dependences	Linear model MSD
Manhattan's distance	
M_{PMBP}= -0.11959-0.2021939 d_{PMBP} + 0.8611193σ_x – – 0.76389 σ_y + 0.0109959 δ_{PMBP}	0.138857
M_{PMBP}= -0.1152137-0.1607265 d_{PMBP} + 0.8572778σ_x – 0.772278 σ_y	0.139636
M_{PMBP}= -0.3369-2.57924 d_{PMBP} + 1.36307 δ_{PMBP}	0.993784
Euclidean distance	
M_{PMBP}= -0.05447-0.022103 d_{PMBP} + 0.778079σ_x – – 0.78622 σ_y + 0.0304211 δ_{PMBP}	0.144857
M_{PMBP}= -0.0540528+0.0060352 d_{PMBP} + 0.7825238σ_x –0.788432 σ_y	0.145470
M_{PMBP}= -0.1912+0.58971 d_{PMBP} – 0.5965 δ_{PMBP}	0.907906
Canberra distance	
M_{PMBP}= 0.05213687-0.2092843 d_{PMBP} + 0.8298258σ_x – – 0.77639 σ_y + 0.0068988 δ_{PMBP}	0.142832
M_{PMBP}= -0.0903373-0.122526 d_{PMBP} + 0.8181384σ_x – 0.785087 σ_y	0.144424
M_{PMBP}= -1.4982 - 3.68793 d_{PMBP} + 0.99132 δ_{PMBP}	1.10174
Proposed algorithm	
M_{PMBP}= -0.03038-0.0057578 d_{PMBP} + 0.0723113σ_x – – 0.05927 σ_y + 0.0125336 δ_{PMBP}	0.128857
M_{PMBP}= -0.027262+ 0.018385 d_{PMBP} – 0.0694287σ_x – 0.58578 σ_y	0.139634
M_{PMBP}= 0.1699-0.11542 d_{PMBP} – 0.6771 δ_{PMBP}	0.993784

Thus, using the estimated values of $\delta_{PMBP},, \sigma_x, \sigma_y$ and d_{PMBP} , we can calculate the PMBP value 2-4 times more precisely, which makes it possible to improve the certainty of pattern recognition.

5. Conclusions

The application of the proposed method for minimizing the error in assessing the state of the object / process of the electric power industry by SCADA systems will improve the reliability and quality of power supply. This is achieved through more accurate diagnostics of the condition of the equipment produced in real time on the equipment of power plants, substations and power lines. Correct analysis will allow you to receive early warning of a possible network failure, establish the causes of equipment failures, predict the volume and timing of repairs, as well as equipment service. Thus, it is possible to improve the efficiency of the power system through a stable supply of electricity to the consumer, as well as reduce repair costs by reducing the number of equipment failures.

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