Method of Periodically Non-stationary Random Signals Demodulation with Hilbert Transform

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Abstract

We discuss the use of the Hilbert transform for the analysis of periodically nonstationary random signals (PNRSs), whose carrier harmonics are modulated by jointly stationary high-frequency random processes. The narrow-band modulations are considered. A representation of the signal in the form of a superposition of high-frequency components is obtained and it is shown that these components are jointly periodically nonstationary random processes (PRNPs). The properties of the band-pass filtered signals are examined, and it is shown that band-pass filtering can reduce both the number of signal variance cyclic harmonics and their amplitudes. We show that it is possible to extract the quadratures of narrow-band high-frequency modulation processes using the Hilbert transform.

Keywords

periodically non-stationary random signals, amplitude-phase high-frequency modulations, Hilbert transform, analytic signal, quadratures, vibration

1. Introduction

The vibration signal in many cases can be described as a superposition of stochastically amplitude- and phase-modulated carrier harmonics with multiple frequencies [1-5]. Note that this representation is the characteristic feature of PNRP [6-8]. Taking this into consideration, we can analyze in more detail the covariance and spectral structures of stochastical modulations of PNRP carrier harmonics. Separation of the individual modulated harmonics and their quadratures can be performed using band-pass filtering and the Hilbert transform [9]. Unfortunately, the properties of the Hilbert transform of vibration signals in most works are analyzed only superficially, without the involvement of the PNRP harmonic series representation. In this paper we consider the use of the Hilbert transform for analysis of the PNRP with high-frequency modulation, i.e., in the case when the spectra of the modulating processes are concentrated in an interval whose lower boundary is higher than the highest carrier frequency of the signal.

2. Model of multicomponent periodically non-stationary random signal

The PNRP mean function $m_\xi(t)=E\xi(t)$, where E is the operator of the mathematical expectation, the covariance function $b_\xi(t,u)=E\overset{\circ}{\xi}(t)\overset{\circ}{\xi}(t,u)$, $\overset{\circ}{\xi}(t)=\xi(t)-m_\xi(t)$, are periodical functions of time, i.e. $m_\xi(t)=m_\xi(t+P)$, $b_\xi(t,u)=b_\xi(t+P,u)$, where P is period. If $m_\xi(t)$ are absolutely integrable time functions over interval $[0,P]$, namely

$$\int_0^P |m_\xi(t)|dt<\infty,\ \int_0^P |b_\xi(t,u)|dt<\infty\ \forall u\in\mathbb{R},$$

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then they can be represented in the form of a Fourier series as follows:

$$m_\xi(t)=\sum_{k\in\mathbb{Z}} m_k e^{ik\omega_0 t}=m_0+\sum_{k\in\mathbb{N}}\left(m_k^c\cos k\omega_0 t+m_k^s\sin k\omega_0 t\right), \tag{1}$$

$$b_\xi(t,u)=\sum_{k\in\mathbb{Z}} B_k^{(\xi)}(u)e^{ik\omega_0 t}=B_0^{(\xi)}(u)+\sum_{k\in\mathbb{N}}\left[C_k^{(\xi)}(u)\cos k\omega_0 t+S_k^{(\xi)}(u)\sin k\omega_0 t\right] \tag{2}$$

where $\omega_0=\frac{2\pi}{P}$, $m_k\frac{1}{2}\left(m_k^c-im_k^s\right)$, $B_k^{(\xi)}(u)=\frac{1}{2}\left[C_k^{(\xi)}(u)-iS_k^{(\xi)}(u)\right]\forall k\neq 0$, $\mathbb{Z}$ is the set of integer numbers, $\mathbb{N}$ is the set of natural numbers and

$$m_k=\frac{1}{P}\int_0^P m_\xi(t)e^{-ik\omega_0 t}dt,\ B_k^{(\xi)}=\frac{1}{P}\int_0^P b_\xi(t,u)e^{-ik\omega_0 t}dt.$$

The mean function in Eq. (1) describes the deterministic part of the vibrations, which is usually associated with the macroscopic defects of mechanical systems, such as imbalance, eccentricity, misalignment, etc. The stochastic part $\overset{\circ}{\xi}(t)$ contains information about the non-linearity and non-stationarity of the vibration signal caused by friction forces, changes in the viscosity of lubricants, surface irregularities, etc. An analysis of the stochastic part, including its periodical non-stationarity characteristics, i.e. the Fourier coefficients $B_k^{(\xi)}(u)$ in Eq. (2), allows defects to be detected in the early stages after their initiation [10-12].

The covariance components $B_k(\tau)$ satisfy the equality:

$$B_k^{(\xi)}(-u)=B_k^{(\xi)}(u)e^{-ik\omega_0 u}.$$

The zero[th] covariance component is an even function: $B_0^{(\xi)}(-u)=B_0^{(\xi)}(u)$. It is also a positive definite function [6, 12, 13]. Thus $B_0^{(\xi)}(u)$ has all the properties of the covariance function of stationary random processes. Therefore, this quantity is called a covariance function of stationary approximation of PNRP [6, 12, 13]. If

$$\int_{-\infty}^{\infty}\left|b_\xi(t,u)\right|du<\infty,$$

then we can introduce the function

$$f_\xi(\omega,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}b_\xi(t,u)e^{-i\omega_0 u}du,$$

which is called the instantaneous spectral density of PNRP. Taking into account the Fourier series in Eq. (2), we have

$$f_\xi(\omega,t)=\sum_{k\in\mathbb{Z}}f_k^{(\xi)}(\omega)e^{ik\omega_0 t},$$

where

$$f_k^{(\xi)}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}B_k^{(\xi)}(u)e^{-i\omega u}du. \tag{3}$$

Proceeding from the PNRP series representation [6, 9, 13],

$$\xi(t)=\sum_{k\in\mathbb{Z}}\xi_k(t)e^{ik\omega_0 t}, \tag{4}$$

where $\xi_k(t)$ are jointly stationary random processes, we can deduce that the properties of the mean in Eq. (1) and covariance function in Eq. (2) are determined by the properties of modulating processes $\xi_k(t)$. The mathematical expectations of $\xi_k(t)$ are equal to the Fourier coefficients of the mean function $m(t):E\xi_k(t)=m_k$. The cross-covariance functions $R_{kl}(u)=E\overline{\overset{\circ}{\xi}_k(t)}\overset{\circ}{\xi}_l(t+u)$, where

$\overset{\circ}{\xi}_k(t)=\xi_k(t)-m_k$, determine the Fourier coefficients of the PNRP covariance function with the number $k=l-r$:

$$B_k^{(\xi)}(u)=\sum_{l\in\mathbb{Z}} R_{l-k,l}^{(\xi)}(u)e^{il\omega_0 u} . \quad (5)$$

It follows from Eq. (5) that the random process in Eq. (3) is periodically non-stationary of the second order only in the case when some of the cross-covariance functions of the modulation processes are not equal to zero. The zero[th] covariance component is defined by the auto-covariance functions of $\xi_l(t)$:

$$B_0^{(\xi)}(u)=\sum_{l\in\mathbb{Z}} R_{ll}^{(\xi)}(u)e^{-il\omega_0 u} .$$

Substituting Eq. (5) into Eq. (3), we obtain the equality:

$$f_k^{(\xi)}(\omega)=\sum_{l\in\mathbb{Z}} f_{l-k,l}^{(\xi)}(\omega-l\omega_0) , \quad (6)$$

where

$$f_{kl}^{(\xi)}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} R_{kl}^{(\xi)}(u)e^{-i\omega u}du .$$

It follows from (6) that the correlations of the PNRP spectral harmonics and the correlations of the modulating processes in series representation in Eq. (4) are equivalent.

3. High-frequency modulation of multi-component signal

Consider PNRS, which are represented by finite stochastic series

$$\xi(t)=\sum_{k=-L}^{L}\xi_k(t)e^{ik\omega_0 t}=\xi_0(t)+\sum_{k=l}^{L}\left[\xi_k^c(t)\cos k\omega_0 t+\xi_k^s(t)\sin k\omega_0 t\right]. \quad (7)$$

We suppose that the power spectral densities of the modulating processes

$$f_{kl}^{(\xi)}(\omega)=\frac{1}{2}\int_{-\infty}^{\infty} R_{kl}^{(\xi)}(u)e^{-i\omega u}du$$

are concentrated in the interval $[\lambda_0-\omega_m,\lambda_0+\omega_m]$ and that $\lambda_0-\omega_m>L\omega_0$. This modulation we shall call high-frequency modulation.

3.1. Signal representation for narrow-band modulation

We assume that the high-frequency quadratures in Eq. (4) are narrow-band (i.e. $\omega_m \ll \lambda_0$) and can be described by the Rice representations:

$$\xi_0(t)=p_0^c(t)\cos\lambda_0 t+p_0^s(t)\sin\lambda_0 t , \quad (8)$$

$$\xi_k^c(t)=p_k^c(t)\cos\lambda_0 t+p_k^s(t)\sin\lambda_0 t , \quad (9)$$

$$\xi_k^s(t)=q_k^c(t)\cos\lambda_0 t+q_k^s(t)\sin\lambda_0 t . \quad (10)$$

Using Eqs. (8)–(10) each component of Eq. (7) can be written in the form:

$$\xi_k^c(t)\cos k\omega_0 t+\xi_k^s(t)\sin k\omega_0 t=\mu_k^c(t)\cos(\lambda_0+k\omega_0)t+\mu_k^s(t)\sin(\lambda_0+k\omega_0)t+$$
$$+\nu_k^c(t)\cos(\lambda_0-k\omega_0)t+\nu_k^s(t)\sin(\lambda_0-k\omega_0)t ,$$

where

$$\mu_k^c(t)=\frac{1}{2}\left[p_k^c(t)-q_k^s(t)\right],\ \mu_k^s(t)=\frac{1}{2}\left[p_k^s(t)+q_k^c(t)\right],$$

$$\nu_k^c(t)=\frac{1}{2}\left[p_k^c(t)+q_k^s(t)\right],\ \nu_k^s(t)=\frac{1}{2}\left[p_k^s(t)-q_k^c(t)\right].$$

Introducing the complex random processes

$$\mu_k(t)=\frac{1}{2}\left[\mu_k^c(t)-i\mu_k^s(t)\right],\ \mu_{-k}(t)=\bar{\mu}_k(t),$$

$$\nu_k(t)=\frac{1}{2}\left[\nu_k^c(t)-i\nu_k^s(t)\right],\ \nu_{-k}(t)=\bar{\nu}_k(t),$$

for the signal representation gives:

$$\xi(t)=\xi_0(t)+\sum_{k=1}^{L}\left[\xi_k^+(t)+\xi_k^-(t)\right],$$

where

$$\xi_k^+(t)=\mu_k(t)e^{i(\lambda_0+k\omega_0)t}+\bar{\mu}_k(t)e^{-i(\lambda_0+k\omega_0)t}, \tag{11}$$

$$\xi_k^-(t)=\nu_k(t)e^{i(\lambda_0-k\omega_0)t}+\bar{\nu}_k(t)e^{-i(\lambda_0-k\omega_0)t}. \tag{12}$$

The time changes in the signal covariance function are defined by the correlations of the narrow-band components Eqs. (11) and (12), where the correlations of the component shifted by ω_0 define the first harmonic of the covariance functions, and the correlations of the components shifted by $2\omega_0$ define the second harmonics, etc. To obtain a compact formula for the signal covariance function, we rename each component $\xi_k^-(t)$ in the following way:

$$\xi_k^-(t)=\mu_{-k}^c(t)\cos(\lambda_0-k\omega_0)t+\mu_{-k}^s(t)\sin(\lambda_0-k\omega_0)t,$$

where $\mu_{-k}^c(t)=\nu_k^c(t),\mu_{-k}^s(t)=\nu_k^s(t)$. We can then represent the signal in the form:

$$\xi(t)=\sum_{k=-L}^{L}\left[\mu_k^c(t)\cos(\lambda_0+k\omega_0)t+\mu_k^s(t)\sin(\lambda_0+k\omega_0)t\right]. \tag{13}$$

3.2. Extraction of quadratures and signal band-pass filtering.

To analyze the structure of the quadrature correlations in more detail, we can separate the narrow-band components:

$$\xi_k^+(t)=\mu_k^c(t)\cos(\lambda_0+k\omega_0)t+\mu_k^s(t)\sin(\lambda_0+k\omega_0)t, \tag{14}$$

$$\xi_k^-(t)=\nu_k^c(t)\cos(\lambda_0-k\omega_0)t+\nu_k^s(t)\sin(\lambda_0-k\omega_0)t, \tag{15}$$

using filtering with the corresponding transfer function, i.e.:

$$H_k(\omega)=\begin{cases}1,\ \omega\in\left[\lambda_0\pm k\omega_0-\dfrac{\omega_0}{2},\lambda_0\pm k\omega_0+\dfrac{\omega_0}{2}\right],\\ 0,\ \textit{for other frequencies}.\end{cases}$$

A Hilbert transform of Eqs. (14) and (15) gives:

$$\eta_k^+(t)=H\left\{\xi_k^+(t)\right\}=\mu_k^c(t)\sin(\lambda_0+k\omega_0)t-\mu_k^s(t)\cos(\lambda_0+k\omega_0)t, \tag{16}$$

$$\eta_k^-(t)=H\left\{\xi_k^-(t)\right\}=\nu_k^c(t)\sin(\lambda_0-k\omega_0)t-\nu_k^s(t)\sin(\lambda_0-k\omega_0)t. \tag{17}$$

From Eqs. (14) and (16), and Eqs. (15) and (17), we obtain:

$$\mu_k^c(t)=\xi_k^+(t)\cos(\lambda_0+k\omega_0)t+\eta_k^+(t)\sin(\lambda_0+k\omega_0)t, \tag{18}$$

$$\mu_k^s(t)=\xi_k^+(t)\sin(\lambda_0+k\omega_0)t-\eta_k^+(t)\cos(\lambda_0+k\omega_0)t, \tag{19}$$

$$\nu_k^c(t)=\xi_k^-(t)\cos(\lambda_0-k\omega_0)t+\eta_k^-(t)\sin(\lambda_0-k\omega_0)t, \tag{20}$$

$$\nu_k^s(t)=\xi_k^-(t)\sin(\lambda_0-k\omega_0)t-\eta_k^-(t)\cos(\lambda_0-k\omega_0)t. \tag{21}$$

The relations in Eqs. (18)–(21) can be used to create techniques for processing experimental data to extract the high-frequency quadratures and to analyze their properties. The covariance and spectral structure of the quadratures may have specific features for a given fault in the mechanism.

The signal power spectrum density is equal to the sum of the power spectral density of the narrow-band components in Eqs. (14) and (15), i.e.

$$f_0^{(\xi)}(\omega) = f_{\xi_0}(\omega) + \sum_{k=1}^{L}\left[f_{\xi_k^+}(\omega) + f_{\xi_k^-}(\omega)\right], \tag{22}$$

where

$$f_{\xi_0}(\omega) = \frac{1}{\pi}\int_0^\infty R_{\xi_0}(u)\cos\omega u du, \tag{23}$$

$$f_{\xi_k^-}(\omega) = \frac{1}{\pi}\int_0^\infty R_{\xi_k^-}(u)\cos\omega u du,\ f_{\xi_k^+}(\omega) = \frac{1}{\pi}\int_0^\infty R_{\xi_k^+}(u)\cos\omega u du. \tag{24}$$

The spectra in Eqs. (23) and (24) contain sharp peaks around the points $k\omega_0$, which are concentrated within the intervals $\left[k\omega_0 - \frac{\omega_0}{2}, k\omega_0 + \frac{\omega_0}{2}\right]$. Then the signal spectrum in Eq. (22) belongs to band $\left[\lambda_0 - \left(L + \frac{1}{2}\right)\omega_0, \lambda_0 + \left(L + \frac{1}{2}\right)\omega_0\right]$.

Let us consider the properties of the signal in Eq. (13) after band-pass filtering; the transfer function for $\omega > 0$ is defined by

$$H_k(\omega) = \begin{cases} 1, \omega \in \left[\lambda_0 - \left(L - \frac{1}{2}\right)\omega_0, \lambda_0 + \left(L + \frac{1}{2}\right)\omega_0\right], \\ 0, \quad \textit{for other frequencies}. \end{cases} \tag{25}$$

The signal in Eq. (13) after filtering is presented by the following expression (where $N < L$):

$$\xi_f(t) = \sum_{k=-N}^{N}\left[\mu_k^c(t)\cos(\lambda_0 + k\omega_0)t + \mu_k^s(t)\sin(\lambda_0 + k\omega_0)t\right]. \tag{26}$$

For its covariance function, we get

$$b_{\xi_f}(t,u) = \sum_{r=-2N}^{2N}\sum_{l\in S_1}\begin{bmatrix} R_{\mu_{l-r}\mu_l}^c(u)\cos\left[r\omega_0 t + (\lambda_0 + l\omega_0)u\right] + \\ + R_{\mu_{l-r}\mu_l}^{cs}(u)\sin\left[r\omega_0 t + (\lambda_0 + l\omega_0)u\right] \end{bmatrix},$$

where $S_1 = \{-N,\ldots,r+N\}$ for $r \le 0$ and $S_1 = \{r-N,\ldots,N\}$ for $r > 0$. The variance of the output signal in Eq. (26) is equal to

$$b_{\xi_f}(t,0) = B_0^{(\xi_f)}(0) + \sum_{r=1}^{2N} C_r^{(\xi_f)}(0)\cos r\omega_0 t, \tag{27}$$

where

$$B_0^{(\xi_f)}(0) = \sum_{l=-N}^{N} R_{\mu_l}^c(0),\ C_r^{(\xi_f)}(0) = 2\sum_{l=r-N}^{N} R_{\mu_{l-r}\mu_l}^c(0).$$

As we can see, the filtering of the signal in Eq. (13) with the bandwidth in Eq. (25) reduces the number of the harmonics for its variance and also changes the value of their amplitudes. This is a consequence of the reduction of the number of the components of the signal in Eq. (13), whose cross-covariances result in time changes of the variance. Note that filtering also reduces the power of the stationary background, which is determined by the zero$^{\text{th}}$ covariance component in Eq. (27).

4. Discussion

It should be noted that the works which devoted to an envelope or square envelope analysis of vibration [14–21], largely stimulates investigations, the results of which are presented in this paper. The envelope technique is empirical, and the results were interpreted as a blind transfer of the definitions and well-known consequences of the Rice representation analysis for the simplest case when the spectra of the deterministic quadratures are narrower than the frequency of the harmonic carrier.

The theoretical analysis performed above illustrates that this interpretation is incorrect in cases where vibrations are modeled as PNRSs, which can generally be represented by a superposition of the amplitude- and phase-modulated carrier harmonics. This multi-component superposition adequately describes the properties of the stochasticity and the recurrence of the numerous natural and man-made processes, including vibrations [6–13]. Using the Hilbert transform, the component quadratures can be extracted and their auto- and cross-covariance functions can be estimated on the basis of the obtained time series. In this way, the quadratures of the high-frequency oscillations that modulate the PNRS carrier harmonics can also be studied.

5. Conclusions

It has been shown that the application of the Hilbert transform to a PNRS, the carrier harmonics of which are amplitude or amplitude-phase modulated by high-frequency, jointly stationary random processes, does not change the structure of the signal covariance; that is, the Fourier coefficients of the covariance functions of the signal and its Hilbert transform (the covariance components) are the same.

The issue of filtering of the raw signal for selection of the informative frequency band must also be re-formulated. It is necessary to consider it in terms of the filtering of a PNRS, which has some special features that must be taken into consideration for a more effective choice of band. In the case of narrow-band modulation, a PNRS is represented by the superposition of the high-frequency, narrow-band components which are stationary (but jointly, periodically non-stationary) random processes. The component quadratures can be extracted using the Hilbert transform. An auto- and cross-covariance analysis of the quadratures allows us to study the covariance structure of a PNRS in more detail.

6. References

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