Adaptation Applying of Economic Growth Theoretical Models

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Abstract

The article emphasizes the importance of the adaptive use of theoretical results in computer modeling of economic growth. Computer models prove to be a powerful tool for analyzing and forecasting economic processes, but they have their advantages and limitations. Positive aspects include the inclusion of various factors in the model, the decomposition of the economic system, the consideration of international trade, and the possibility of modification. The limitations include unrealistic assumptions, the absence of some aspects (such as the shadow economy), and the failure to take into account economic cycles. It is concluded that for practical application it is important to get rid of unrealistic assumptions and develop system models based on mathematical validity.

Keywords

Economic growth, adaptive application, nonlinear evolution, digital economy.

1. Introduction

Economic growth, being an important concept for the economy, does not always accurately reflect the real situation, as not all the available factors of the evolutionary process are taken into account [1, 2]. At the same time, a coherent understanding of the nature of economic growth is key to improving the standard of living of society and its place in the international arena. To date, a significant number of one-dimensional Mathematical Models (MM) and their modifications have been developed that explain economic growth in different ways, since they take into account one or more interrelated factors, i.e. there is no systematic approach [3–4].

The purpose of the article. By testing the above-mentioned classical (one-dimensional) MM of economic dynamics based on real data, the main factors influencing the level of economic growth are to be experimentally determined. A secondary goal is to find out why orthodox models of economic growth are mainly of theoretical importance, and their practical application is very limited.

The statement of basic material. **Theories and models of economic growth from the perspective of IT.** The problem of economic growth raises the question of what forces drive growth and economic development. Whether the same factors and in the same proportions will remain decisive for future economic growth as they have been in the past.

The earliest studies on the causes of differences in the level of prosperity between countries date back to the end of the 18th century. The most famous work of this period is an essay by Thomas Malthus, which later grew into a theoretical movement called Malthusianism. According to this work, the population grows exponentially, while production capacity grows arithmetically, which will sooner or later lead to a shortage [1].

Classical economists saw the main determinants of economic growth in investment and improvements in productive capacity, according to Adam Smith [2].

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The first significant model of economic growth was the dynamic Harrod-Domar equation.

It is worth noting that the model is linear. The prerequisites for its derivation are as follows: firms in the country operate in perfect competition; only one product is produced, which is used exclusively for consumption and investment; population growth, savings rate, and labor efficiency are constant and externally determined; there is no fiscal policy, foreign trade or investment lag in the economy.

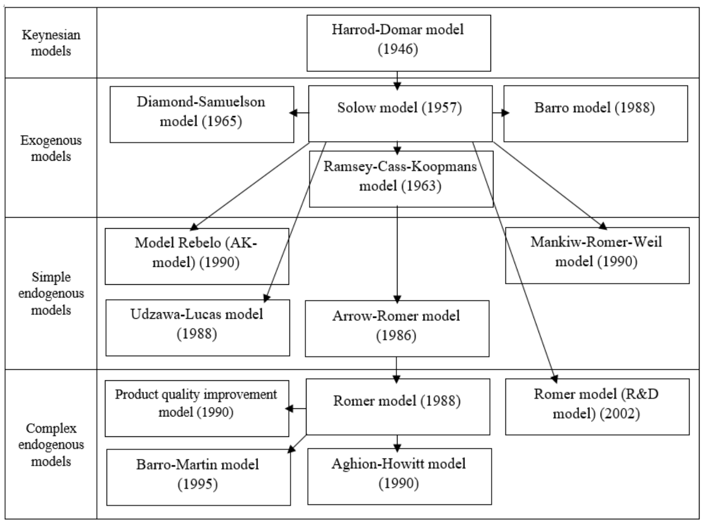

Figure 1: Genesis of economic growth models (developed by the authors based on [3–23])

The production function is two-factor and is described by the Leontief function:

$$Y = min(AK; BL)k, \quad (1)$$

where Y is GDP, K is capital, L is labor. Since the labor supply is excessive, GDP depends only on the level of capital in the economy. The Harrod-Domar model is described by the differential equation

$$\mathrm{BY}^{\bullet} + \mathrm{Y} = \mathrm{C}_0 * \mathrm{e}^{\mathrm{rt}}. \quad (2)$$

The analytical solution of which is written:

$$Y_{HD}(t) = \left[Y_0 - \frac{\mathrm{C}_0}{1 - Br}\right] * e^{\frac{1}{B}t} + \frac{\mathrm{C}_0}{1 - Br} * e^{rt}, \quad (3)$$

where Y_0 is the initial value of the output, C_0 is initial consumption value, B is capital intensity ratio, and r is consumption rate [3–4].

The dynamic model (2) explains the high growth rates of economies that initially have low domestic savings and capital-output ratios and a negative trade balance that is formed by capital imports. However, the model has the following shortcomings: it is based on a closed economy, which means that there is no explanation for the emergence of capital and labor flows in case of disequilibrium; the model cannot describe the phenomena of divergence and convergence between regions when the economy is a net exporter of only capital or only labor; the trajectory of balanced growth is not stable, as there are no stabilizers to dampen external influences on the economy; the assumption of no interaction between labor and capital, in the long run, is not valid. [3–4]

The nonlinear model of economic growth, being the first of the neoclassical models of this kind, was developed by Robert Solow. It was based on the Cobb-Douglas production function, where the economy was described by two factors: capital and labor, in comparison to the Harrod-Domar model and others like it. The formula for the equation is as follows:

$$Y = AK^{\alpha}L^{\beta}, \quad (4)$$

where α and β are the elasticities of change of capital and labor, respectively, and the coefficient A is responsible for technical progress. Additional prerequisites for the model are: capital intensity, $k = K/L$ is not a constant, as in Keynesian models, but varies depending on the macroeconomic situation in the country; the price of goods, services, and resources is set by the market mechanism; the growth rate of labor resources is equal to the average growth rate of population, but the dynamism of wages is not taken into account; there is a hypothesis that population growth and technical progress are absent at the initial stage; the rates of saving, depreciation, technical progress, elasticities of capital and labor, and population growth are constants, although they vary [5–7].

Finally, Solow's equation takes the form:

$$k^{\bullet} = sAk^{\alpha} - (d + n)k, \quad k_0 = k(t_0), \qquad (5)$$

where the variable *k = k(t)* corresponds to the capital intensity, $k^{\bullet} = \frac{dk}{dt}$ is its first derivative, coefficient *s* is capital accumulation rate, constants *A* and α are Cobb-Douglas functions, accordingly *A* reflects indirect costs, and the value of α is the elasticity, coefficient, *s* is capital accumulation rate, *d* is level of capital disposal, *n* is the average growth rate of the employed population. And what's more *d+n = λ* [5–7].

By integrating Solow's differential equation (5), its analytical solution is written:

$$k(t) = \left[(k_0^{1-\alpha} - \frac{s(1-\alpha)}{\lambda})e^{-\lambda t(1-\alpha)} + \frac{sA(1-\alpha)t}{\lambda}\right]^{\frac{1}{1-\alpha}}. \qquad (6)$$

In theoretical terms, the Solow model better explains the dynamics of GDP under its predecessors, explains the phenomenon of convergence/divergence, and describes the relationship between capital and labor, but has several unresolved problems. First of all, two main problems should be highlighted: the exogeneity of the rate of accumulation and the rate of technological progress.

A solution to the problem of exogeneity of the savings rate was proposed in the model of the same name by Frank Ramsey, Tjaling Koopmans, and David Cass. The preconditions and production functions of the model are similar to the Solow model, except for the homogeneity of the rate of accumulation. According to the model, economic agents in the system seek to maximize their utility when consuming a good, and the utility function takes the form:

$$U = \int_0^{\infty} u(c)e^{-(\rho-n)t}dt, \qquad (7)$$

where *c* is the consumption rate per unit of labor, or *c = C/L*, *ρ* is the coefficient of intertemporal preference of the consumer. Utility function $u(c)$ is separable, i.e., past and future consumption does not affect current utility, only current consumption does. Then the equation of the Solow model takes the form:

$$k^{\bullet} = Ak^{\alpha} - c - (d + n)k, \quad k_0 = k(t_0), \qquad (8)$$

Thus, the Ramsey-Cass-Koopmans model theoretically solves the problem of an exogenous rate of accumulation by transforming it into an endogenous one, but the problem of an exogenous rate of technological progress remains [8–10].

The authors of the overlapping generations model (Diamond-Samuelson) proposed a different method of finding the endogenous rate of saving. In this model, additional preconditions are added: agents live in two periods: in the first period they work, consume, and save, in the second period they only consume, spending the savings accumulated in the first period; there are no altruistic ties between generations; time changes discretely with a period of 20–25 years, which corresponds to the change of generations. For the Diamond-Samuelson model, the production function is similar to the Solow and Ramsey-Cass-Koopmans models. The dynamic equation of the model can be written as:

$$k^{\bullet} = \frac{s}{(1+n)(1+g)}(1-\alpha)Ak_{-1}^{\alpha}, \qquad (9)$$

where *s*—the accumulation rate, which is calculated by the formula:

$$s = \frac{(1-r)^{(1-\theta)/\theta}}{(1-\rho)^{1/\theta}(1+r)^{(1-\theta)/\theta}}, \qquad (10)$$

where θ is the time elasticity of consumption, ρ is the consumption discount rate, and *r* is the market interest rate. According to expression (10), the market interest rate affects the saving rate: its increase increases the available funds for investment, reducing the demand for credit.

In practice, the length of the period is a significant drawback of the model, as technological change is much faster, capital is actively renewed, and the impact of long economic cycles with a gradually decreasing period [11, 12].

In the period from 1986 to 1990, several mathematical models of nonlinear economic dynamics appeared, offering a solution to the problem of exogeneity of technological progress. One of the first was the Sergio Rebelo model (AK model), which got its second name from the way the production function looks. There are two common forms of the model: original and simplified.

The simplified model is a one-sector model, with the production function taking the form:

$$Y = AK. \qquad (11)$$

An important step towards endogeneity was the expanded interpretation of capital to include not only physical but also human capital, which is a set of knowledge, skills, and abilities used to meet the various needs of individuals and society as a whole. The utility function has undergone some changes compared to the Ramsey-Cass-Koopmans model:

$$U = \int_0^\infty \frac{c^{1-\theta} - 1}{1-\theta} e^{-(\rho-n)t} dt. \quad (12)$$

In this case, the basic equation of the model takes the form:

$$k^\bullet = sAk - (d+n)k, \quad k_0 = k(t_0), \quad (13)$$

and the accumulation rate is calculated by the formula:

$$s = \sigma + \frac{(d+n) - \sigma(d+\rho)}{A}, \quad (14)$$

where σ is the constant replacement rate. [13] From expression (14), we can conclude that the relationship between the constant rate of substitution (the elasticity of substitution of one factor for another) and the saving rate is that when the constant rate of substitution is high, it is easy to substitute one factor for another, i.e. capital easily replaces labor and, as a result, when the constant rate of substitution is high, the capital intensity of production grows faster.

The original model is two-sectoral: the consumption sector and the investment sector. The consumption sector has a Cobb-Douglas production function of the form:

$$C = BK_C{}^\alpha L^\beta, \quad (15)$$

and the investment sector has a production function of the form:

$$I = AK_S, \quad (16)$$

where A and B are technological parameters, K_C and K_S are capital in the consumer and investment sectors, respectively.

For the original model, the basic equation can be derived similarly to the simplified one:

$$k^\bullet = s(Bk_C{}^\alpha + Ak_S) - (d+n)k, \quad k_0 = k(t_0). \quad (17)$$

The model has the advantage of being simple and does not include transitional dynamics. However, the consequence of its simplicity is that the concept of “capital” includes many different types of activities: physical capital, human capital, education, creation of new goods, which makes the model rather limited. At the same time, the model does not explicitly account for technological progress and does not reveal the purposeful activity of economic agents to invest in new technologies to make a profit [13].

Gregory Mankiw, David Romer, and David Weil took a different approach, proposing their solution to the problem of exogeneity of technological progress. The Mankiw-Romer-Weil model is essentially a modification of the Solow model with the addition of human capital (H) to the model. Thus, the production function was transformed to the following form:

$$Y = AK^\alpha H^\beta L^{1-\alpha-\beta}, \quad (18)$$

and the dynamic model itself takes the form of a system of equations

$$\begin{aligned} k^\bullet &= s_k A k^\alpha h^\beta - (d+n)k, \quad k_0 = k(t_0), \\ h^\bullet &= s_h A k^\alpha h^\beta - (d+n)h, \quad h_0 = h(t_0), \end{aligned} \quad (19)$$

where s_k is the rate of accumulation of physical capital, a s_h is the rate of human capital accumulation, and h is human capital per unit of labor [14].

Since the model does not take into account the achievements of the Ramsey-Cass-Koopmans model and others, the advantage of endogenous technological progress is offset by other limitations inherent in the Solow model.

Another solution to the problem of exogenous technological progress was presented in the Uzawa-Lucas model. The production function for the model takes the form:

$$Y = K^\alpha [uhL]^\beta h_{\alpha t}^\delta, \quad (20)$$

where u is part of labor costs in human capital, H is human capital reserve, $h_{\alpha t}$ is the average level of human capital in the economy over a period of time t. The main equation of the model:

$$\begin{aligned} k^\bullet &= sk^\alpha h^{1-\alpha+\delta} u^{1-\alpha} - nk, \quad k_0 = k(t_0) \\ h^\bullet &= \varphi h(1-u), \quad h_0 = h(t_0), \end{aligned} \quad (21)$$

where φ is learning effectiveness. The utility function is similar to the equation (12) [15–17]. It is noteworthy that there is a transition from a one-dimensional to a systematic mathematical description of the economy.

Empirical studies have shown a very weak impact of human capital on aggregate output. Therefore, the model did not provide an exhaustive answer to the question of the causes of economic growth, although it contributed to their understanding.

The next model is the Paul Romer and Kenneth Arrow model or the activity-based learning model. The premises of the model are similar to the Uzawa-Lucas model. The production function is the unmodified Cobb-Douglas equation (4). The model assumes that technical progress depends on the knowledge acquired by employees in practice (hence the name of the model). And knowledge depends on the total amount of capital employed in the economy. The technical progress coefficient from the Cobb-Douglas function is calculated as:

$$A = BK^{\emptyset}, \quad (22)$$

where B is capital efficiency in knowledge generation, $\emptyset$ is elasticity of capital in knowledge generation. The main equation of the model takes the form:

$$K^{\bullet} = sB^{1-\alpha}K^{\alpha+\emptyset(1-\alpha)}L^{1-\alpha} - dK, \quad K_0 = K(t_0). \quad (23)$$

A significant drawback of the model is the direct dependence on the growth rate of labor resources, which the authors explain by the effect of knowledge spillovers, which allows each firm to receive an external effect from the volume of capital in the economy. In practice, there is a different level of economic connectivity between regions, which requires the inclusion of a certain coefficient for the level of knowledge spillovers in the model. In addition, the direct dependence of growth rates on labor resources implies that large countries should grow much faster than small ones, which has not been empirically confirmed [18].

The development of the Arrow-Romer model was facilitated by Paul Romer's research—a model of increasing product diversity, which is the key basis for further generalization.

According to the model, there are three sectors in the economy: intermediate goods, final goods, and R&D. The final goods sector operates under conditions of perfect competition. The intermediate goods sector operates under monopolistic competition. The R&D sector sells its patents on invented products to the intermediate goods sector. The production function has been replaced by the Dixit-Stiglitz function, which has the form:

$$Y = AL^{1-\alpha}\int_0^N x_j^{\alpha}dj = AL^{1-\alpha}(N\underline{X})^{\alpha}N^{1-\alpha}, \quad (24)$$

where x is the volume of intermediate product j, N is the total number of intermediate products, $\underline{X}$ is average volume of intermediate product. The modeling usually uses a simplified power form of the production function. The consumer's utility function is similar to expression (12). The basic equation of the model is:

$$K^{\bullet} = K^{\alpha}AL^{1-\alpha}N^{1-\alpha} - C, \quad K_0 = K(t_0). \quad (25)$$

The growth rates of the main macroeconomic indicators can be found in the formula:

$$\gamma = \frac{1}{\theta}\left(\frac{\pi}{\mu} - p\right), \quad (26)$$

where π is firm profit, μ is R&D sector expenditure, p is the cost of industrial products, θ>0 is the time elasticity of consumption [19]. From expression (26) it follows that the growth rate of GDP and other indicators depends on firms' profits excluding costs and adjusted for the elasticity of consumption.

Significant disadvantages of the model are the lack of technology transfer between countries, the lack of dependence on product quality, and the dependence of growth rates on the labor force from the previous model.

The problem of the lack of dependence on product quality is solved in the model of product quality improvement, which is almost completely similar to Paul Romer's result, except for the addition of the product quality coefficient q to the model. Expressions (24) and (26) are similar for the above model. The basic equation is as follows:

$$K^{\bullet} = K^{\alpha}AL^{1-\alpha}N^{1-\alpha}q^{\alpha} - C, \quad K_0 = K(t_0). \quad (27)$$

A solution to another problem of Paul Romer's model, namely the lack of technology spillovers between countries, was proposed by Robert Barro and Xavier Sala-i-Martin in their model of technology diffusion.

Similarly, the model had the same premises as Paul Romer's model. According to the model, countries are divided into leading and following countries. For the leading countries, there are no differences from the Paul Romer model, and for the following countries, the growth rates of the main macroeconomic indicators can be found from the formula:

$$\gamma = \frac{1}{\theta}\left(\frac{\pi}{v} + \frac{v^{\bullet}}{v} - p\right), \quad (28)$$

where v is the cost of imitating technologies, θ>0 [21]. According to expression (28), the costs of imitating a technology are usually lower than full development, so imitator countries should have faster economic growth, but the growth rate will slow down steadily as they approach the level of development of the innovator countries.

Robert Barro also developed a modification of the Solow model with government spending. The production function of the model is as follows:

$$Y = AK^{\alpha}G^{1-a}L^{1-\alpha}, \quad (29)$$

where G is the amount of public spending. The main equation of the model is derived similarly to most economic growth models [20]:

$$K^{\bullet} = sAK^{\alpha}G^{1-a}L^{1-\alpha} - dK. \quad (30)$$

Aghion and Howitt proposed a model where they focused on the fact that old types of goods are regularly gradually replaced by new ones. The development of new technologies destroys old ones, so the life cycle of innovations should be limited, and the monopoly power gained after the development of a new product is temporary. Mathematically, this is expressed in the form of the consumer's utility function as follows:

$$U = \int_0^{\infty} y_{\tau}e^{-r\tau}d\tau, \quad (31)$$

where r is the rate of intertemporal preference of the consumer, which is equal to the interest rate. The consumer utility function of the model is chosen so that intertemporal preferences are linear. The production function in the model is the Dixit-Stiglitz function (24) with the condition that the coefficient A, and the number of intermediate products N=*const*. The basic equation of the model is similar to the expression (25) [22–23].

One common drawback of these models is the dependence of growth rates on the size of the economy. However, in this case, we are talking about the impact of the number of skilled workers on the growth of the quality of goods, which can be justified by the fact that the more skilled workers there are in the economy, the faster economic growth occurs.

The model proposed by D. Romer is a multi-sector model. Thus, there are two sectors in the economy: a sector that produces goods using some of the (1–A_L) i (1–A_K) labor and capital resources, respectively, and the sector that produces knowledge (scientific and technological innovations) using shares of A_L i A_K labor and capital resources. In the model, the Cobb-Douglas equation is modified as follows:

$$Y = A[(1-A_K)K]^{\alpha}[(1-A_L)L]^{\beta}. \quad (32)$$

The basic equation of the model can be expressed in the following differential equations:

$$K^{\bullet} = sA(1-A_K)^{\alpha}(1-A_L)^{1-\alpha}K^{\alpha}L^{1-\alpha}, \quad K_0 = K(t_0),$$
$$A^{\bullet} = B(KA_K)^{\gamma}(LA_L)^{1-\gamma}A^{\theta}, \quad A_0 = A(t_0), \quad (33)$$

where B is the efficiency of the combination of factors in the R&D sector, γ is capital elasticity in the R&D sector, θ is in the model is a parameter that accelerates or slows down the STD.

In addition, the study included some modifications to the Solow model:

- Model with foreign trade. The idea is to adjust the rate of economic growth by aˆ=ak, where a is the trade balance per unit of labor [24].
- Model with public capital means that total capital is divided into two parts: public (infrastructure, public goods) and private [25].
- Model with a land factor—the formula is similar to the Mankiw-Romer-Weil model.
- Model with taxes—tax burden slows down economic growth by g is taxes per unit of labor.
- Model with a time lag where the labor growth rate is equal to $n = a - bL_b$, where L_b is employment in one of the previous periods [26].
- -Solow's multisectoral model is based on the division into primary sector (agriculture and mining), secondary sector (heavy and light industry), and tertiary sector (services).

Adaptive methodology for computer-aided research. The methodology of studying based on computer modeling and theoretical developments of economists is distinguished by its distinct innovation and powerful potential for understanding complex economic processes. This approach addresses the limitations of classical research methods, allowing for a deeper and more accurate analysis of the impact of various factors on the economy.

The use of computer models allows us to create a virtual environment for economic experimentation, where we can study various scenarios and options for economic development. This innovative method allows us to make more accurate and informed forecasts, as well as to identify unexpected relationships and opportunities for effective development. Combining the theoretical developments of economists with computer modeling allows us to create the basis for new research and innovations in the field of economics.

The algorithm for a numerical experiment with economic growth models includes defining models, reducing them to a comparable form, processing data, replacing gaps, conducting a numerical experiment, and evaluating the quality of models to obtain and interpret results.

Economic growth models have much in common with each other, but each model differs to some extent in terms of certain assumptions, interpretation of economic processes, and their explanations. Comparing models on a scale of worse/better would be incorrect, given their preconditions and specifics, which overlap in many aspects. Comparison means determining the cause-and-effect relationships between the main object of model modification and the dynamics of the error size in practice.

To ensure the correctness of the modeling and interpretation of the results, economic growth models need to be reduced to a "common denominator." This includes consideration of continuous and non-linear models, accounting for depreciation of physical capital, reduction to a common indicator, and consideration of the period from 1960 to 2021. The Diamond-Samuelson and Harrod-Domar models were excluded due to their limited applicability and outdated assumptions. All the models considered (Table 1) were brought to a single form using the following transformation:

$$k^{\bullet} = \left(\frac{K}{L}\right)^{\bullet} = \frac{K^{\bullet}L - KL^{\bullet}}{L^2} = \frac{K^{\bullet}L}{LL} - \frac{KL^{\bullet}}{L^2} = \frac{K^{\bullet}}{L} - \frac{K}{L}\left(\frac{L^{\bullet}}{L}\right) = \frac{K^{\bullet}}{L} - nk. \tag{34}$$

Table 1

Summary models of economic growth

№	Model name	Production function	$K^{\bullet}=$	$k^{\bullet}=$
1	Solow-Swan	$Y = AK^{\alpha}L^{1-\alpha}$	$sAK^{\alpha}L^{1-\alpha} - dK$	$sAk^{\alpha} - (d+n)k$
2	Ramsey-Cass-Koopmans	$Y = AK^{\alpha}L^{1-\alpha}$	$AK^{\alpha}L^{1-\alpha} - C - dK$	$Ak^{\alpha} - c - (d+n)k$
3	Rebelo simplified	$Y = AK$	$sAK - dK$	$sAk - (d+n)k$
4	Rebelo original	$Y = BK_C{}^{\alpha}L^{1-\alpha}+AK_S$	$s(BK_C{}^{\alpha}L^{1-\alpha} + AK_S)$ $-dK$	$s(Bk_C{}^{\alpha} + Ak_S)$ $-(d+n)k$
5	Mankiw-Romer-Weil	$Y = AK^{\alpha}H^{\beta}L^{1-\alpha-\beta}$	$sAK^{\alpha}H^{\beta}L^{1-\alpha-\beta} - dK$	$s_k Ak^{\alpha}h^{\beta} - (d+n)k$
6	Uzawa-Lukas	$Y = K^{\alpha}[uhL]^{1-\alpha}h_{\alpha t}^{\delta}$	$sK^{\alpha}[uhL]^{1-\alpha}h_{\alpha t}^{\delta} - dK$	$sk^{\alpha}h^{1-\alpha+\delta}u^{1-\alpha}$ $-(d+n)k$
7	Arrow-Romer	$Y =$ $B^{1-\alpha}K^{\alpha+\emptyset(1-\alpha)}L^{1-\alpha}$	$B^{1-\alpha}K^{\alpha+\emptyset(1-\alpha)}L^{1-\alpha}$ $-C - dK$	$B^{1-\alpha}K^{\emptyset(1-\alpha)}k^{\alpha} - c$ $-(d+n)k$
8	Romer	$Y = AL^{1-\alpha}\int_0^N x_j^{\alpha}dj$	$K^{\alpha}AL^{1-\alpha}N^{1-\alpha} - C$ $-dK$	$AN^{1-\alpha}k^{\alpha} - c$ $-(d+n)k$
9	Improving product quality	$Y =$ $AL^{1-\alpha}\int_0^N x_j^{\alpha}q_j^{\alpha}dj$	$K^{\alpha}q^{\alpha}AL^{1-\alpha}N^{1-\alpha} - C$ $-dK$	$AN^{1-\alpha}k^{\alpha}q^{\alpha} - c$ $-(d+n)k$
10	Barro	$Y = AK^{\alpha}G^{1-\alpha}L^{1-\alpha}$	$sAK^{\alpha}G^{1-\alpha}L^{1-\alpha} - dK$	$sAk^{\alpha}G^{1-\alpha} - (d+n)k$
11	Barro-Martin	$Y = AL^{1-\alpha}\int_0^N x_j^{\alpha}dj$	$K^{\alpha}AL^{1-\alpha}N^{1-\alpha} - C$ $+\frac{v^{\bullet}}{v\emptyset}K - dK$	$AN^{1-\alpha}k^{\alpha} - c$ $-\left(d + n - \frac{v^{\bullet}}{v\emptyset}\right)k$
12	Aghion-Howitt	$Y =$ $A\gamma^{t}L^{1-\alpha}\int_0^N x_j^{\alpha}dj$	$K^{\alpha}\gamma^{t}AL^{1-\alpha}N^{1-\alpha} - C$ $-dK$	$AN^{1-\alpha}k^{\alpha}\gamma^{t} - c$ $-(d+n)k$
13	Romer R&D	$Y = A[(1-A_K)K]^{\alpha}$ $[(1-A_L)L]^{\beta}$	$sA(1-A_K)^{\alpha}(1-A_L)^{1-\alpha}$ $K^{\alpha}L^{1-\alpha} - dK$	$sA(1-A_K)^{\alpha}(1-A_L)^{1-\alpha}$ $k^{\alpha} - (d+n)k$
14	Modification with international trade	$Y = AK^{\alpha}L^{1-\alpha}$	$sAK^{\alpha}L^{1-\alpha} - dK - \hat{a}$	$sAk^{\alpha} - (d+n)k - \hat{a}$
15	Modification with state capital	$Y = AK_p{}^{\alpha}K_g{}^{\beta}L^{1-\alpha-\beta}$	$sAK_p{}^{\alpha}K_g{}^{\beta}$ $L^{1-\alpha-\beta} - dK$	$s_k Ak_p{}^{\alpha}k_g{}^{\beta}$ $-(d+n)k$
16	Modification with the land factor	$Y = AK^{\alpha}N^{\beta}L^{1-\alpha-\beta}$	$sAK^{\alpha}N^{\beta}L^{1-\alpha-\beta} - dK$	$s_k Ak^{\alpha}n_N{}^{\beta} - (d+n)k$
17	Modification with taxes	$Y = AK^{\alpha}L^{1-\alpha}$	$sAK^{\alpha}L^{1-\alpha} - G - dK$	$sAk^{\alpha} - g - (d+n)k$
18	Modification with a time lag	$Y = AK^{\alpha}L^{1-\alpha}$	$sAK^{\alpha}L^{1-\alpha} - dK$	sAk^{α} $-(d + a - bL_b)k$
19	Multi-sector modification	$Y = AK_1{}^{\alpha}L_1{}^{1-\alpha} + \cdots$	$sAK_1{}^{\alpha}L_1{}^{1-\alpha} - dK + \cdots$	$L_1/L(sAk_1{}^{\alpha} -$ $(d+n)k_1) + \cdots$

To evaluate the quality of economic growth models, 218.5 thousand models were built for the long-term, medium-term, and short-term periods. The assessment included a comparison of capital intensity growth rates with the real rate based on R^2, MAE, MRE, MSE, MSLE, RMSE, RMSLE, and risk. Standardization of calculation approaches was important for obtaining objective and comparable results between different economic growth models.

For modeling economic growth, real data is an important component that provides the basis for successful quantitative analysis. The main focus in selecting data sources was on their reliability, relevance, and availability, including the use of official statistics, research, and databases of international and national organizations. The interest in careful selection of sources and coordination of data ensures the quality and objectivity of the research.

However, it is important to take into account possible problems such as random or systematic errors in the collected data, as well as the completeness of information that may affect the accuracy and adequacy of the modeling results. All these aspects pose challenges for the study and require a careful approach to ensure the objectivity and reliability of the results. Thus, due to the lack of data, the factor of the number of intermediate products is replaced by the factor of the number of firms, since it can be assumed at the macro level that each intermediate product is produced by a separate firm.

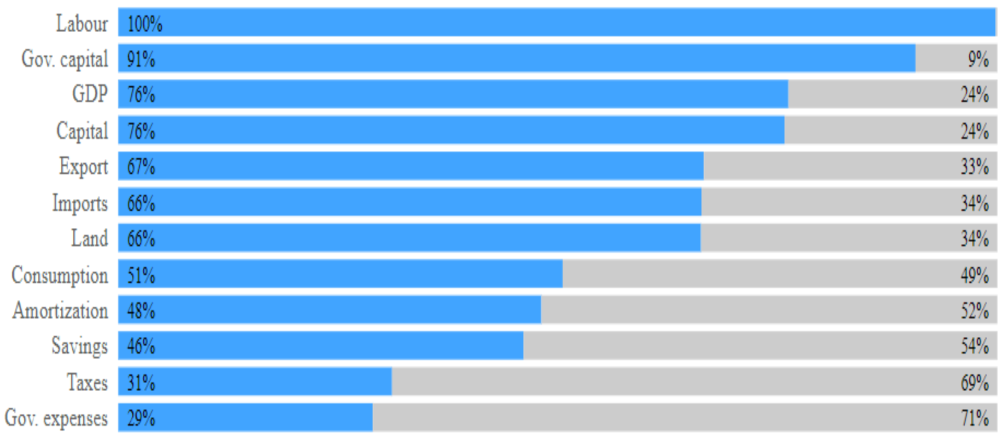

Figure 2: Completeness of data in the period from 1960–2021 (calculated based on [28–30])

The lack of data can significantly complicate the construction of economic growth models, especially when different indicators have different completeness. In such cases, methods such as neural networks and machine learning algorithms, mathematical modeling (regression, time series forecasting), or replacement with averages can be used. However, it is important to keep in mind that the use of neural networks can be difficult when there is limited data for the training set, while replacement with averages can lead to unreliable models, especially in the field of macroeconomics.

To interpolate and extrapolate the missing data (Fig. 2), we used an econometric model:

$$y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + a_5 x_5 + E, \quad (35)$$

where x_1 is trend sequence number *, x_2 is the country's population, x_3 is group average target (e.g., for GDP, this is the average GDP per capita of the group of countries to which the country whose indicator is modeled belongs), x_4 is a similarly known indicator for the country under consideration (e.g., GDP for capital), x_5 is the previous/next value of the indicator under review (if any).

*Note: the trend number takes the group of values that result in the lowest total absolute error: a linear relationship ($y = x$), parabolic relationship ($y = x^2, y = \sqrt{x}$), hyperbolic relationship ($y = \frac{1}{x}$) and logarithmic relationship ($y = ln\ ln\ x$).

2. Research Results

Economic growth models, grounded in economic theory, identify the interaction of sectors and factors of production, helping to understand the impact on economic growth and develop policies to improve living standards. Despite the theoretical basis, models are used in a limited way due to unrealistic conditions, such as ideality (full competition, constant technology), which make it difficult to predict in the real world. Restrictive conditions, such as a closed economy or ignoring the role of the state, also affect the quality of the models, and their consideration depends on the level of integration into the global economic system, or the level of state regulation, respectively.

Thus, from Fig. 3, we can conclude that all economic growth models on average give a high error since the values of the coefficient of determination are negative and significantly exceed the error from the mean values.

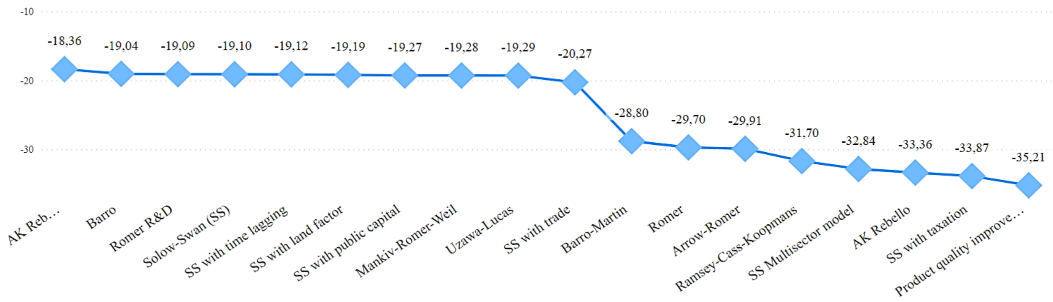

Figure 3: Coefficient of determination by economic growth models

We can also divide the models into two groups: the models that use the savings rate have a much smaller error than the models that use consumption per unit of labor. This is because although the indicators correlate, they do not fully do so in practice, the conditions of a closed economy $Y = C + S$ та $S = I$ are not fulfilled precisely because the active movement of capital, goods, services, and labor in the international market, as well as the existence of the "grey" and "shadow" economy.

On the one hand, gross savings do not reflect the full potential of capital growth due to the impact of foreign investment, government spending, and imports, which leads to an underestimation of savings. Even a comparison of the absolute values of gross accumulation and gross savings often shows a predominance of accumulation, especially in less developed countries.

On the other hand, the analysis of the relationship between the quality of the model and the length of the period (Fig. 4) on which it is based shows an increase in model errors, which is explained by the growing multifactoriality and reduction of the shadow economy, while this is partially offset by globalization processes.

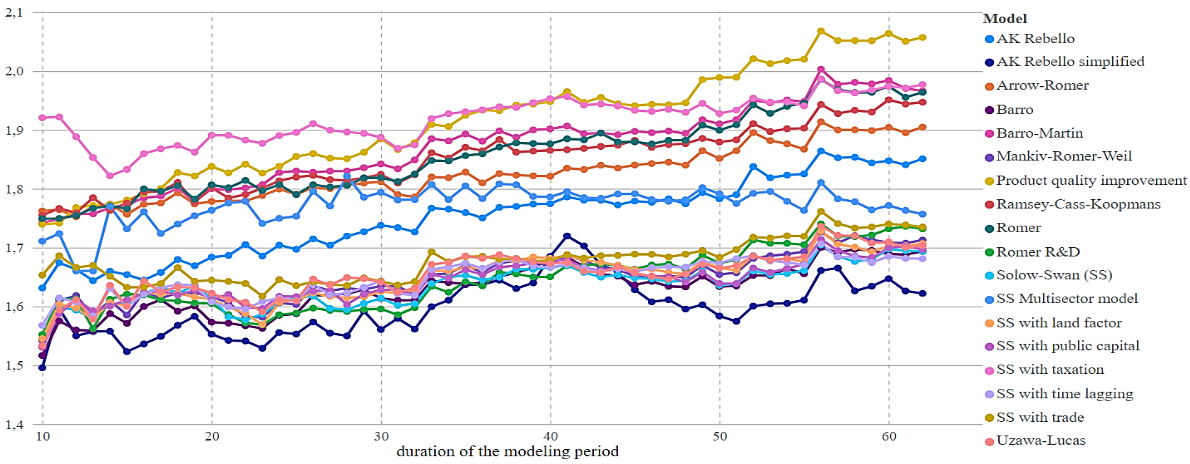

Figure 4: RMSLE by increasing the duration of the modeling period by economic growth models

After compensating for the shadow economy factor and building static models for each year separately (Fig. 5), the model errors have been increasing over time. This confirms the hypothesis that the processes of globalization and the intensification of international trade and investment make the restrictions of a closed economy less effective. Sharp increases in errors are observed during the phases of revival and boom in international investment and trade, and then the error decreases sharply, which is associated with a crisis/depression when the impact of international investment is minimized.

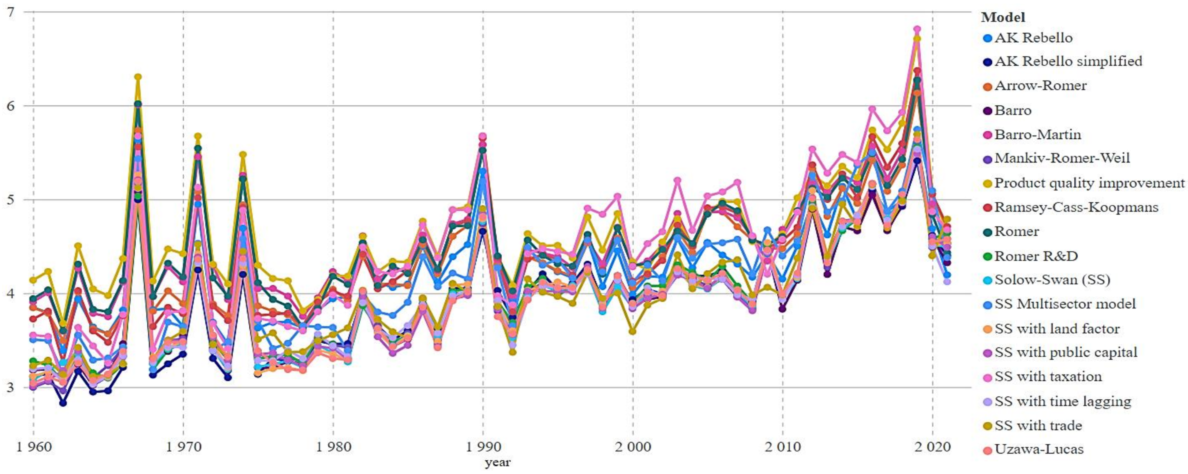

Figure 5: RMSLE by economic growth models

Fig. 6 shows that the relative error is on average higher for low-income countries. The main reasons for this are that: low-income countries often face more difficult economic conditions, including lack of access to sufficient capital, poor infrastructure, and limited opportunities for innovation; economies may be more vulnerable to external shocks, such as commodity price fluctuations, international financial crises, or changes in global trade, and may be vulnerable to internal political crises; and the shadow economy is much larger than in highly developed countries; structure and organization of such economies differs significantly from developed countries in terms of disproportionate prevalence of the primary sector of the economy and monopolistic risks.

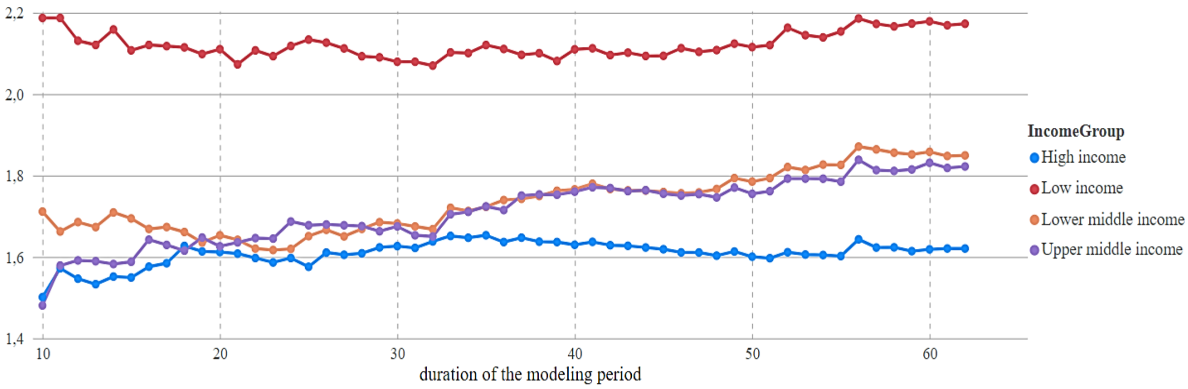

Figure 6: RMSLE by income group (based on the World Bank methodology)

A ranking approach based on quality indicators was used to compare economic growth models. The ranks were assigned in ascending order from 1 to 19, where 1 corresponds to the model with the lowest score and 19 to the highest. It is important to keep in mind that such comparisons are conditional due to the limitations of the models and the failure to take into account many of the significant factors described earlier.

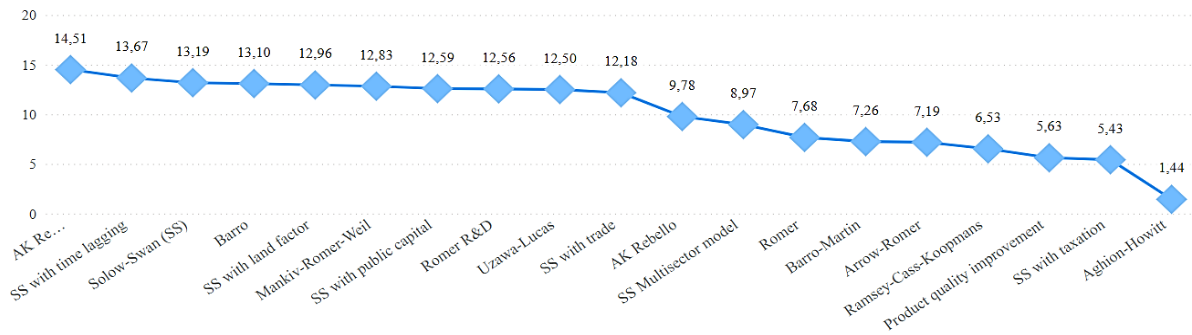

Figure 7: Average rank by economic growth model

According to Fig. 7, there are three key aspects of the comparison between the models.

First, models with a simpler production function perform better on average than those based on more complex functions. This does not mean that simpler models better reflect the economic situation (Fig. 8) but rather emphasizes that the subsequent modeling algorithm multiplies the error due to unrealistic assumptions.

It is worth noting that there is a time lag between the transition from savings to domestic investment, usually several years, but it can be longer. The dynamics of the transition of savings to investment is influenced by the phase of the economic cycle, so during recessions and depressions, investment activity is minimized, and during recoveries and booms, on the contrary, the share of domestic investment to foreign investment increases significantly. For developed countries, a significant share of savings is used to invest in other countries, where higher rates of return are available due to cheaper labor and means of production.

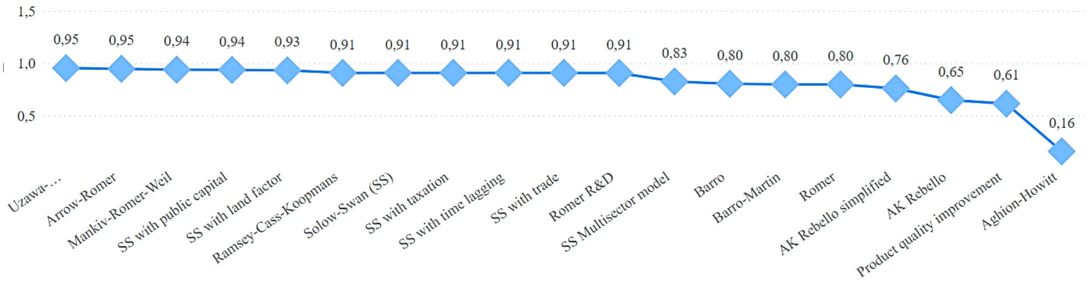

Figure 8: Average coefficient of determination of the production function by economic growth models

From Fig. 8, we can draw the following conclusions: the factors of human capital, land, and the division of capital into public and private have a positive impact on the quality of the production function; each production factor has an individual impact on gross output, and combining factors under a common parameter often only worsens the quality of the model; the presence of constant or trend factors in the production function significantly reduces the quality of the model; the inclusion of the innovation sector in the production function has a minimal positive impact on the final quality.

Secondly, the method of decomposing the economic system into sectors does not produce an unambiguous effect. Typically, such a model produces a higher error than its single-sector counterpart. For underdeveloped countries, the multisectoral approach yields better results, which is influenced by the specifics of such a model and the underlying production function: low-income countries have a predominant primary or secondary sector, thus these sectors are better modeled by the two-factor Cobb-Douglas production function; in highly developed countries, the service sector is predominant, so capital and labor factors are not enough to model it, and public and human capital must be taken into account; for the primary and secondary sectors, the labor factor loses its influence over time due to the processes of mechanization and automation of labor, which may lead to additional errors in forecasting; all sectors are closely interconnected by flows of goods and services, so this should be taken into account in the decomposition.

Third, the impact of foreign trade on capital growth is determined by the fact that imports affect consumption in the first place, while exports affect savings. Thus, a positive trade balance increases savings and investment.

Practical results show that the inclusion of trade in the model improves quality in the long run and worsens it in the short run (Fig. 9). The high level of a country's involvement in international trade increases its sensitivity to external crises and shocks, and often leads to an inadequate response in terms of the ratio of investment to foreign trade. Countries with different export structures react differently to external fluctuations. If the export structure is dominated by raw materials, such countries are less sensitive to risks, and if the export structure is dominated by final technological products, they are more sensitive.

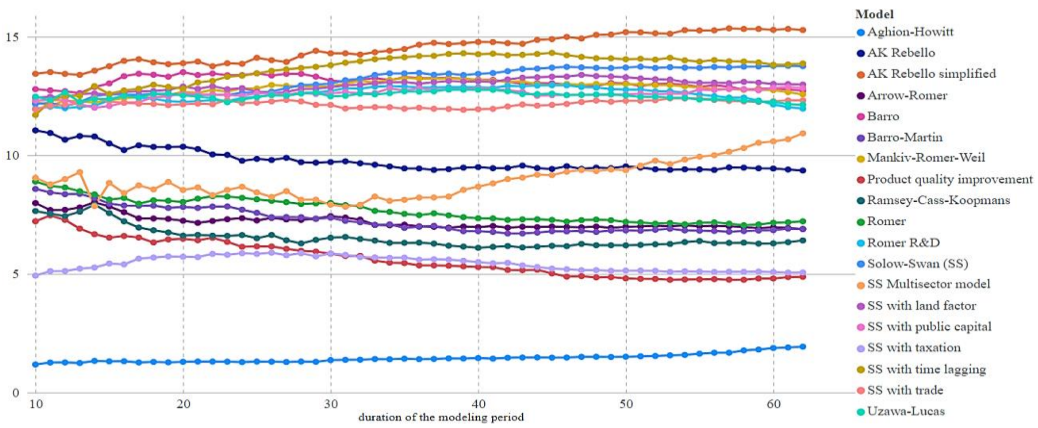

Figure 9: Average rank of economic growth models by period length

The modification of the Solow model with taxes is based on the hypothesis that taxes compensate for savings, but this does not reflect reality, as taxes also affect government spending, investment, and consumption. It is more efficient to include taxes in the production function, balancing the impact on private and public capital.

Among other things, several important conclusions can be drawn: the innovation sector has a decisive impact on economic growth, especially for highly developed countries that can import technology; the primary and secondary sectors have a significant impact on low-and middle-income countries; and the human capital factor is crucial for developed countries, as production is more demanding on the educational level of workers; for underdeveloped countries, an important factor in product quality due to large differences in quality levels; public capital plays a key role in low-income countries, where it is often significant in transitional or authoritarian systems.

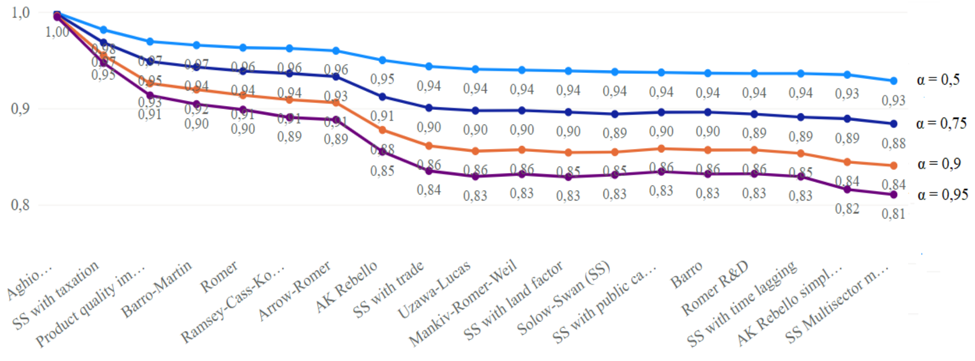

Figure 10: Level of risk by economic growth models with different parameters

According to Fig. 10, the lowest level of risk (the probability that the forecast value of the economic growth model will not fall within the confidence interval with a certain probability of a type II error) is associated with the multi-sector modification of the Solow model, which indicates that the decomposition method is reliable in terms of risk reduction. Multisectoral models of economic growth, due to their enhanced ability to take into account many factors and interrelationships, can reduce the risks of implausible or inaccurate forecasts and provide a more accurate analysis of economic processes.

3. Conclusion

It is important to emphasize the adaptive use of theoretical results in computer modeling of economic growth. Computer models, using a practical approach, are proving to be a powerful tool for analyzing and forecasting economic processes, adapting theoretical concepts to real-world conditions. Research conducted with the help of computer modeling of economic growth has significant potential for the future, especially in the development of a systematic approach that can take the analysis of economic processes to a new level. This opens up opportunities for accurate and realistic forecasts, which is key to achieving sustainable economic growth.

The positive aspects of the considered models are the inclusion of human capital, public capital, and land factors in the model—in addition to the main labor and physical capital; decomposition of the overall economic system into several simpler systems (sectors), such as the main production sectors and the innovation sector; consideration of international trade in modeling the dynamics of capital intensity; the dynamism of indicators and production factors; the possibility of a simple modification of existing models.

The limitations include: the requirement of a closed economy is not realistic in the context of increasing globalization and international division of labor; multidimensional models are more comprehensive and can better explain economic processes, while most of the models reviewed are unidimensional; savings in a given period do not equal investment even if the economy is closed; models do not include foreign investment and government and international transfers; and ignore the phenomenon of the "shadow" economy; economic cycles have a direct and significant impact on the amount of capital in the economy, which is not taken into account in the models; incorrect use of taxation and lack of mathematical mechanisms for the impact of taxes on public capital; market typology is an important factor in shaping supply and demand, which is practically not considered in growth models.

Considering the advantages and limitations of economic growth models, it can be concluded that they have limited practical application due to the size of the error and unrealistic assumptions. The analysis of factors in the process of assessing the economy allows for a more accurate consideration of the complex interrelationships that affect the development of the country, increasing the practical significance of economic growth models. For further development of the study, it is important to get rid of unrealistic assumptions and create systematic models based on mathematical validity, testing their effectiveness in practice.

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