Composition of L-Fuzzy contexts

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Abstract. In this work, we introduce and study the composition of two L-fuzzy contexts that share the same attribute set. Besides studying its properties, this composition allows to establish relations between the sets of objects associated to both L-fuzzy contexts.
We also define, as a particular case, the composition of an L-fuzzy context with itself.
In all the cases, we show some examples that illustrate the results.

Key words: Formal contexts theory, L-fuzzy contexts, Contexts associated with a fuzzy implication operator

1 Introduction

In some situations we have information that relates two sets X and Z to the same set Y and we want to know if these relations allow us to establish connections between X and Z. In the present work we will try to deal with the study of this problem using as tool the L-fuzzy Concepts Theory.

The Formal Concept Analysis developed by Wille ([13]) tries to extract some information from a binary table that represents a formal context (X, Y, R) with X and Y being two finite sets (of objects and attributes, respectively) and $R \subseteq X \times Y$. This information is obtained by means of the formal concepts which are pairs (A, B) with $A \subseteq X$, $B \subseteq Y$ fulfilling $A^* = B$ and $B^* = A$ (where $*$ is the derivation operator which associates to each object set A the set B of the attributes related to A, and vice versa). A is the extension and B the intension of the concept.

The set of the concepts derived from a context (X, Y, R) is a complete lattice and it is usually represented by a line diagram.

In some previous works ([4],[5]) we defined the L-fuzzy context (L, X, Y, R), where L is a complete lattice, X and Y the sets of objects and attributes respectively and $R \in L^{X\times Y}$ an L-fuzzy relation between the objects and the attributes, as an extension to the fuzzy case of the Wille's formal contexts when the relation between the objects and the attributes that we want to study takes values in a complete lattice L. When we work with these L-fuzzy contexts we use the derivation operators 1 and 2 defined by: For every $A \in L^X, B \in L^Y$

$$A_1(y) = \inf_{x \in X} \{I(A(x), R(x,y))\}, \quad B_2(x) = \inf_{y \in Y} \{I(B(y), R(x,y))\}$$

where I is a fuzzy implication operator defined in $(L, \leq)$, $I : L \times L \longrightarrow L$, which is decreasing in its first argument, and, A_1 represents, as a fuzzy set, the attributes related to the objects of A and B_2 the objects related to the attributes of B.

The information of the context is visualized by means of the L-fuzzy concepts which are pairs $(A, A_1) \in (L^X, L^Y)$ with $A \in \text{fix}(\varphi)$ the set of fixed points of the operator φ, being this one defined by the derivation operators 1 and 2 mentioned above as $\varphi(A) = (A_1)_2 = A_{12}$. These pairs, whose first and second components are the extension and the intension respectively, represent, as a fuzzy set, the set of objects that share some attributes.

The set $\mathcal{L} = \{(A, A_1) : A \in \text{fix}(\varphi)\}$ with the order relation $\leq$ defined as:

$$(A, A_1), (C, C_1) \in \mathcal{L}, \quad (A, A_1) \leq (C, C_1) \text{ if } A \leq C$$

(or equiv. $C_1 \leq A_1$) is a complete lattice that is said to be the L-fuzzy concept lattice ([4],[5]).

On the other hand, given $A \in L^X$, (or $B \in L^Y$) we can obtain the derived L-fuzzy concept applying the defined derivation operators. In the case of the use of a residuated implication operator (as it holds in this work), the associated L-fuzzy concept is (A_{12}, A_1) (or (B_2, B_{21})).

Other extensions of the Formal Concept Analysis to the fuzzy area are in [14], [12], [3], [8], [10], [11] and [6].

2 Composed formal contexts

The composition of formal contexts allows to establish relations between the elements of two sets of objects that share the same attribute set.

Definition 1. *Let $(X, Y, R1)$ and $(Z, Y, R2)$ be two formal contexts, the composed formal context is defined as the context $(X, Z, R1 \star R2)$, where $\forall (x,z) \in X \times Z$:*

$$R1 \star R2(x,z) = \begin{cases} 1 & \textit{if } R2(z,y) = 1, \ \forall y \textit{ such that } R1(x,y) = 1 \\ 0 & \textit{in other case} \end{cases}$$

That is, the object x is related to z in the composed context if z shares all the attributes of x in the original contexts.

Proposition 1. *The relation of the composed context, $R1 \star R2$, can also be defined as:*

$$R1 \star R2(x,z) = \min_{y \in Y} \{\max\{R1'(x,y), R2(z,y)\}\} \quad \forall (x,z) \in X \times Z$$

where $R1'$ is the negation of the relation $R1$, that is, $R1'(x, y) = (R1(x, y))'$ $\forall(x, y) \in X \times Y$.

This property will be helpful in the following sections.

Remark 1. Given the formal contexts $(X, Y, R1)$ and $(Z, Y, R2)$, the relation of the composed context $R1 \star R2$ is not necessarily the opposed of the relation $R2 \star R1$, that is, in general,

$$\text{There exists } (x, z) \in X \times Z \text{ such that } \; R1 \star R2(x, z) \neq R2 \star R1(z, x)$$

Example 1. Let us consider the formal contexts $(X, Y, R1)$ and $(Z, Y, R2)$, where $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3, y_4, y_5\}$, $Z = \{z_1, z_2, z_3, z_4\}$, and the respective relations are the following ones:

$R1$	y_1	y_2	y_3	y_4	y_5
x_1	0	1	1	0	1
x_2	1	1	0	1	0
x_3	0	0	1	0	1

$R2$	y_1	y_2	y_3	y_4	y_5
z_1	1	1	0	1	0
z_2	0	1	0	0	1
z_3	1	1	0	1	1
z_4	0	1	1	1	1

If we calculate the composition of the contexts defined above in the two possible orders, then the obtained relations are:

$R1 \star R2$	z_1	z_2	z_3	z_4
x_1	0	0	0	1
x_2	1	0	1	0
x_3	0	0	0	1

$R2 \star R1$	x_1	x_2	x_3
z_1	0	1	0
z_2	1	0	0
z_3	0	0	0
z_4	0	0	0

and, as can be seen, $(R1 \star R2)^{op} \neq R2 \star R1$.

This property will be helpful in the following sections.

2.1 Particular case: when a formal context is composed with itself

Let us analyze a particular case where some interesting results are obtained.

Proposition 2. *Let (X, Y, R) be a formal context. If (X, Y, R) is composed with itself, then the obtained context is $(X, X, R \star R)$ where the sets of objects and attributes are coincident and the relation $R \star R$ is a binary relation defined on X as follows:*

$$R \star R(x_1, x_2) = \min_{y \in Y}\{\max\{R'(x_1, y), R(x_2, y)\}\} \; \forall(x_1, x_2) \in X \times X$$

Remark 2. The object x_1 is related to attribute x_2 in the composed context, if in the original context the object x_2 has at least the same attributes than the object x_1.

Example 2. Returning to the formal context (X, Y, R) that we studied in the previous example, where the relation R was:

R	y_1	y_2	y_3	y_4	y_5
x_1	0	1	1	0	1
x_2	1	1	0	1	0
x_3	0	0	1	0	1

The composition of this context with itself is the context $(X, X, R \star R)$, and relation is given by the table:

$R \star R$	x_1	x_2	x_3
x_1	1	0	0
x_2	0	1	0
x_3	1	0	1

Proposition 3. *The relation $R \star R$ obtained by the composition of the formal context (X, Y, R) with itself is a preorder relation defined on the object set X.*

Proof. As a consequence of the definition, it is immediate to prove that:

1. The relation $R \star R$ is reflexive.
2. The relation $R \star R$ is transitive.

□

Remark 3. It is a simple verification to see that:

- The relation $R \star R$ is not, in general, a symmetric relation. To be symmetric it is necessary that whenever an object x_2 in the original context (X, Y, R) has all the attributes of another object x_1, both objects have the same set of attributes.
- The relation $R \star R$ is not antisymmetric either. Therefore, $R \star R$ is not, in general, an order relation.

3 Extension to the *L*-fuzzy context case

The expression given in proposition 1 can be generalized to the fuzzy case substituting the maximum operator by a t-conorm S and taking a strong negation $'$. In this way, we can define the compositions of two L-fuzzy contexts as follows:

Definition 2. *Let* $(L, X, Y, R1)$ *and* $(L, Z, Y, R2)$ *be two L-fuzzy contexts, we define the composed L-fuzzy context* $(L, X, Z, R1 \star R2)$*, where:*

$$R1 \star R2(x, z) = \inf_{y \in Y} \{S(R1'(x, y), R2(z, y))\} \quad \forall (x, z) \in X \times Z$$

with S *being a t-conorm defined in the lattice* L*.*

If we remind the definition of a fuzzy S-implication, the previous one can be expressed in this way:

Definition 3. *Let* $(L, X, Y, R1)$ *and* $(L, Z, Y, R2)$ *be two L-fuzzy contexts, and* I *an S-implication operator. We define the composed L-fuzzy context* $(L, X, Z, R1 \star R2)$*, where:*

$$R1 \star R2(x, z) = \inf_{y \in Y} \{I(R1(x, y), R2(z, y))\} \quad \forall (x, z) \in X \times Z$$

We can generalize this definition to any fuzzy implication as we will see next.

3.1 Composition of L-fuzzy contexts associated with an implication operator

Definition 4. *Let* $(L, X, Y, R1)$ *and* $(L, Z, Y, R2)$ *be two L-fuzzy contexts, and let* I *be a fuzzy implication operator, we define the* composed L-fuzzy context associated with the implication I *as the L-fuzzy context* $(L, X, Z, R1 \star_I R2)$*, where:*

$$R1 \star_I R2(x, z) = \inf_{y \in Y} \{I(R1(x, y), R2(z, y))\} \quad \forall (x, z) \in X \times Z$$

Remark 4. If we remind the definition of the *triangle subproduct* operator $\triangleleft$ given by [9], one of the standard operators in the fuzzy relation theory which has been previously used in diverse works [1, 2], we can see that the composed relation defined here can be written as:

$$R1 \star_I R2 = R1 \triangleleft (R2)^{op}$$

As can be observed, also in this case a similar result to the crisp case is obtained.

Proposition 4. *Let* $(L, X, Y, R1)$ *and* $(L, Z, Y, R2)$ *be two L-fuzzy contexts. Then, the relation of the composed L-fuzzy context* $(L, X, Z, R1 \star_I R2)$ *is not, in general, the opposite of the relation of the composed L-fuzzy context* $(L, Z, X, R2 \star_I R1)$*.*

$$(R1 \star_I R2)^{op} \neq R2 \star_I R1$$

That is, if we change the order of the composition, the obtained relation between the elements of X *and* Z *is different.*

Proof. Given two L-fuzzy contexts $(L, X, Y, R1)$ and $(L, Z, Y, R2)$, and a fuzzy implication operator I, the relation of the composed L-fuzzy context $(L, X, Z, R1 \star_I R2)$ is:

$$R1 \star_I R2(x, z) = \inf_{y \in Y} \{I(R1(x, y), R2(z, y))\} \quad \forall (x, z) \in X \times Z$$

On the other hand, the relation of the composed L-fuzzy context $(L, Z, X, R2 \star_I R1)$ is defined as:

$$R2 \star_I R1(z, x) = \inf_{y \in Y} \{I(R2(z, y), R1(x, y))\} \quad \forall (z, x) \in Z \times X$$

As, in general, given a fuzzy implication $I(a, b) \neq I(b, a)$, then these relations are not opposed. □

Example 3. We have a company of temporary work in which we want to analyze the suitability of some candidates to obtain some offered employments. The company knows the requirements of knowledge to occupy each one of the positions, represented by means of the L-fuzzy context $(L, X, Y, R1)$, where the set of objects X is the set of employments, the attributes Y the necessary knowledge, and the relation among them appears in Table 1 with values in the chain $L=\{0, 0.1, 0.2, \ldots, 1\}$.

Table 1. The requirements of knowledge to obtain each one of the employments.

$R1$	computer science	accounting	mechanics	cooking
domestic helper	0.1	0.3	0.1	1
waiter	0	0.4	0	0.7
accountant	0.9	1	0	0
car salesman	0.5	0.7	0.9	0

On the other hand, we have the knowledge of some candidates for these positions, represented by the L-fuzzy context $(L, Z, Y, R2)$ in which the objects are the different candidates to occupy the jobs, the attributes the necessary knowledge and the relation among them is given by Table 2.

A candidate will be suitable to obtain a job if he owns all the knowledge required in this position. Therefore, to analyze what candidate is adapted for each job, we would use the composed L-fuzzy context $(L, X, Z, R1 \star R2)$. The relation of this composed context, calculated using the Lukasiewicz implication operator, is the represented in Table 3.

To obtain the information of this L-fuzzy context we will use the ordinary tools of the L-fuzzy Concept Theory to analyze the associated L-fuzzy concepts. Thus, for example, if we want to find the best candidate to occupy the job of *waiter*, we take the set:

Table 2. Knowledge of the candidates.

$R2$	computer science	accounting	mechanics	cooking
C1	0.5	0.8	0.3	0.6
C2	0.2	0.5	0.1	1
C3	0	0.2	0	0.3
C4	0.9	0.4	0.1	0.5
C5	0.7	0.5	0.2	0.1

Table 3. Suitability of each candidate for each position.

$R1 \star R2$	C_1	C_2	C_3	C_4	C_5
domestic helper	0.6	1	0.3	0.5	0.1
waiter	0.9	1	0.6	0.8	0.4
accountant	0.6	0.3	0.1	0.4	0.5
car salesman	0.4	0.2	0.5	0.2	0.3

$$\{\text{domestic helper}/0, \text{waiter}/1, \text{accountant}/0, \text{car salesman}/0\}$$

and we obtain the derived L-fuzzy concept, whose intension is:

$$\{C_1/0.9, C_2/1, C_3/0.6, C_4/0.8, C_5/0.4\}$$

If we look at the attributes with the highest membership degree, we can deduce that the most suitable candidate for the job of *waiter* is C_2, followed by C_1 and C_4.

If, for instance, we want to find the best person to be *accountant* in a restaurant that also could work as a *waiter*, we take the set

$$\{\text{domestic helper}/0, \text{waiter}/1, \text{accountant}/1, \text{car salesman}/0\}$$

and the derived L-fuzzy concept is

$$\{C_1/0.6, C_2/0.3, C_3/0.1, C_4/0.4, C_5/0.4\}$$

where we can see that the most suitable candidate is C_2.

On the other hand, if our interest is to analyze which of the jobs is the most suitable for each candidate, we do the composition in the contrary order, obtaining the L-fuzzy context $(L, Z, X, R2 \star R1)$, where the composed relation is represented in Table 4.

We can see in this example that both compositions are different: A candidate can be the best to occupy a concrete job, but that job need not be the most appropriate for this candidate.

Table 4. Suitability of each employment for each candidate.

$R2 \star R1$	domestic helper	waiter	accountant	car salesman
C_1	0.5	0.5	0.4	0.4
C_2	0.8	0.7	0	0
C_3	0.3	0.2	0.2	0.7
C_4	0.2	0.1	0.5	0.5
C_5	0.4	0.3	0.8	0.8

The following result will be of interest to study the L-fuzzy concepts associated to the objects of the composed L-fuzzy context.

Before to proceed with the proposition, we are going to introduce a new notation: If the subscripts point out the derivation operators and the superscripts the L-fuzzy contexts where they are applied, then $A_1^{\circledast}$ is the derived set from A obtained in the composed L-fuzzy context, $A_1^{\textcircled{1}}$ is the derived set obtained in the L-fuzzy context $(L, X, Y, R1)$, and $(A_1^{\textcircled{1}})_2^{\textcircled{2}}$ the derived set of the last one in the L-fuzzy context $(L, Z, Y, R2)$

Proposition 5. *If the implication operator I is residuated and we consider the set:*

$$A(x) = \begin{cases} 1 & \textit{if } x = x_i \\ 0 & \textit{in other case} \end{cases}$$

then, the intension of the L-fuzzy concept obtained in the composed L-fuzzy context $(L, X, Z, R1 \star_I R2)$ from the set A, is equal to the extension of the L-fuzzy concept obtained in $(L, Z, Y, R2)$ from the intension of the L-fuzzy concept obtained in $(L, X, Y, R1)$ from A. That is, we obtain the same fuzzy set Z applying the derivation operators twice (once in each one of the contexts that make up the composition), or once in the composed context.

Moreover, it is verify that:

$$\forall z \in Z, \quad A_1^{\circledast}(z) = (A_1^{\textcircled{1}})_2^{\textcircled{2}}(z) = R1 \star_I R2(x_i, z)$$

That is, the membership degrees obtained are the values of the row of $R1 \star_I R2$ that corresponds to the object x_i.

Proof. Let be $A(x) = \begin{cases} 1 & \text{if } x = x_i \\ 0 & \text{in other case} \end{cases}$, the intension of the L-fuzzy concept obtained from A in the context $(L, X, Y, R1)$ is the L-fuzzy subset of Y:

$$A_1^{\textcircled{1}}(y) = \inf_{x \in X} \{I(A(x), R1(x, y))\}, \quad \forall y \in Y.$$

As the implication I is residuated, $\forall a \in L$ it is verified that $I(0, a) = 1$ and $I(1, a) = a$, thus,

$$A_1^{\textcircled{1}}(y) = R1(x_i, y), \quad \forall y \in Y.$$

Taking now the set $A_1^{\textcircled{1}}$, we obtain the derived L-fuzzy concept in the L-fuzzy context $(L, Z, Y, R2)$, the extension of which is:

$$(A_1^{\textcircled{1}})_2^{\textcircled{2}}(z) = \inf_{y\in Y}\{I(A_1^{\textcircled{1}}(y), R2(z, y))\} =$$
$$= \inf_{y\in Y}\{I(R1(x_i, y), R2(z, y))\} = R1 \star_I R2(x_i, z), \quad \forall z \in Z.$$

On the other hand, the intension of the obtained L-fuzzy concept in the composed L-fuzzy context from A is:

$$A_1^{\circledast}(z) = \inf_{x\in X}\{I(A(x), R1 \star_I R2(x, y))\} = R1 \star_I R2(x_i, z), \quad \forall z \in Z.$$

□

Example 4. If we come back to example 3, we have analyzed which candidate is the most suitable for the job of *waiter*.

To do this, in the L-fuzzy context $(L, X, Z, R1 \star R2)$ (see Table 3) we have taken the set

$$A = \{\text{domestic helper}/0, \text{waiter}/1, \text{accountant}/0, \text{car salesman}/0\}$$

and we have calculated the closed L-fuzzy concept, where the fuzzy intension is:

$$A_1^{\circledast} = \{C_1/0.9, C_2/1, C_3/0.6, C_4/0.8, C_5/0.4\}$$

And here, if we look at those attributes whose membership degrees stand out from the others, we deduce that the most suitable candidates to be good *waiters* were, C_2, C_1 and C_4, in this order.

The same result is obtained if we take the L-fuzzy context $(L, X, Y, R1)$ (see Table1) and we calculate the L-fuzzy concept from A, which intension is:

$$A_1^{\textcircled{1}} = \{\text{computer science}/0, \text{accounting}/0.4, \text{mechanics}/0, \text{cooking}/0.7\}$$

And, from this fuzzy set we obtain in the L-fuzzy context $(L, Z, Y, R2)$ (see Table2) the derived L-fuzzy concept the extension of which is:

$$(A_1^{\textcircled{1}})_2^{\textcircled{2}} = \{C_1/0.9, C_2/1, C_3/0.6, C_4/0.8, C_5/0.4\}$$

As can be seen, the result is the same that the obtained in the composed L-fuzzy context.

3.2 Composition of an *L*-fuzzy context with itself

The composition of an L-fuzzy context (L, X, Y, R) with itself will allow us to set up some relationships between the elements of the object set X.

Proposition 6. *If I is a residuated implication associated with a left continuous t-conorm T, then the relation $R\star_I R$ that results of the composition of (L, X, Y, R) with itself, associated with the implication I, constitutes a fuzzy preorder relation defined in the object set X.*

Proof. 1. First, we prove that it is a reflexive relation, that is, the relation verifies:

$$\forall x \in X,\ R \star_I R(x,x) = 1.$$

By the definition of the composition associated with an implication operator, we have

$$\forall x \in X, \quad R \star_I R(x,x) = \inf_{y \in Y}\{I(R(x,y), R(x,y))\},$$

and, as any residuated implication verifies that $I(a,a) = 1,\ \forall a \in L$, then

$$\forall x \in X, \quad R \star_I R(x,x) = 1.$$

2. To see that $R \star_I R$ is a T-transitive relation, we have to prove that

$$\forall x,t,z \in X, \quad T(R \star_I R(x,t), R \star_I R(t,z)) \leq R \star_I R(x,z),$$

that is, the following inequality must be verified:

$$T\left(\inf_{\alpha \in Y}\{I(R(x,\alpha), R(t,\alpha))\}, \inf_{\beta \in Y}\{I(R(t,\beta), R(z,\beta))\}\right) \leq \inf_{\alpha \in Y}\{I(R(x,\alpha), R(z,\alpha))\}.$$

By the monotony of the t-norm, we have:

$$T\left(\inf_{\alpha \in Y}\{I(R(x,\alpha), R(t,\alpha))\}, \inf_{\beta \in Y}\{I(R(t,\beta), R(z,\beta))\}\right) \leq \inf_{\alpha \in Y}\left\{T\left(I(R(x,\alpha), R(t,\alpha)), \inf_{\beta \in Y}\{I(R(t,\beta), R(z,\beta))\}\right)\right\} \leq \inf_{\alpha \in Y}\left\{T\left(I(R(x,\alpha), R(t,\alpha)), I(R(t,\alpha), R(z,\alpha))\right)\right\}.$$

As the used t-norm T is left-continuous, we know that [7]

$$\forall a,b,c \in [0,1],\ T(I(a,b), I(b,c)) \leq I(a,c),$$

and it is verified that:

$$T\left(\inf_{\alpha \in Y}\{I(R(x,\alpha), R(t,\alpha))\}, \inf_{\beta \in Y}\{I(R(t,\beta), R(z,\beta))\}\right) \leq \inf_{\alpha \in Y}\{I(R(x,\alpha), R(z,\alpha))\}.$$

□

Remark 5. The relation $R \star_I R$ is neither symmetric nor antisymmetric and then, is neither an equivalence nor an order relation. For instance, if we take the relation R given by the table:

R	y_1	y_2	y_3	y_4
x_1	0.1	0.3	0.5	0.1
x_2	0.8	0.2	0.8	0.2
x_3	0.4	0.7	0	0.1

then the relation $R \star_I R$ associated with the Lukasiewicz implication operator is:

$R \star_I R$	x_1	x_2	x_3
x_1	1	0.9	0.5
x_2	0.3	1	0.2
x_3	0.5	0.5	1

and, as can be seen, is neither a symmetric nor an antisymmetric relation.

Remark 6. If we are using a non residuated implication operator, not always a fuzzy preorder relation is obtained. For instance, if we take the previous relation R and we do the composition $R \star_I R$ associated with the Kleene-Dienes implication (that does not verify $I(x, x) = 1$), then we obtain the following relation:

$R \star_I R$	x_1	x_2	x_3
x_1	0.5	0.7	0.5
x_2	0.2	0.2	0.2
x_3	0.3	0.3	0.3

that is neither a reflexive nor a fuzzy preorder relation.

The application of this composition can be very interesting in social or work relations as we can see next:

Example 5. There are four different manufacture processes in a factory and we want to organize the workers so that each of them is subordinate of another one if its capacity to carry out each one of the processes of manufacture is smaller.

To model this problem, we are going to take the L-fuzzy context (L, X, Y, R), where the set of objects X is formed by the workers $\{O_1, O_2, O_3, O_4, O_5\}$, the attributes are the different manufacture processes $\{P_1, P_2, P_3, P_4\}$, and the relation R represents the capacity of each one of the workers to carry out each one of the processes, in a scale of 0 to 1 (See Table 5).

The L-fuzzy context that results of the composition of this context with itself allow us to define relations boss-subordinate between the workers so that the relation $R \star R(x, y)$ of the compound context (associated with the Lukasiewicz implication) gives the degree in which the worker x is subordinate of the worker y. (See Table 6).

Table 5. Capacity of the workers to carry out each one of the manufacture processes

R	P_1	P_2	P_3	P_4
O_1	0.7	1	0.3	0
O_2	0.3	0.8	0.9	0.4
O_3	0.1	0.2	1	0.5
O_4	0.5	0.3	0.2	0.4
O_5	1	0.5	0.8	1

Table 6. Relation "be subordinate of".

$R \star R$	O_1	O_2	O_3	O_4	O_5
O_1	1	0.6	0.2	0.3	0.5
O_2	0.4	1	0.4	0.3	0.7
O_3	0.3	0.9	1	0.2	0.8
O_4	0.6	0.8	0.6	1	1
O_5	0	0.3	0.1	0.4	1

This will allow us, for example, to choose bosses in the group watching the columns of the obtained relation: In this case, we could choose as bosses of the workers to O_2 and O_5 because both have as subordinate O_3 and O_4 and the subordination degrees are the biggest values of the columns.

4 Conclusions and future work

This work constitutes the first approach to the problem of composition of L-fuzzy contexts. In future works we will use these results in the resolution of other problems that seem interesting to us:

- First, this composition will be useful to study the chained L-fuzzy contexts, that is, to find relations between two defined contexts where the set of attributes of the first context is the same that the set of objects of the second one.

- On the other hand, we think that it will be useful to define the composition of L-fuzzy contexts in the interval-valued case in order to study certain situations.

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References

1. C. Alcalde, A. Burusco and R. Fuentes-González, "Analysis of certain L-Fuzzy relational equations and the study of its solutions by means of the L-Fuzzy Concept

Theory." *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems.* 20 No.1 (2012), pp. 21–40.
2. E. Bartl and R. Bělohlávek, "Sup-t-norm and inf-residuum are a single type of relational equations." *International Journal of General Systems.* 40 No.6 (2011), pp. 599–609.
3. R. Bělohlávek, "Fuzzy Galois connections and fuzzy concept lattices: from binary relations to conceptual structures", in: Novak V., Perfileva I. (eds.): *Discovering the World with Fuzzy Logic*, Physica-Verlag (2000), pp. 462–494.
4. A. Burusco and R. Fuentes-González, "The Study of the L-Fuzzy Concept Lattice." *Mathware and Soft Computing.* 1 No.3 (1994), pp. 209–218.
5. A. Burusco and R. Fuentes-González, "Construction of the L-Fuzzy Concept Lattice." *Fuzzy Sets and Systems.* 97 No.1 (1998), pp. 109–114.
6. Y. Djouadi, D. Dubois and H. Prade, "On the possible meanings of degrees when making formal concept analysis fuzzy." *EUROFUSE workshop. Preference Modelling and Decision Analysis.* Pamplona, Sep 2009, pp. 253–258.
7. J. Fodor, M. Roubens. *Fuzzy Preference Modelling and Multicriteria Decision Support.* Theory and Decision Library (Kluwer Academic Publishers), Dordrecht/Boston/London (1994).
8. A. Jaoua, F. Alvi, S. Elloumi, S. B. Yahia. "Galois Connection in Fuzzy Binary Relations." *Applications for Discovering Association Rules and Decision Making.* RelMiCS (2000), pp. 141–149.
9. L. J. Kohout, W. Bandler, Use of fuzzy relations in Knowledge representation, acquisition, and processing, in: *L. Zadeh, J. Kacprzyk (Eds.), Fuzzy Logic for Management of Uncertainty*, 1992, pp. 415-435.
10. S. Krajči. "A generalized concept lattice." *Logic J. IGPL* 13 (5) (2005) pp. 543–550.
11. J. Medina, M. Ojeda-Aciego, and J. Ruiz-Calviño. "On multi-adjoint concept lattices: definition and representation theorem." *Lect. Notes in Artificial Intelligence*, 4390,(2007), pp 197–209.
12. S. Pollandt, *Fuzzy Begriffe: Formale Begriffsanalyse unscharfer Daten*, Springer (1997).
13. R. Wille. "Restructuring lattice theory: an approach based on hierarchies of concepts",in: Rival I.(ed.),*Ordered Sets*, Reidel, Dordrecht-Boston (1982), pp. 445–470.
14. K.E. Wolff. "Conceptual interpretation of fuzzy theory", in: Proc. *6th European Congress on Intelligent techniques and Soft computing*, 1, (1998), pp. 555–562.