Structural properties and algorithms on the lattice of Moore co-families

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Abstract. A collection of sets on a ground set U_n (U_n denotes the set $\{1, 2, ..., n\}$) closed under intersection and containing U_n is known as a Moore family. The set of Moore families for a fixed n is in bijection with the set of Moore co-families (union-closed families containing the empty set) denoted $\mathbb{M}_n$. In this paper, we show that the set $\mathbb{M}_n$ can be endowed with the quotient partition associated with some operator h. This operator h is the main concept underlying a recursive description of $\mathbb{M}_n$. By this way each class of the partition contains all the families which have the same image by h. Then we prove some structural results linking any Moore co-family to its image by h. From these results we derive an algorithm which computes efficiently the image by h of any given Moore co-family.

Key words: Moore co-families, Formal Concept Analysis, lattices

References

1. Barbut, M., Monjardet, B.: *Ordre et classification.* Hachette (1970)
2. Birkhoff, G.: *Lattice Theory.* Third edn. American Mathematical Society (1967)
3. Birkhoff, G.: Rings of sets. *Duke Mathematical Journal* **3** (1937) 443–454
4. Caspard, N., Monjardet, B.: The lattices of closure systems, closure operators, and implicational systems on a finite set: a survey. *Discrete Appl. Math.* **127** (2003) 241–269
5. Cohn, P.: *Universal Algebra.* Harper and Row, New York (1965)
6. Colomb, P., Irlande, A., Raynaud, O.: Counting of Moore families on n=7. In: *ICFCA, Lecture Notes in Artificial Intelligence 5986, Springer.* (2010)
7. Colomb, P., Irlande, A., Raynaud, O., Renaud, Y.: About the recursive décomposition of the lattice of moore co-families. In: *ICFCA.* (2011)
8. Davey, B.A., Priestley, H.A.: *Introduction to lattices and orders.* Second edn. Cambridge University Press (2002)
9. Demetrovics, J., Molnar, A., Thalheim, B.: Reasoning methods for designing and surveying relationships described by sets of functional constraints. *Serdica J. Computing* **3** (2009) 179–204
10. Demetrovics, J., Libkin, L., Muchnik, I.: Functional dependencies in relational databases: A lattice point of view. *Discrete Appl. Math.* **40(2)** (1992) 155–185
11. Doignon, J.P., Falmagne, J.C.: *Knowledge Spaces.* Springer, Berlin (1999)
12. Duquenne, V.: Latticial structure in data analysis. *Theoretical Computer Science* **217** (1999) 407–436

13. Ganter, B., Wille, R.: *Formal Concept Analysis.* mathematical foundations, Berlin-Heidelberg-NewYork, Springer (1999)
14. Habib, M., Nourine, L.: The number of Moore families on n=6. *Discrete Mathematics* **294** (2005) 291–296
15. Sierksma, G: Convexity on union of sets. *Compositio Mathematica* **volume 42** (1981) 391–400
16. van de Vel, M.L.J.: *Theory of convex structures.* North-Holland, Amsterdam (1993)