Improving Emergency Department Processes Using Coloured Petri Nets

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Abstract. With increasing demand for medical services, emergency departments (ED) are facing problems such as overcrowding and dissatisfaction. Improving the key performance indicators of EDs has been the focal point of healthcare management. This paper addresses performance analysis of ED of a general hospital. To this aim, a discrete event dynamic modeling approach is used to model the ED processes. The model employs a hierarchical timed Coloured Petri net framework in a concise and detailed way to capture patient flow and care processes within the ED. The simulation model was validated against historical data and then different types of scenarios were used to assess, compare and improve ED key performance indicators, such as patients waiting time, length of stay (LOS), and resource utilization rate. The proposed model helped the hospital policy makers to configure the ED in a way to improve its efficiency and staff satisfaction.

Keywords: Healthcare System, Emergency Department, Coloured Petri Net**,** Performance Analysis

1 Introduction

Emergency departments (ED) are facing different problems which affect their performance. Of these, overcrowding is a common issue around the world which provides EDs patient with long length of stay (LOS), waiting times for receiving services and then dissatisfaction [1]. Although most emergency departments are under growing demand, they often face with insufficient staffing and budget constraints. One solution to this problem is to increase capacity of ED, providing adequate facilities and manpower, but this is not the best approach for solving the problem, and perhaps not achievable [2]. Recently, the need for improvement in ED processes due to cost, overcrowding and safety of patients admitted to a large extent [3].To improve the efficiency and quality of ED processes, different methods were used which include process mapping, demand management, critical path identification, queuing systems, statistical forecasting, balanced scorecard and computer simulation [4].

In the last years, the use of computer simulation to help effective decision making in health care and to improve the medical operations has been rising [5]. One of the main reasons that simulation has become a common practice in solving medical problems is its ability to dynamically analyze situations and present to the stakeholders a more realistic view of the system [6]. The main purpose of the use of simulation studies in health care is to reduce waiting times and length of stay for patients, better use of resources and reducing operating cost [7]. Among the various methods for simulation in health care, discrete event simulation is the most used method especially in EDs, and it seems to be a better alternative with less time and cost compared to more traditional statistical methods [8].

This study is intended to present a general simulation model for studying hospital emergency department. For this purpose, we used Coloured Petri Nets modeling and simulation formalism for making a general model of emergency department of a general hospital. In addition to internal processes, external relations between ED and other hospital wards, such as Radiology and Laboratory, is also considered. The main objective of this paper is, hence, to improve ED processes. The problem we are dealing with is ED overcrowding which provide patient with long length of stay and waiting times.

The remainder of the paper is organized as follows. Section 2 covers a brief literature review of the application of simulation in emergency departments. Research methodology, simulation model, input data and variables are presented in Section 3. Section 4 focuses on improvement scenarios and results of simulation runs. Finally, the paper is concluded in Section 5.

2 Literature review

In the literature, the main focal point of discrete-event simulation models that are used for analysis of hospital emergency department is improving the flow of patients to reduce waiting times. Examples of this include studying patient flow and forecasting ED overcrowding using simulation by Hoot et al. [9], defining buffer concept to reduce waiting times and increase throughput, and comparing amount of improvement gain by buffers by Kolb et al. [10], and Khandekar et al. [11] paper on rearranging sequence of activities of care process in order to reduce waiting times. Another area of ED simulations study focus is on capacity estimation which determines the optimum number of personnel and physical resources such as bed [12], [13] and also ED layout [14]. Improving quality of services and ED processes is another area of study [5], [15].

Although the use of Petri nets in the health sector is less than other fields such as computer networks and production system, but it can be a useful method in this area. Here some related works in this area are presented. Xiong et al. [16], apply petri nets for modeling and analysis of health care process. They used a Petri net model to examine the effect of changes in arrival pattern and resources on performance metrics such as waiting times and resource utilization. Chockalingam et al [17] used Petri nets to model patient and resource flow in a hospital system. Using the Petri net model they obtained a stochastic representation of a metric termed distance to divert which measure the proximity of a hospital to a divert state. Dotoli et al. [18] focused on pulmonology department workflow and drug distribution system and used simulation as a decision support system. They employed a timed Petri net (TPN) framework to describe the workflow in the department. Another example is the work done by Ronny Man et al [19] of using process mining and Petri nets for pre hospital stroke care. In the paper, process mining is used to extract process related information e.g. timing information. Jorgensen et al [20] have used CPN for implementing a new Electronic Patient Record Workflow System at two stages. The first CPN model is used as an execution engine for a graphical animation called EUC and the second CPN model is a Coloured Workflow Net (CWN). Together, the EUC and the CWN are used to close the gap between the given requirements specification and the realization of these requirements with the help of an IT system.

In this paper, care processes of ED are modeled using hierarchical timed CPN. The main focus of the model is on patient flows. The model also concentrates on the inter-department care processes. It is aimed to find a suitable operating scenario to improve some performance metrics of the department. Similar to [17], this paper has considered the relationship between different departments (Labs, Radiology) of the hospital. Performance metrics including waiting time, patient length of stay, and resource utilization are calculated under different operating scenarios. Compared to existing literature, this paper puts more emphasis on using features of CPN (color, time, and hierarchy) to capture the complex nature of the system.

3 Research methodology

In this paper, CPN Tools is utilized to create the Coloured Petri net model of the system and to simulate the model to produce desired outputs. CPN Tools [21] is a powerful software tool for modeling and simulation of discrete event systems modeled in CPN. Our choice of using CPNs to model ED patient flow stems from the fact that PNs capture structural properties of the underlying system which we can study and use. Petri nets provide the foundation of the graphical notation and the basic primitives for modeling concurrency and synchronization, conditions which are common in our model. After reviewing a wide range of related literature, an initial model was prepared. Based on the initial model, the generic conceptual model was developed. The generic model aimed to capture the characteristics of an emergency department of a general hospital in Iran. Information required for the modeling and simulation of processes were collected using hospital information system, sampling in ward, and also open interview with employees. In order to simulate the model under different configurations, different types of improvement scenarios were defined and compared against performance criteria. Please note that here the term “Generic” as Gunal and Pidd [22] mentioned means that the model has a defined structure with probability distributions that can be parameterized by the user.

The hospital under study is a general hospital in the city of Yazd of Iran. The emergency department of the hospital consists of one triage room, one primary visit room, admission and discharge unit, CPR room, and two inpatient areas with 24 inpatient bed. It works 3 shifts a day with 1 triage nurse, 1 general practitioner (GP), 1 emergency medicine specialist (SP), 1 admission staff and 6 nurses.

3.1 Process flow chart

Fig.1. depicts an overall patient flow of Emergency Department of the hospital. This flow chart is depicted based on researcher observation of the process and also domain expert opinion. The process diagram was drawn in such a way that in addition to our hospital it could also be used in other Iranian hospitals. Patients are triaged on arrival at the emergency department. The triage nurse makes an initial evaluation of patient symptoms. Then, according to Emergency Severity Index (ESI), she classifies the patient in one of the 5 levels of emergency. A patient in level 1 is with more acuity while a patient in level 5 is in fact

an outpatient with less acuity. After this stage, patient is referred to the GP. The GP determines whether or not the patient requires other care services. Usually, patients with acuity level 5 will leave the ED as soon as the payment cleared. Other patients who need more medical services, such as diagnostic tests, need to be registered and will be directed through the other processes. The final decision about the patient including discharge, inpatient at ED, or being referred to other wards is taken by SP.

3.2 Performance variables

For each process improvement project, establishing quantitative measures to implement changes and develop monitoring system for continuous improvement is crucial. In this paper, we investigate three key performance metrics including patients waiting times, length of stay, and ED resource utilization.

- Waiting times [min]. It is the mean duration a patient need to spend in the ED waiting room.
- Length of stay (LOS) [min]. The total time of staying at ED, from arrival to the time of final decision made by SP.
- Resource utilization [%]. Represents the total busy time of resources compared with total working time.

3.3 Data collection

Model inputs are distribution functions of ED activities. Random sampling was used to estimate required data for patient's arrival times and service time for all resources. All distributions determined from the data and used in the model were validated by using Kolmogorov Smirnov goodness of fit test with a 5% significance level. Using statistical goodness of fit method, the distribution of processing time of different activities have been defined.

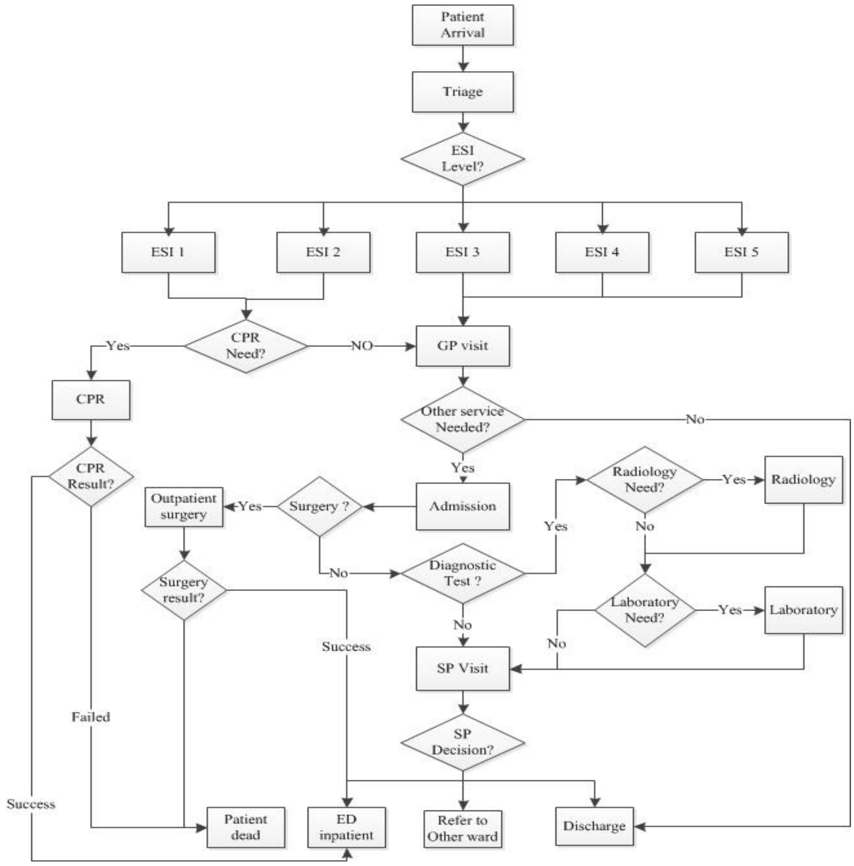

Fig. 1. Process flow of emergency department

Table 1. Simulation input distribution functions

Input parameter	Distribution
Patients inter arrival pattern	Exponential , expel(9)
Triage time	Lognormal , 0.21 + LOGN(0.875, 0.613)
GP visit time	Gamma , 1 + GAMM(0.732, 2.2)
admission	Lognormal , 0.16 + LOGN(1.11,0.729)
SP visit time	Triangular (1,2,3)
CPR time	Triangular (5,15,30)
ED outpatient surgery(OR)	Triangular (10,20,30)

In cases where there was no possibility of sampling, based on information available in the hospital information system and also hospital staff experience, minimum, average and maximum duration of each activity were chosen as the statistical distribution.

3.4 The Hierarchical Timed Petri Net Model of the ED

We used Color Petri-nets (CPNs) to model patient flow in an emergency department of a hospital. A PN consists of a set of places, and a set of transitions and arcs that connect place(s) to a transition and vice-versa. Non-negative integers assigned to every place in the net are known as tokens.

Overview of Colored Petri Nets. Coloured Petri nets are a discrete-event modeling language combining the capabilities of Petri nets with the capabilities of a high-level programming language. Petri nets are directed, bipartite graphs that can be used to model discrete distributed systems. A CPN as defined by [23] is a nine-tuple $CPN = (P, T, A, \Sigma, V, C, G, E, I)$, where P is a finite set of place T, is a finite set of transitions T such that $P \cap T = \emptyset$, $A \subseteq P{\times}T \cup T{\times}P$ is a set of directed arcs, Σ is a finite set of non-empty color sets, V is a finite set of typed variables such that type $[v] \epsilon \Sigma$ for all variable $v \epsilon V$ s, $C: P \rightarrow \Sigma$ is a color set function that assigns a color set to each place, $G: T \rightarrow EXPR_v$ is a guard function that assigns a guard to each transition t such that type $[G(t)] = Bool$, $E: A \rightarrow EXPR_v$ is an arc expression function that assigns an arc expression to each arc α such that type, $[E(a)] = c(p)_{MS}$ where p is the place connected to the arc α, $I: P \rightarrow EXPR_0$ is an initialization function that assigns an initialization expression to each place p such that type. $[I(p)] = c(p)_{MS}$. A CP-net has a distinguished initial marking, denoted by M_0, and obtained by evaluating the initialization expressions. The marking can be viewed as a 'snapshot' of how tokens are distributed in the PN [24].

The simulation model of ED. Fig.2. shows the key structures of the model. The top layer of the ED model is illustrated in this figure. This layer is the core part of the model. In the model each place (circle) represents the state where patients may to be exposed there (table 2). Entry of each patient to the ED is modeled by a token on the place `New Patient` (Fig.2). This place has the color set `PAT`, whose elements are 5-tuples `(ESI, at, qtr., wt, pt)` consisting of patient Emergency Severity Index (`ESI=1,...,5`), patient arrival time to the ED (at), an intermediate variable for Calculating wait time (`qt`), patient wait time for receiving services (`wt`) and activities process time (`pt`). In the initial marking, the `New Patient` has a random integer ESI number Between 1 to 5, an arrival time based on an exponential distribution with mean 9, `qt` is equal to at and `wt` and `pt` are equal to 0. In the ED layer (Fig.2) there are 8 transitions (the rectangles) with tag beside them which called substitution transitions. Each of this transition has a subnet page belong to it that corresponds to one of the considered tasks in the process. To know about the model mechanism in each subnet consider GP visit subnet page as an example (Fig.3).The occurrence of the transition `Start visit` models the situation where a general Practitioner (resource) changes from being ready to being busy until the transition `End visit` occurs. Patients wait to seize GP and after stochastic delay (GP visit time) and receiving GP orders, they release that resource and come back to top layer to carry on rest of the process. The other page is as the illustrated mechanism.

Table 2. Place Description of the ED core layer

Place	description
P1	State for non urgent patient
P2	State for urgent patient (CPR needed)
P3	Patients visited and need extra services
P4	Patients visited and will be discharge
P5	Patients admitted
P6	Patients admitted and need surgery
P7	Number of patients waiting for radiology
P8	Number of patients waiting for laboratory
P9	Patients have done radiology test
P10	Patients have done radiology and also need lab test
P11	Surgery result with success
P12& P13	Patients with their Lab test result
P14	Patients have done radiology and do not need Lab

The model contains resources (shown in Table 3) like nurses; physicians etc. in the form of tokens, and patients use these resources according to model logic to receive care. These resource tokens are held by the patients until they move to the next stage. The delay in availability of a resource is represented as non-availability of tokens to advance the patient tokens through the net. This delay increases the number of patient tokens in the system waiting for a resource. Also Sets of variables used on the transitions in Fig. 2 are showed in Table 4.

Table 3. Initial resource-token distribution

resources places	tokens
Triage nurse	1
GP Doc	1
admission	1
SP Doctor	1
Radiology staff	6
Laboratory staff	12
CPR equipment	1

Table 4. Model variables and description

variable	description
`p`	Represent each patient and carry five attribute: *ESI*, arrival time, *qt*, wait time, process time which are assigned to them
`l`	determine that patient need laboratory test or not
`r`	determine that patient need radiology test or not
`LR`	Laboratory result
`RR`	Radiology result
`gp`	General Practitioner (resource)
`sp`	Emergency medicine specialist (resource)

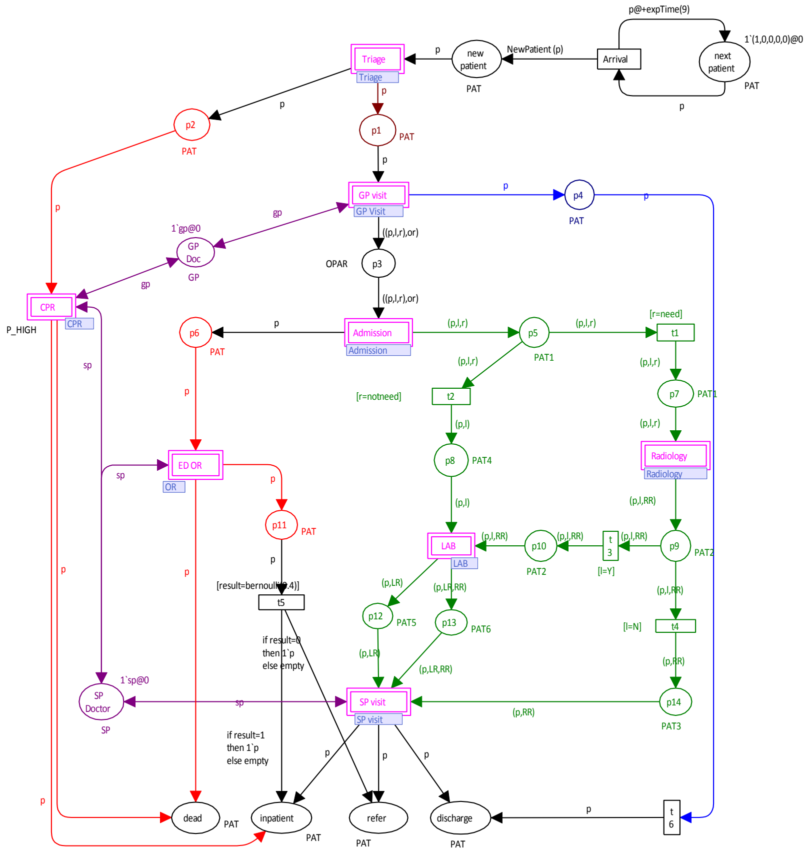

Fig. 2. Core page of ED simulation model using Coloured Petri net (top layer)

In Fig.3 the function on output arc from `end visit` transition determines whether patients need diagnostic tests (laboratory or radiology) or surgery or to be discharged. In SP visit subnet page shown in Fig.4, the SP Doctor place is a common resource that is shared by three activities in the page. Patients with Lab result, Radiology result or both of them are coming to SP, because SP should decide about them. Patients may need to be inpatient in ED. So go to the ED inpatient place or maybe it is necessary to go to other hospital ward for special care, then go to the refer place. Finally, patient after visit by SP may be discharged, so they go to discharge place in SP visit page.

Model validation. We validate final results of the simulation model at first by interviewing ED senior managers and nursing staff in order to validate the final results of the simulation model. Secondly output of the simulation model is compared with real performance indicators (Table 5) and it shows the validation of our model

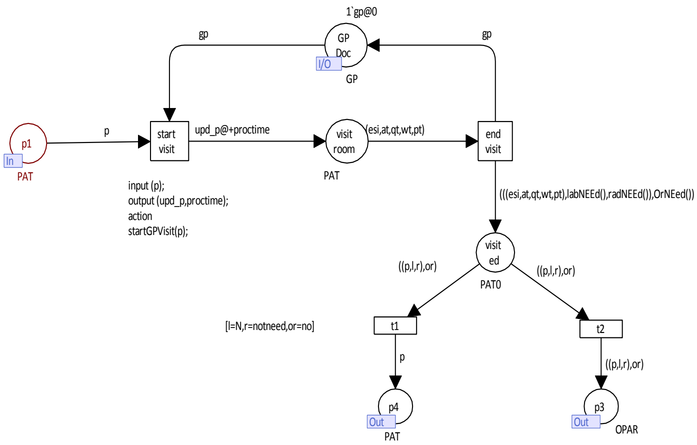

Fig. 3. GP visit page of substitution transition GP visit in ED page (subnet layer)

Finally we also used CPN Tools state space graph to investigate whether the model works truly or not. The state spaces tools are used to calculate state spaces and to generate state space reports. Because our graph is very large, it is not possible to show it here.

Table 5. Comparison between simulated and real data.

Performance criteria	Simulation output	real data
Wait for GP visit	3.2	2.9
Wait for Admission	5.12	5.5
Wait for SP visit	1.15	1
ESI 1&2 LOS	38.2	37.5
ESI 3 LOS	185	187
ESI 4 LOS	140	136
ESI 5 LOS	11.5	12

The performance metrics are investigated using 5 different replications with 95% confidence interval. In each case the system is simulated by a long simulation run of 3 years.

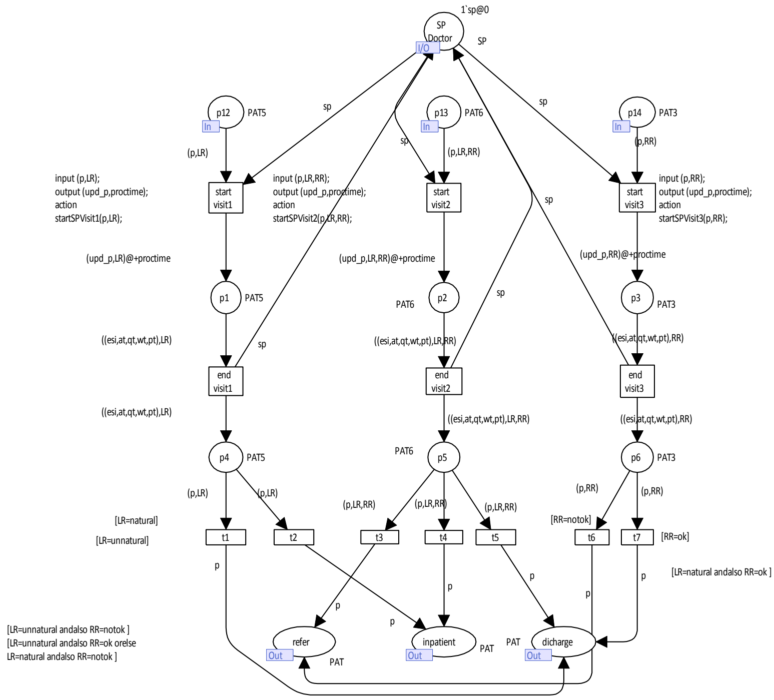

Fig. 4. SP visit page of substitution transition SP visit in ED page (subnet layer)

4 Improvement scenarios

In order to improve processes in terms of system performance metrics, four types of scenarios were defined. These alternate scenarios are validated with domain experts and then were implemented in simulation model. They are as fallow:

- Current scenario (benchmark):

  A – Current state of the ED as a basis for comparisons.

- **Increase or decrease scenarios.** It is, in fact, the most common type of scenario associated with simulation studies. In this scenarios (increase or decrease) number of resources, the number of emergency room doctors, nurses, beds and other physical resources will be changed. In this view, one scenario is defined here:

  B – Increase an emergency medicine specialist (SP)

- **Displacement Scenarios.** In these scenarios, if possible, an alternative resource will be replaced with available resource.

  C – Putting in place an emergency medicine specialist instead of GP

- **Structural scenarios.** The purpose of these scenarios is change of the process activities and even delete or add new activities as part of the process.

D – Remove triage unit and refer patients for triage and visit to GP

- **Hybrid scenarios.** These scenarios are defined as a combination of two or more than two of the above scenarios. For example, displacement scenarios, and a structural scenario combined to make a hybrid one.

E – Replace GP visit and triage activities with a substitute emergency medicine specialist who does these two.

In scenario B, we have added 1 specialist to `SP Doctor` place and then run the simulation model. To implement scenarios D and E, we have to change our basic model. In these scenarios triage and visit is done simultaneously by a substitute doctor (GP or SP). The difference is on the time of visit done by each of them. It's less for SP than GP.

The results of running simulation model with scenarios A to E alongside with their improvement are shown in Table 6. We ran each of the improvement options individually as separate scenario for this purpose.

Table 6. Improvement rate of each scenario considering its resources

Scenarios	Resources			Improvement rate (%)									
				Resource utilization rate (%)									
	GP	SP	Admission Staff	Admission Staff	SP	GP	Wait for GP	Wait for SP	Wait for Admission	ESI 1&2 LOS	ESI 3 LOS	ESI 4 LOS	ESI 5 LOS
A	1	1	1	62	63.7	58	3.2	5.12	1.1	38	185	140	11.5
B	1	2	1	+9.67	-10.5	0	0	+23	0	+5.2	+2.7	+2.65	0
C	-	2	1	+7.25	-1.8	-	-	-21	-7.2	+5.2	+0.5	+1	+8.7
D	1	1	1	+8	-0.3	+14.5	-65	+0.3	-81	+8.6	+1	+1.42	+19.1
E	-	2	1	+11.3	-0.3	-	-	-2	-81	+9.2	+1.35	+2.15	+21.7

Benchmark scenario, A, represent current situation in terms of three performance measures. Waiting time is one of the effective measures of patient satisfaction. Here are three main areas of patients waiting for service. Current scenario has the lowest waiting for GP. Scenario B and C reduce SP waiting by about 45% and 0.4%. Scenario B reduces admission waiting by 10%. Patients' length of stay is a measure of ED efficiency and very important in hospital performance evaluation. We compare LOS for patients with different ESI level. ESI 1 and 2 include those patients who need CPR and then go to inpatient. In this level, E has 9.2% improvement. D reduces LOS by about 8.6% and B and C by about 5.2%. Patients with ESI 3 are patients who need two diagnostic tests here include Laboratory and

Radiology. In this level B, C, D and E reduced LOS by about 2.7%, 0.5%, 1% and 1.35%. Patients with ESI 4 just need one diagnostic test, laboratory or radiology. In this level B, C, D and E reduced LOS by about 2.65%, 1%, 1.42% and 2.15%. Finally, patients with ESI 5 are outpatient and leave ED after GP visit. In this level B, C, D and E reduced LOS by about 0%, 8.7%, 19.1% and 21.7%. Resource utilization represents total busy time of resources to available working time under the simulated conditions.it is a good measure for ED manager in the allocation of resources. Scenario A improves GP utilization by 18%. Scenario B, C, D and E improve admission staff utilization by about 9.67%, 7.25%, 8% and 11.3%. Scenario A has the most SP utilization and other scenarios reduced it. Substituted SP utilization for scenario E is more than C by about 2.1 minute. Also ED staff reaction to our work was positive and they helped us through the work but due to the reluctance of ED managers, we failed to implement the proposed changes in reality.

5 Conclusions and future work

In this research a CPN model was developed to analyze the performance of an emergency department. To evaluate the system under different conditions and improve processes, improvement scenarios were defined. These scenarios may not greatly improve the performance of the model parameters, but could be considered as an existing and potential alternative. We compare 5 scenarios by three variables.

In table 7, improvement rate of each scenario, considering its resources, presented.
To have a better analysis in choosing scenarios, it is necessary to see cost and benefit of each scenario simultaneously. Another option which should be considered is ED's mission, saving patients with high acuity (ESI 1&2), and scenarios that aim to facilitate this purpose even if they cost more than other, are selected. Among defined scenarios, E and D have more improvement especially about patient with ESI 1&2. Although the cost of scenario E to scenario D is some more, but given the purpose of the improvements resulting from the scenario E, this scenario is selected as major one. In the next stage scenario D due to lower cost and also more overall improvement than the other two scenarios have been chosen as the second better scenario.

Based on the model proposed in this paper, it is possible to translate the flow diagram into a Generalized Stochastic Petri net. It would be interesting to compare the results of the two modeling approaches. That is, the exact values of different performance indices can be compared with the simulated values. Because of the hierarchical nature of the model and that every activity has a separate page belong to it; acceptance and use of this model in various conditions may seem easy and by just few change it could be localized. Using Coloured Petri net, we were able to assign different attributes to patients entered into the ED and therefore, the model traces them to calculate performance metrics. The tools and features that are available for simulating CPN models in CPN Tools e.g. hierarchy, functions, guards, state space analysis etc. made it a useful option in simulating complex systems specially healthcare. Future development to this work would be to add other attributes to tokens color such as cost of each activity in the process and engage other wards. Also, it would be of value to consider other resources including beds, facilities and equipment.

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