

# Quantum Mechanics of the Diatomic Molecule (Second Edition)

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## **IOP Series in Coherent Sources, Quantum Fundamentals, and Applications**

### **About the Editor**

F J Duarte is a laser physicist based in Western New York, USA. His career has covered three continents while contributing within the academic, industrial, and defense sectors. Duarte is editor/author of 15 laser optics books and sole author of three books: *Tunable Laser Optics*, *Quantum Optics for Engineers*, and *Fundamentals of Quantum Entanglement*. Duarte has made original contributions in the fields of coherent imaging, directed energy, high-power tunable lasers, laser metrology, liquid and solid-state organic gain media, narrow-linewidth tunable laser oscillators, organic semiconductor coherent emission,  $N$ -slit quantum interferometry, polarization rotation, quantum entanglement, and space-to-space secure interferometric communications. He is also the author of the generalized multiple-prism grating dispersion theory and pioneered the use of Dirac's quantum notation in  $N$ -slit interferometry and classical optics. His contributions have found applications in numerous fields, including astronomical instrumentation, dispersive optics, femtosecond laser microscopy, geodesics, gravitational lensing, heat transfer, laser isotope separation, laser medicine, laser pulse compression, laser spectroscopy, mathematical transforms, nonlinear optics, polarization optics, and tunable diode-laser design. Duarte was elected Fellow of the Australian Institute of Physics in 1987 and Fellow of the Optical Society of America in 1993. He has received various recognitions, including the *Paul F Foreman Engineering Excellence Award* and the *David Richardson Medal* from the Optical Society.

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# Quantum Mechanics of the Diatomic Molecule (Second Edition)

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*To John, Anna, and Melissa*

*To Justin, Jason, and Jola*





# Contents

<b>Preface</b>	<b>xvii</b>
<b>Acknowledgements</b>	<b>xxi</b>
<b>Author biographies</b>	<b>xxii</b>
<b>Part I Fundamentals of the diatomic molecule</b>	
<b>1 Primer on diatomic spectroscopy</b>	<b>1-1</b>
1.1 Overview	1-1
1.2 Reversed angular momentum	1-2
1.3 Exact diatomic eigenfunction	1-4
1.4 Computation of diatomic spectra	1-5
References	1-6
<b>2 Formal quantum mechanics of diatomic molecular spectroscopy</b>	<b>2-1</b>
2.1 Introduction	2-1
2.2 Theory details	2-2
2.3 Results	2-4
2.3.1 Angular momentum commutators	2-4
2.3.2 Diatomic wave function	2-6
2.3.3 Selected diatomic spectra	2-7
2.4 Summary	2-7
References	2-8
<b>3 Line strength computations</b>	<b>3-1</b>
3.1 Introduction	3-1
3.2 Idealized computation of spectra	3-4
References	3-5
<b>4 Framework of the Wigner–Witmer eigenfunction</b>	<b>4-1</b>
References	4-4
<b>5 Derivation of the Wigner–Witmer eigenfunction</b>	<b>5-1</b>
5.1 Outline of the derivation	5-1
5.2 Time translation symmetry	5-2

5.3	Spatial translation symmetry	5-4
5.4	Two-body symmetry	5-7
5.5	Time and spatial translations together	5-8
5.6	Rotational symmetry	5-9
	References	5-10
<b>6</b>	<b>Diatomic formula inferred from the Wigner–Witmer eigenfunction</b>	<b>6-1</b>
	References	6-3
<b>7</b>	<b>Hund’s cases (a) and (b)</b>	<b>7-1</b>
7.1	Introduction	7-1
7.2	Case (b) basis functions	7-2
7.3	Case (a) eigenfunctions	7-3
	References	7-5
<b>8</b>	<b>Basis set for the diatomic molecule</b>	<b>8-1</b>
	References	8-3
<b>9</b>	<b>Angular momentum states of diatomic molecules</b>	<b>9-1</b>
9.1	Introduction	9-1
9.2	The standard $ JM\rangle$ angular momentum representation	9-2
9.3	Rotations	9-3
9.4	Generators of coordinate transformations	9-5
	References	9-6
<b>10</b>	<b>Diatomic parity</b>	<b>10-1</b>
10.1	Parity details	10-1
	10.1.1 Parity is rotationally invariant	10-2
	10.1.2 Spin is immune to the parity operator	10-2
	10.1.3 Parity operates on Cartesian coordinates, not angles	10-3
	10.1.4 Intrinsic parity and $\Lambda$ doublets	10-3
	10.1.5 Summary of parity details	10-3
10.2	Parity designation	10-3
10.3	The parity operator	10-4
10.4	Parity and angular momentum	10-6

10.5	Diatomic parity	10-7
10.6	$\Lambda$ doublets	10-8
	References	10-8
<b>11</b>	<b>The Condon and Shortley line strength</b>	<b>11-1</b>
	Reference	11-2
<b>12</b>	<b>Hönl–London line-strength factors in Hund’s Cases (a) and (b)</b>	<b>12-1</b>
12.1	Case (a) basis functions	12-1
12.2	Case (b) basis functions	12-2
12.3	Mathematical properties of case (a) and case (b) basis functions	12-2
12.4	Diatomic parity operator	12-3
12.5	Hönl–London line-strength factors	12-4
12.6	Triple integral of three rotation matrix elements	12-5
12.7	Calculation of the Hönl–London line-strength factors for cases (a) and (b)	12-5
12.8	Hund’s case (b) Hönl–London line-strength factors	12-7
12.9	The electronic–vibrational strength	12-9
	Reference	12-10
<b>13</b>	<b>Using the Morse potential in diatomic spectroscopy</b>	<b>13-1</b>
13.1	Introduction	13-1
13.2	Morse eigenfunctions	13-2
	13.2.1 Computation of Morse eigenfunctions	13-4
13.3	Morse eigenfunctions as a vibrational basis	13-5
	References	13-6
<b>Part II Selected applications of diatomic spectroscopy</b>		
<b>14</b>	<b>Introduction to applications of diatomic spectroscopy</b>	<b>14-1</b>
	References	14-5
<b>15</b>	<b>Computation of selected diatomic spectra</b>	<b>15-1</b>
15.1	Introduction	15-1
15.2	Computation details	15-2
	15.2.1 MATLAB scripts	15-3

15.3	Results	15-5
15.4	Discussion	15-7
	References	15-11
<b>16</b>	<b>Experimental arrangement for laser-plasma diagnosis</b>	<b>16-1</b>
16.1	Spectroscopy	16-1
16.2	Shadowgraphy	16-3
16.3	Summary	16-3
	References	16-5
<b>17</b>	<b>Methylidyne, CH, cavity ring-down spectroscopy in a microwave plasma discharge</b>	<b>17-1</b>
17.1	Introduction	17-1
17.2	Experiment details	17-2
17.3	Diatomic spectra computation details	17-2
17.4	Results and discussion	17-3
	17.4.1 Methylidyne overview spectra	17-3
	17.4.2 Emission- and cavity ring-down- spectra of the A–X and B–X bands	17-3
17.5	Conclusions	17-9
	References	17-10
<b>18</b>	<b>Cyanide, CN</b>	<b>18-1</b>
18.1	Analysis of CO <sub>2</sub> laser plasma	18-1
18.2	Analysis of CN in Nd:YAG laser plasma	18-2
18.3	Spatially and temporally resolved CN spectra	18-4
	18.3.1 Laser-beam focusing	18-5
	18.3.2 Shadowgraphs	18-6
	18.3.3 Raw CN spectra	18-6
	18.3.4 Abel-inverted CN spectra	18-7
	References	18-12
<b>19</b>	<b>Cyanide molecular laser-induced breakdown spectroscopy with current databases</b>	<b>19-1</b>
19.1	Introduction	19-1
19.2	Computation of diatomic spectra	19-2

19.2.1	Traditional simulation of diatomic molecular spectra	19-3
19.2.2	Line positions and strengths of diatomic spectra	19-3
19.3	Results	19-4
19.3.1	Analysis of the 0.033 nm spectral resolution data	19-5
19.3.2	Analysis of the 0.11 nm spectral resolution data	19-10
19.4	Discussion	19-13
	References	19-15
<b>20</b>	<b>Diatomic carbon, C<sub>2</sub></b>	<b>20-1</b>
20.1	Analysis of C <sub>2</sub> in Nd:YAG laser-plasma	20-1
20.2	Detailed fitting of C <sub>2</sub> spectra	20-2
20.3	Superposition spectra of hydrogen and carbon	20-4
	References	20-8
<b>21</b>	<b>Laser plasma carbon Swan bands fitting with current databases</b>	<b>21-1</b>
21.1	Introduction	21-1
21.2	Experiment and analysis overview	21-2
21.3	Results	21-3
21.3.1	Analysis of $\Delta v = -1$ Swan spectra with NMT program and C <sub>2</sub> -Swan-lsf line strengths	21-3
21.3.2	Analysis of $\Delta v = -1$ Swan spectra with NMT program and ExoMol C <sub>2</sub> line strengths	21-3
21.3.3	Swan spectra $\Delta v = 0, \pm 1$ : ExoMol C <sub>2</sub> and C <sub>2</sub> -Swan-lsf data comparisons	21-5
21.3.4	Laser-induced fluorescence and C <sub>2</sub> -Swan line strengths	21-7
21.4	Discussion	21-7
	References	21-8
<b>22</b>	<b>Aluminum monoxide, AlO</b>	<b>22-1</b>
22.1	Laser-induced breakdown spectroscopy	22-2
22.2	Experimental details for AlO measurements	22-3
22.3	Selected results	22-3
	References	22-6
<b>23</b>	<b>AlO laser-plasma emission spectra analysis with current databases</b>	<b>23-1</b>
23.1	Introduction	23-1

23.2	Experimental and analysis details	23-2
23.3	Results	23-3
23.3.1	Analysis with NMT program and ExoMol line strengths	23-3
23.3.2	ExoMol AIO and AIO-lsf data comparisons	23-3
23.4	Discussion	23-7
	References	23-8
<b>24</b>	<b>Hydroxyl, OH</b>	<b>24-1</b>
	References	24-3
<b>25</b>	<b>Hydroxyl laser-plasma emission spectra analysis with current databases</b>	<b>25-1</b>
25.1	Summary for computation of line-strength data	25-1
25.1.1	Wigner–Witmer diatomic eigenfunction	25-1
25.1.2	Diatomic line position fitting algorithm	25-2
25.2	Hydroxyl analysis example	25-3
25.3	Analysis comparisons	25-4
	References	25-5
<b>26</b>	<b>OH laser-induced breakdown spectroscopy and shadowgraphy</b>	<b>26-1</b>
26.1	Introduction	26-1
26.2	Experiment results	26-2
26.3	Summary	26-5
	References	26-5
<b>27</b>	<b>Titanium Monoxide, TiO</b>	<b>27-1</b>
27.1	Introduction	27-1
27.2	Experiment	27-3
27.3	Results	27-3
	References	27-8
<b>28</b>	<b>Nitric Oxide, NO</b>	<b>28-1</b>
28.1	Experimental details	28-3
28.2	Results	28-3
28.3	Comparison with overview spectra	28-3
	References	28-8

<b>29</b>	<b>Radial electron density measurements in laser plasma from Abel-inverted hydrogen Balmer beta line profiles</b>	<b>29-1</b>
29.1	Introduction	29-1
29.2	Experimental details	29-4
29.3	Results	29-6
	29.3.1 Spatially resolved line-of-sight spectra	29-6
	29.3.2 Abel-inverted spectra	29-10
29.4	Discussion	29-15
	References	29-15
<b>30</b>	<b>Hypersonic imaging and emission spectroscopy of hydrogen and cyanide following laser-induced optical breakdown</b>	<b>30-1</b>
30.1	Introduction	30-1
30.2	Shock waves	30-2
30.3	Electron density	30-5
	30.3.1 Atomic carbon line interference	30-5
	30.3.2 Line broadening and deconvolution	30-6
	30.3.3 Computation of electron density	30-7
30.4	Molecular spectra analysis	30-8
30.5	Abel inversion	30-8
30.6	Results	30-10
	30.6.1 Shadowgraphs	30-10
	30.6.2 Emission spectra	30-11
	30.6.3 Shock wave and plasma expansion	30-15
	30.6.4 Electron density	30-18
	30.6.5 Cyanide temperature	30-19
	30.6.6 Abel inverted spectra	30-20
30.7	Discussion	30-22
	References	30-23
<b>Part III Appendices</b>		
	<b>Appendix A: Review of angular momentum commutators</b>	<b>A-1</b>
	<b>Appendix B: Effects of raising and lowering operators</b>	<b>B-1</b>
	<b>Appendix C: Modified Boltzmann plots</b>	<b>C-1</b>

<b>Appendix D: Aspects of nitric oxide computations</b>	<b>D-1</b>
<b>Appendix E: Parity in diatomic molecules</b>	<b>E-1</b>
<b>Appendix F: Rotational line strengths for the CN BX (5,4) band</b>	<b>F-1</b>
<b>Appendix G: Intrinsic parity of the diatomic molecule</b>	<b>G-1</b>
<b>Appendix H: Review of diatomic laser-induced breakdown spectroscopy</b>	<b>H-1</b>
<b>Appendix I: Program MorseFCF.for</b>	<b>I-1</b>
<b>Appendix J: Boltzmann equilibrium spectrum (BESP) and Nelder–Mead temperature (NMT) scripts</b>	<b>J-1</b>
<b>Appendix K: Abel-inversion scripts</b>	<b>K-1</b>
<b>Appendix L: LIBS: 2018 to 2023 publications that include C.G.P.</b>	<b>L-1</b>



# Preface

## 0.1 First edition

The book notes from J O Hornkohl and extensive scientific discussions and research engagements in my work at the University of Tennessee Space Institute, Center for Laser Applications, motivate completion of this ebook. Communication exchanges occurred since the spring of 1987, and continued regularly until winter 2017 [1]. Over the years, several colleagues and postgraduate MSc and PhD students have contributed to applications of fundamental insights in the physics of the diatomic molecule. Thanks go to David Plemmons, Guoming Guan, Ying-Ling Chen, Wenhong Qin, Ivan Dors, Alexander Woods, David Surmick, Michael Witte, Ghaneshwar Gautam, and Christopher Helstern.

Significant emphasis has been placed on the application of the diatomic spectroscopy predictions in analysis of experimental data. For this reason, this ebook includes several chapters on applications in studies of diatomic molecules, especially important molecules such as cyanide (CN), aluminum monoxide (AlO), diatomic carbon (C<sub>2</sub>), titanium monoxide (TiO), hydroxyl (OH), but also selected work on other diatomic molecules.

This text introduces insights that are essential in utilizing the inherent symmetries associated with diatomic molecules. Consequently, line positions and strengths associated with transitions from lower and upper state-manifolds are determined without invoking approximations that separate vibrations and rotations of diatomic nuclei from electron motion based on mass. The approach utilized in this work makes use of the separation of angular coordinates from electronic vibrational coordinates. Consequently, the volley of selection rules for diatomic spectroscopy is no longer required, including methodologies that rely on so-called reversed angular momentum techniques.

This work summarizes well over 30 years of quantitative analysis of temporally and spatially resolved experimental records, almost all of the experiments discussed in this ebook were conducted at the Center for Laser Applications (CLA) at the University of Tennessee Space Institute. Applications include understanding on nonequilibrium fluid and plasma physics and interpretation of stellar astrophysics spectra. In several cases of laser-induced plasma investigations, both atomic and molecular signatures or superposed spectral characteristics from molecules and atoms can be identified. Analysis of such superposition spectra requires accurate knowledge of wavelength positions and transition strengths. The revival and replacement of electrical-spark spectroscopy with laser-spark or laser-plasma spectroscopy for quantitative elemental composition analysis since the mid-1990s, viz. laser-induced breakdown spectroscopy (LIBS), extends into increased interests in molecular LIBS since (give-or-take) the mid-2000s. From an analytical and practical point of view, the requirements can be reduced to the availability of a set of diatomic line-strengths in tabular form along with programs that are designed to appropriately read the records. However, this ebook provides a reasonable account

of the quantum mechanics of the diatomic molecule, along with selected applications that were important for motivating a consistent approach and for analyzing recorded data sets from various experiments in the CLA laboratories.

The challenge of this work has been the prediction of spectra with a focus on diatomic spectroscopy. The aim of the lifetime work of Jim Hornkohl is the design of an algorithm to predict and fit computed and measured molecular spectra to provide inferences on parameters such as excitation temperature. The means to accomplish goals for various diatomic molecules are the consistent application of standard quantum theory of angular momentum. During his career, Jim engaged in efforts to overcome techniques such as Van Vleck's reversed angular momentum approach based on angular momentum commutators. The apparent difficulties included the battles with the established practice to predict and compute spectra and design programs despite the mathematical inconsistencies associated with the reversed angular momentum practice. The experimental investigations, and again the stimulating discussions, motivated refinements such as enlarging the data sets for the CN, C<sub>2</sub>, or TiO diatomic molecules. In turn, the discussed applications in this book are intended to alleviate analysis of diatomic spectra composed of superpositions of a significant amount of transition lines within typical resolution for laser-plasma emission spectroscopy, to name but one example.

Christian Parigger  
August 2019

## 0.2 Second edition

The second edition includes 10 additional chapters, one on the fundamentals and nine on the applications parts. Three additional appendices are included, namely: communication of NMT and BESP scripts for computation of diatomic spectra, Abel inversion scripts with one specific example, and an appendix on select recent publications that include C.G.P. as author. The additions primarily address communication of spatial profiles analyses, including Abel inversion and communication of scripts for diatomic spectroscopy and Abel inversions. However, comparisons with other existing databases clearly reveal the significance of the line strengths for the selected electronic transitions of diatomic molecules. The comparisons also include a section of C<sub>2</sub> laser-induced fluorescence. The existing databases comparisons include PGOPHER, LIFBASE, and ExoMol databases that are compared with line-strength data of diatomic molecules of interest, particularly for laser-induced plasma that is generated in gases and gas mixtures.

- New chapter 2 addresses the foundations of quantum mechanics and the mathematical implementation of specific symmetries. Application of the correspondence principle, relating classical and quantum mechanics, leads to the occurrence of the infamous sign-reversal. This chapter addresses formal treatment of symmetries in quantum mechanics. Quantum theory contradicts sign changes of the fundamental angular momentum algebra.

Reversed angular momentum sign changes are of a heuristic nature and are actually undesirable in the analysis of diatomic spectra.

- New chapter 15 communicates line-strength data and associated scripts for the computation and spectroscopic fitting of selected transitions of diatomic molecules. The scripts for data analysis are designed for inclusion in various software packages or program languages. Selected results demonstrate the applicability of the program for data analysis in laser-induced optical breakdown spectroscopy, primarily at the University of Tennessee Space Institute, Center for Laser Applications. Representative spectra are calculated and referenced to measured data records. Comparisons of experiment data with predictions from other tabulated diatomic molecular databases confirm the accuracy of the communicated line-strength data.
- New chapter 17 discusses cavity ring-down spectroscopy of methylidyne in a chemiluminescent plasma that is produced in a microwave cavity. Of interest are the rotational lines of selected vibrational transitions for the A–X and B–X bands. This chapter also includes recent analysis that shows excellent agreement of measured and computed data, and it communicates CH line-strength data. The CH radical is an important diatomic molecule in hydrocarbon combustion diagnosis and analysis of stellar plasma emissions, to name just two examples for analytical plasma chemistry.
- New chapter 19 discusses diatomic molecular spectroscopy of laser-induced plasma and analysis of data records, specifically signatures of cyanide (CN). Line-strength data from various databases are compared for simulation of the cyanide spectra. Of interest are recent predictions using an astrophysical database, i.e., ExoMol, a laser-induced fluorescence database, i.e., LIFBASE, and a program for simulating rotational, vibrational, and electronic spectra, i.e., PGOPHER.
- New chapter 21 presents analysis of carbon Swan bands laser-plasma emission records using line-strength data and the ExoMol database. The temperature inferences are elaborated when using nonlinear fitting with both databases. The line-strength data are also utilized for analysis of laser-induced fluorescence experiments that employ a spectral resolution of the order of 5 pm. Accurate diatomic carbon databases show many applications in laboratory diagnosis and interpretation of astrophysical plasma records.
- New chapter 23 elaborates on analysis of aluminum monoxide (AlO), laser-plasma emission records using line-strength data, and the ExoMol astrophysical database. A nonlinear fitting program computes comparisons of measured and simulated diatomic molecular spectra. This work also presents a comparison of the AlO line strength and of ExoMol data for the AlO diatomic molecule. Accurate AlO databases show a volley of applications in laboratory and astrophysical plasma diagnosis.
- New chapter 25 applies NMT and BESP scripts for the fitting of recorded experimental hydroxyl data. The fitting program also incorporates a slight, overall wavelength offset. The ExoMol and line-strength data yield close to identical temperature with a slightly different linear background.

The databases for specific hydroxyl transition yield similar predictions of the recorded laser-plasma spectra for time delays of the order of one hundred microseconds after optical breakdown initiation.

- New chapter 26 communicates measurement and analysis of diatomic molecular hydroxyl spectra after generation of laser-induced plasma, and it also shows details of the expanding plasma including associations of shadowgraphs with spectroscopy. Formation of OH is clearly discernible at time delays of several dozen microseconds after plasma initiation. Optical emissions are dispersed by a Czerny–Turner spectrometer and an intensified charge-coupled device records the data along the wavelength and slit dimensions.
- New chapter 29 combines time-resolved emission spectroscopy with Abel integral inversion techniques to obtain radial electron density values in laser-induced plasma. This chapter also includes details of the Abel transforms. Hydrogen beta line profiles are recorded following optical breakdown in ultra-high-pure hydrogen gas. Asymmetric Abel inversion techniques are utilized in the analysis of collected, time-resolved data. The averaged, line-of-sight electron densities are found to be in of the order of one hundredth of an amagat for time delays close to one-half microseconds. The electron densities indicate variations across the laser-induced plasma.
- New chapter 30 elucidates the connection of measured shadowgraphs from optically induced air breakdown with emission spectroscopy in selected gas mixtures. Spectroscopic analysis explores well-above hypersonic expansion dynamics using primarily diatomic molecule cyanide and atomic hydrogen emission spectroscopy. Analysis of the air breakdown and selected gas breakdown events permits the use of Abel inversion for inference of the expanding species distribution. Typically, species are prevalent at higher density near the hypersonically expanding shock wave, measured by tracing cyanide and a specific carbon atomic line.
- New appendix J presents NMT and BESP MATLAB-scripts for computation of diatomic spectra.
- New appendix K presents Abel inversion MATLAB-scripts with one specific example.
- New appendix L summarizes select recent publications that include C.G.P. as author.

Christian Parigger  
February 2024

## Reference

- [1] Parigger C G and Nemes L 2017 *Int. J. Mol. Theor. Phys.* **1** 00105

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CGP and JOH appreciate the support by the Center for Laser Applications (CLA) at the University of Tennessee Space Institute during over 30 years of research engagement in spectroscopy. The research led to this book and its second edition that describe how quantum mechanics can be used to predict diatomic molecule spectra. The book and its second edition provide a comprehensive overview on diatomic molecule fundamentals and emphasize the applications of spectroscopy predictions in analysis of experimental data. Both CGP and JOH are happy to report interests and contributions by postgraduate students. Moreover, CGP is delighted about publication of the first and second editions in memoriam to JOH, a long time collaborator in analysis of laser-induced plasma spectra recorded at the CLA. Last but not least, CGP thanks researchers and colleagues at the CLA and international collaborators throughout the world for extensive discussions and motivation towards completion of both book editions.

# Author biographies

## Christian G. Parigger

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Christian G. Parigger (Courtesy UTSI photo albums: L Horton.)

The research interests of Dr Christian Parigger include fundamental and applied spectroscopy, nonlinear optics, quantum optics, ultrafast phenomena, ultrasensitive diagnostics, lasers, combustion and plasma physics, optical diagnostics, biomedical applications, and, in general, atomic and molecular and optical physics. His Mag. rer. nat. degree shows work on optical bistability at the University of Innsbruck, Austria, with guidance by Dr Peter Zoller.

His PhD degree studies are on the subject of polarization spectroscopy and magnetically induced switching at the University of Otago, Dunedin, New Zealand, with guidance by Drs Wes Sandle and Rob Ballagh. He also holds the Dr rer. nat. degree in Physics from the University of Innsbruck, Austria. Since 1987, his work encompasses experimental, theoretical and computational research, together with teaching, service, and outreach at the Center for Laser Applications at The University of Tennessee Space Institute, Tullahoma, Tennessee, USA.

## James O and Jeri Hornkohl

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James O and Jeri Hornkohl (Courtesy UTSI photo albums: L Horton.)

The contributions of James Hornkohl, or ‘Jim’, encompass the spectroscopy of diatomic molecules, and the application of such spectroscopy in diagnosis of combustion, plasmas, rocket propulsion, and related problems. His support of student theses and dissertations has been especially significant, including the application of numerical methods in analysis of experiments.

Moreover, his help has been greatly appreciated by his collaborators in the design of computational and experimental methods to record digital data. During the last 30 years prior to his death on February 7, 2017, Jim had been strongly engaged in the description of the very details on diatomic spectroscopy. The challenge to Jim’s work has been the prediction of spectra with a focus on diatomic spectroscopy. The aim of his lifetime work was the design of an algorithm for the prediction and fitting of computed to measured molecular spectra and to provide inferences of parameters such as excitation temperature. The means to accomplish goals for various transitions of diatomic molecules are the consistent application of standard quantum theory of angular momentum.

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# Part I

Fundamentals of the diatomic molecule





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# Chapter 1

## Primer on diatomic spectroscopy

### 1.1 Overview

This book describes how one uses quantum mechanics to predict the spectra of diatomic molecules in their gaseous state. The two most important attributes of a spectral line are its position in the electromagnetic spectrum and the strength with which the molecule can interact with the radiation field to produce spectral lines. Thus, a book that discusses the calculation of positions and intensities of spectral lines of a diatomic molecule equally communicates the application of quantum theory to the diatomic molecule.

The theoretically convenient measure of spectral line position is its vacuum wave number  $\tilde{\nu}_{ul}$ , which is the difference between the upper term  $T_u$  (i.e., upper energy eigenvalue expressed in the units of  $\text{cm}^{-1}$ ) and the lower term  $T_l$ ,

$$\tilde{\nu}_{ul} = T_u - T_l. \quad (1.1)$$

In the optical region, the term difference corresponds to a specific color. However, experiments usually measure the wavelength positions in a laboratory setting at standard ambient temperature and pressure. For typical laser spectroscopy investigations of, say, optical emission spectroscopy subsequent to generation of a laser spark, spectral resolutions of the instrument spectrometer and detector amount to 0.1–0.01 nm, rarely to 0.001 nm or 1 pm. At the wavelength,  $\lambda$ , of 400 nm, a spectral resolution,  $\Delta\lambda$ , of better than 1 pm corresponds to a resolving power,  $R$ ,

$$R = \lambda/\Delta\lambda \geq 400\,000, \quad (1.2)$$

or a wave number resolution of better than  $0.05 \text{ cm}^{-1}$ . The spectral resolution of diatomic molecular data computed in this book is better than  $0.05 \text{ cm}^{-1}$ . For laser-induced optical breakdown experiments, which is a recent application of diatomic molecular spectroscopy, resolving powers are of the order of 4000–10 000. For high-resolution, absorption measurements of stellar astrophysical objects, resolving powers of the order of 40 000 are quite common.

The theoretically most convenient measure of a molecule's ability to interact with electromagnetic radiation is its Condon and Shortley [1] line strength,  $S_{u\ell}$ , which describes transitions between an upper,  $u$ , and a lower level,  $\ell$ . The line strength represents a summation over individual states that comprise upper and lower levels. Both the vacuum wave number  $\tilde{\nu}_{u\ell} = \tilde{\nu}_{\ell u}$  and the line strength  $S_{u\ell} = S_{\ell u}$  are symmetric with regard to the upper and lower levels. In addition, the symbols  $u$  and  $\ell$  represent a collection of quantum numbers. In diatomic spectroscopy, upper state quantum numbers are normally denoted with a single prime, while lower states are denoted with the absence of a prime or a double prime. The absence of a double prime has become the standard way of denoting a lower state diatomic quantum number.

## 1.2 Reversed angular momentum

Historically, the reversed-angular-momentum (RAM) methodology has successfully predicted diatomic spectra without the use of modern digital computers. The RAM method establishes a reduced set of basis states; in other words, works with an a priori approximation. Sets of rules are introduced when applying a transformation to a molecular-fixed from the laboratory-fixed coordinate system. These rules utilize a supposed reversal of sign in the application of quantum mechanical angular momentum algebra. This section provides a brief historic account of the challenges associated with the RAM method.

The reversed-angular momentum approach is mentioned first in an article on the quantization question of the asymmetric top [2]. Klein writes in the introduction that the paper might be of interest for methods of quantization. The reversed sign is introduced for the equations of the components of angular momentum in the molecular-fixed coordinate system in order to obtain agreement with the well-established classical equations for the symmetric top. Conversely, the application of the standard, laboratory-fixed angular momentum equations would lead to the wrong classical result. This article also makes reference to canonical conjugate Euler angles that are interpreted as references to dual space.

The RAM methodology is embraced by Van Vleck in his work on the coupling of angular momentum vectors in molecules [3]. Notably, Sir Harold Kroto communicates in his acceptance lecture for the 1996 Nobel Prize in Chemistry, 'Symmetry, Space, Stars and C<sub>60</sub>' [4], the importance of 'Symmetry, the Key to the Theory of Everything'. With reference to the RAM work, Sir Kroto quotes Van Vleck: '*Practically every-one (!) knows that the components of total angular momentum (NB the angular momentum operator is usually denoted by  $\mathbf{J}$  and the associated quantum number by  $j$ ) of the molecule relative to the axes  $[x, y, z]$  fixed in space satisfy the commutation relation of the form*

$$J_x J_y - J_y J_x = iJ_z \quad (1.3)$$

*Klein discovered the rather surprising fact that when total angular momentum is referred to axes mounted in the molecule which we will denote by  $[x', y', z']$  the sign of  $i$  in the commutation relation is reversed i.e.*

$$J_x' J_y' - J_y' J_x' = -iJ_z' \quad (1.4)$$

Sir Kroto goes on to say: *Does practically everyone know this?—I wondered whether to check this claim out by asking everyone on the main street in Brighton whether they did. I hardly knew—or more accurately—really understood the first relation, let alone the second. However I did know that angular momentum was quantised and governed by the fundamental relations*

$$\langle j | J^2 | j \rangle = \hbar^2 j(j+1) \quad (1.5)$$

$$M_J = -j \dots +j \quad (1.6)$$

which means that  $J$  has  $2j + 1$  possible orientations, and

$$\Delta j = 0, \pm 1 \quad (1.7)$$

which indicates that when a transition occurs,  $j$  may only change by one unit or on occasion remain unchanged.' Previously, in 1975 and then in 1992, Sir Kroto discussed the molecule-fixed angular momentum following Van Vleck [3], leading to the reversed-angular momentum equations in his Nobel laureate lecture [4] and in his book on molecular rotation spectra [5].

However, an accurate review shows that there is no reversal of the sign when moving from a laboratory-fixed to a molecule-fixed coordinate system; in other words, there is no mathematical support of the reversed sign. Sustainance of the angular momentum equations can be explained as follows. In terms of classical mechanics, reversal of motion occurs as one goes from a rotating system to a fixed system, or vice versa. For example, motion reversal can be experienced by looking at the surroundings while on a rotating merry-go-round versus observing the rotation in the fixed reference frame. The quantum mechanical implementation of motion reversal or time reversal changes the sign and takes the conjugate complex, leading to the preservation of the sign. Reference to dual space would confuse things because clearly the standard angular momentum operator equations are not affected by a transformation from laboratory-fixed to molecule-fixed coordinates (see appendix A).

A reasonably concise treatment shows preservation of the commutator relations under a unitary transformation. Consider the operators  $A$ ,  $B$ , and  $C$  which satisfy the commutation formula

$$AB - BA = iC \quad (1.8)$$

and subject these three operators to the unitary transformation  $U$ ; that is,

$$A' = U^\dagger A U \quad (1.9a)$$

$$A = U A' U^\dagger \quad (1.9b)$$

with similar equations holding for  $B$  and  $C$ . Then,

$$AB - BA = U A' U^\dagger U B' U^\dagger - U B' U^\dagger U A' U^\dagger \quad (1.10a)$$

$$= U A' B' U^\dagger \quad (1.10b)$$

$$= iC \quad (1.10c)$$

$$iU^\dagger C U = A'B' \quad (1.10d)$$

$$iC' = A'B' - B'A'. \quad (1.10e)$$

The above result, e.g., see Davydov [6], holds for all commutators, including those for angular momentum. Thus,

$$J_x' J_{y'} - J_{y'} J_x' = i J_z' \quad (1.11a)$$

$$J_{y'} J_z' - J_z' J_{y'} = i J_x' \quad (1.11b)$$

$$J_z' J_x' - J_x' J_z' = i J_{y'} \quad (1.11c)$$

In summary, the RAM method is not utilized in this book for the computation of diatomic molecular spectra. RAM is avoided due, in part, to not needing approximations thanks to the availability of modern digital computers and due in part to the mathematical inconsistency of the supposed change of sign, as implied by the ‘reversed-angular momentum’ descriptive nomenclature.

### 1.3 Exact diatomic eigenfunction

An exact expression of the diatomic eigenfunction is essential for prediction of spectra. The major difference between this book and other treatments of the diatomic molecule is the use of the Wigner–Witmer diatomic eigenfunction [7] in place of invoking the Born–Oppenheimer approximation [8] from the very beginning of a theory description. In the Wigner–Witmer approach, angular coordinates are exactly separated from the electronic–vibrational coordinates. In this book, the Wigner–Witmer eigenfunction is employed for computation of the vacuum wave numbers and the rotational line strengths. If one were to instead adopt the Born–Oppenheimer approximation, then the rotational line strengths would be labeled as Hönl–London factors. The Born–Oppenheimer approximation breaks the electronic–vibrational strength into electronic and vibrational parts that correspond to *r*-centroids and Franck–Condon factors, and both may be functions of the total angular momentum in the upper and lower levels.

The expression *spectroscopic accuracy* refers to the accuracy with which line position measurements can be performed. Whereas wavelength measurements having an accuracy of 1 part per million are routinely performed, achieving an accuracy of 1 part per hundred in the measurement of relative intensities of a group of spectral lines is fully adequate for many purposes. Thus, one may elect to directly use the Born–Oppenheimer approximation for many practical calculations of molecular line intensity; namely, approximating the diatomic eigenfunction as a product of electronic, vibrational, and rotational factors. However, the Born–Oppenheimer approximation cannot produce diatomic term values with

spectroscopic accuracy without generalization. To achieve spectroscopic accuracy within the Born–Oppenheimer approximation, one must include sums over the many electronic states of the molecule and sums over the many vibrational states of each electronic state. Van Vleck transformations [9] or other mathematical procedures reduce the dimension of the Hamiltonian matrix prior to numerically diagonalization [10–15].

In this book, only one diatomic selection rule is used. A spectral line, i.e., a term difference, is allowed if the angular momentum part of its line strength is nonvanishing. However, a modification of the line strength computation is required if the diatomic molecule in question is homonuclear, i.e., the two nuclei are identical. An unresolved hyperfine structure in the spectrum of a homonuclear molecule causes states of positive parity and negative parity to have different nuclear spin statistical weights,  $g_+$  and  $g_-$ . If the nuclear spin is zero, then either  $g_+$  or  $g_-$  will be zero. Thus, exchange symmetry, the symmetry associated with the exchange of identical particles, rigorously forbids certain spectral lines, even when the rotational line strength is nonzero. However, if the rotational line strength factor vanishes, then the spectral line is rigorously forbidden.

## 1.4 Computation of diatomic spectra

The required steps for computation of spectra can be summarized as follows:

- An angular momentum coupling model must be chosen because angular momentum theory does not tell us how the total angular momentum is formed from the orbital and spin momenta.
- The eigenfunctions for everything in the system except the total angular momentum are computed.
- With the eigenfunctions obtained in the previous step and the chosen angular momentum coupling model, upper and lower Hamiltonians are computed and diagonalized.
- From the orthogonal matrices that diagonalize the upper and lower Hamiltonians, the line strengths are computed for various possible types of transitions, e.g., electric dipole, magnetic dipole, electric quadrupole, etc. Typically, one knows precisely what type of transition dominates in the spectrum, but this is not invariably the case.
- The nonvanishing of the rotational angular momentum part of the line strength selects the subset of allowed spectral lines from the computed term differences.

Consequently, the minimal information required for computation of a spectrum includes selected term differences  $\tilde{\nu}_{ul}$  and the computed line strengths  $S_{ul}$ . A description of a diatomic molecule having  $N$  electrons and residing in field free space requires  $3N + 6$  spatial or angular coordinates, the time  $t$ ,  $N$  electronic spin variables, and two nuclear spin variables. In the case of the diatomic molecule, the only exactly separable variables are the time  $t$ , the coordinates of the total mass, and three Euler angles which describe the total angular momentum. The Wigner–Witmer

diatomic eigenfunction provides the exact separation of three Euler angles, but  $3N$  internal spatial coordinates and the numerous spins remain. Unless the number of electrons  $N$  is very small, the diatomic problem remains unsolvable with spectroscopic accuracy because there are  $3N$  independent variables that cannot be treated with mathematical exactness.

Despite the challenges mentioned in the previous paragraph, one can, with two stringent caveats, apply the above algorithm to the diatomic molecule. The first caveat is that one must have extensive experimentally recorded wave number tables,  $\tilde{\nu}_{ul}^{\text{exp}}(J', J)$ , versus upper and lower total angular momenta,  $J'$  and  $J$ , respectively, for many vibrational bands in the spectrum of a molecule of interest. The second caveat is associated with using trial values of semiempirical molecular parameters for each vibrational level,  $v$ , such as  $B_v$ ,  $D_v$ ,  $A_v$ ,  $\lambda_v$ ,  $\gamma_v$ , and so on. One computes term differences,  $\tilde{\nu}_{ul}(J', J)$ , from numerically diagonalized upper and lower Hamiltonians, calculates corrections to the trial values of the parameters from differences  $\tilde{\nu}_{ul}(J', J) - \tilde{\nu}_{ul}^{\text{exp}}(J', J)$ , and iterates the computations until the errors in the computed line positions are comparable to the estimated errors in the experimental line positions. When successful, this procedure yields working models for the upper and lower Hamiltonians and sets of molecular parameters that predict the measured line positions.

The practical significance of molecular parameters was their appearance in term value equations, semiempirical equations with which one can compute the upper  $T_u$  and lower  $T_l$  terms, and thereby the vacuum wave number  $\tilde{\nu}_{ul}$ . Herzberg [16] gives many examples of term value equations, but note that when Herzberg wrote his book the numerical diagonalization of thousands of matrices was impractical. The current significance of the molecular parameters is that they can be used to compute diatomic Hamiltonian matrix representations in one of the Hund's bases.

In this book the computation of  $\tilde{\nu}_{ul}(J', J)$  and  $S_{ul}(J', J)$  is based upon the Wigner–Witmer diatomic eigenfunction instead of the eigenfunction associated with the Born–Oppenheimer approximation, but computations of the electronic–vibrational strengths utilize separation of electronic from vibrational contributions familiar from the Born–Oppenheimer approximation.

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