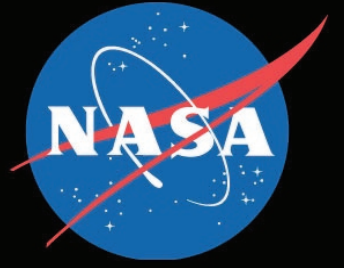
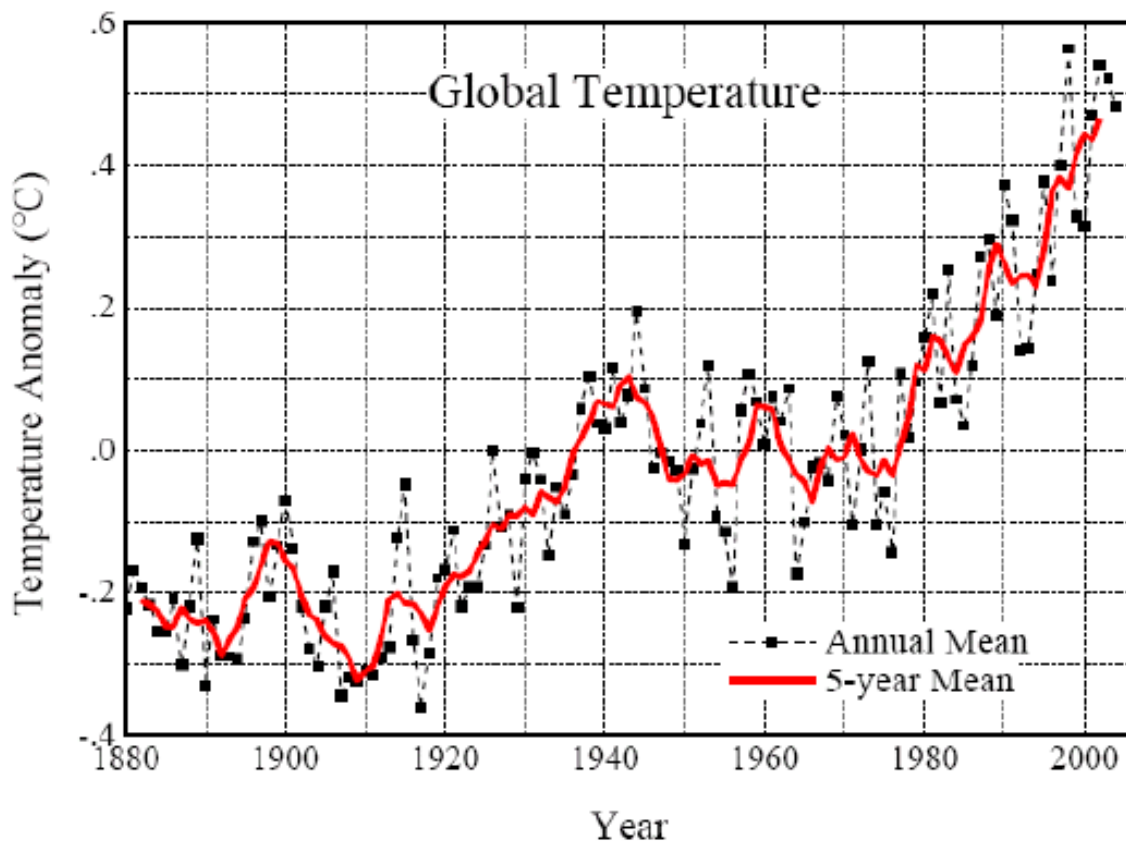


National Aeronautics and Space Administration



Earth Math

Earth Math



A Brief Mathematical Guide to Earth Science and Climate Change

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2009-2010 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 9 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

Reviewers:

Mr. Jim Bean (Carson High School, NV)
Mr. Graham Coleman (Wymondham College, UK)
Mr. Cris DeWolf (Michigan Earth Science Teachers Association, MI)
Dr. David Dillon (Colorado State University - Pueblo)
Ms. Kathy East (Challenger Learning Center, AK)
Ms. Wendy Ehnert (Lathrop High School, AK)
Dr. Mike Inglis (Suffolk County Community College)
Ms. Dorian Janney (Parkland Middle School, MD)
Ms. Elaine Lewis (NASA/Honeywell, MD)
Ms. Leidy Luciani (Institute for Mathematics and Computer Science, FL)
Ms. Sally O'Leary (Murphysboro High School, IL)
Ms. Wendy Sheridan (Ottawa Township High School, IL)

For more weekly classroom activities about astronomy and space visit the NASA website,

<http://spacemath.gsfc.nasa.gov>

To suggest math problem or science topic ideas, contact the Author, Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

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Alignment with Standards (AAAS Project:2061 Benchmarks).

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1

(6-8) Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1

(9-12) - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

Mathematics Topic Matrix

Topic	Problem Numbers																															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
Inquiry		X				X				X			X									X										
Technology, rulers																														X	X	
Numbers, patterns, percentages	X	X	X	X	X				X	X	X		X		X	X						X										
Averages									X																							
Time, distance, speed							X		X																							
Areas and volumes																						X										
Scale drawings																						X									X	X
Geometry																																
Probability, odds																																
Scientific Notation																																
Unit Conversions				X	X						X	X		X	X	X	X	X	X	X	X				X	X	X	X				
Fractions																																
Graph or Table Analysis						X	X	X							X	X	X				X	X		X	X	X	X	X	X	X		
Pie Graphs															X																	
Linear Equations																															X	
Rates & Slopes														X						X	X		X	X						X		
Solving for X	X	X	X		X																											
Evaluating Fns					X		X																									
Modeling						X		X														X									X	
Trigonometry																																
Integration																																
Differentiation																										X						

Mathematics Topic Matrix (cont'd)

Topic	Problem Number															
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	
Inquiry																
Technology, rulers	X	X	X	X	X	X				X	X		X	X		
Numbers, patterns, percentages																
Averages																
Time, distance, speed												X	X	X	X	
Areas and volumes			X						X					X	X	
Scale drawings	X	X	X	X	X	X	X	X	X	X	X			X		
Geometry																
Probability, odds																
Scientific Notation																
Unit Conversions						X	X	X		X	X	X	X	X	X	
Fractions																
Graph or Table Analysis										X	X	X	X			
Pie Graphs																
Linear Equations												X	X			
Rates & Slopes										X	X		X			
Solving for X																
Evaluating Fns																
Modeling										X	X	X	X	X		
Trigonometry	X															
Integration																
Differentiation																

How to Use this Book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Earth Math offers math applications through strong motivation of discovery. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Earth Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Earth Math**. Read the scenario that follows:

Ms. Green decided to pose a new activity using Earth Math with her students. She challenged each student team with math problems from the Earth Math book. She copied each problem for student teams to work on. Beginning with the introductory math problems at the beginning of the book she provided background and basic understanding of Global Climate Change. She then had each team work through problems 25-29 that were developed so that students could study long-term historical trends in the carbon dioxide levels and solar insolation spanning up to 400,000 years. This issue is of considerable importance, not only as a test of sophisticated climate models, but also as a measure of whether current changes are caused by humans (anthropogenic) or simply part of natural trends spanning decades, centuries or even millennia. Student teams were then divided to debate, human impact vs. natural trends. One student from each team was the lead for the debate, while other team members could supply information during the debate to support their team's conclusion.

Earth Math can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Teacher Comments

"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that..." (Associate Professor of Physics)

"Space Math has more up-to-date applications than are found in any textbook. Students enjoy real-world math problems for the math they have already learned. Doing Space Math problems has encouraged some of my students to take pre-calculus and calculus so they can solve the more advanced problems. I learned about Space Math through an email last year. I was very impressed with the problems. I assigned some of the problems to students in my Physics classes, printing them out to put in their interactive notebooks. I displayed other problems for group discussion, assigned some for homework and used some for group class work. I like the diversity, the color format and having the solutions. I expect to use them even more next year in our new space science class. We will have 50 students in two sections." (Alan. High School Science Teacher)

"It took time for them to make the connection between the math they learned in math class and applying it in the science classroom. Now I use an ELMO to project them. I have used them for class work and/or homework. The math activities were in conjunction with labs and science concepts that were being presented. The math helped "show" the science. Oftentimes students were encouraged to help and teach each other. Students began to see how math and science were connected. I knew the students were making the connections because they would comment about how much math they had to do in science. Their confidence in both classes increased as they were able practice the concepts they learned in math in my science class." (Brenda, Technology Resource Teacher)

Introduction to Earth Math

We are at the dawn of a new era when our familiar 'Nine Planets' have been replaced in a matter of a decade by a catalog of over 400 known planets orbiting other stars. They go by crude names such as HD209458b or CoRoT-7a, but through the advance of new technologies and techniques in spectroscopy, we are coming to know the atmospheres of these worlds nearly as well as we do Venus, Mars or distant Pluto.

At the same time, and at long last, we are coming to know our own world just in time to appreciate how fragile it is under the action of over 6 billion humans. Wildlife biologists speak of the carrying capacity of an eco system. We now appreciate that, given the manner in which we interact with Nature, the carrying capacity of Earth for the Genus Homo Sapiens may be dramatically smaller than we had once imagined.

Although scientists have been sounding the alarm about Global Climate Change (GCC) for over 40 years, only in the last five years has this realization penetrated through the thick haze of social and political denial, and has even become a *cause celebre* among the popular media. There are still naysayers, but the growing weight of consistent data and sophisticated modeling has increasingly isolated 'disbelievers'. GCC is no longer a 'belief system' but a political, social and economic reality and fact of our times. It will increasingly rob us of fresh water, coastal real estate, and even good health in the face of diseases migrating out of tropical breeding grounds and into the mid-latitudes.

As a means to the end of understanding our environment, this book explores a few of the many concepts that frequently come up in the study of Earth systems and GCC. It is not meant to be exhaustive, but it does aspire to show how the underlying basis for many of the issues we are now confronted by both socially and politically have simple mathematical underpinnings. This book is meant to be used as a companion guide to standard Earth Science and Mathematics courses at the middle and high school-level. Ironically, both of these themes seldom offer enough applications problems in mathematics to allow students to quantitatively understand typical news media reports about GCC. The problems need not be executed in the order presented.

This book begins with some 'warm up' problems that have to do with basic terminology used in GCC news and research. Among these is the concept of 'part per million' and 'part per billion' (1,2,3,4) and temperature (5). Also covered are problems featuring a few of the general properties of Earth such as its length-of-day (6,7), polar wander (8), gravity (35), magnetism (36). There are also simple exercises that let students use satellite photos of familiar scenes such as Las Vegas (30), Washington DC (31), Paris (32) to determine image scales and perform simple photogrammetry. This is an important skill for completing projects involving satellite studies of glacial retreat (33) and biomass loss (34). Also offered are problems dealing with the concept of reflectivity and absorption (9, 10, 11). The intent is that students will be able to think quantitatively about how the ability of a substance to reflect light influences how hot the body is. Although not extensively covered in this book, reflectivity explains why asphalt

soil and concrete have different temperatures on a sunny day, and why cities are hotter than forests and other green spaces. This is related to why carbon dioxide is a 'green house gas' and the role of urban planning and 'green space' in making a city more comfortable in the summertime.

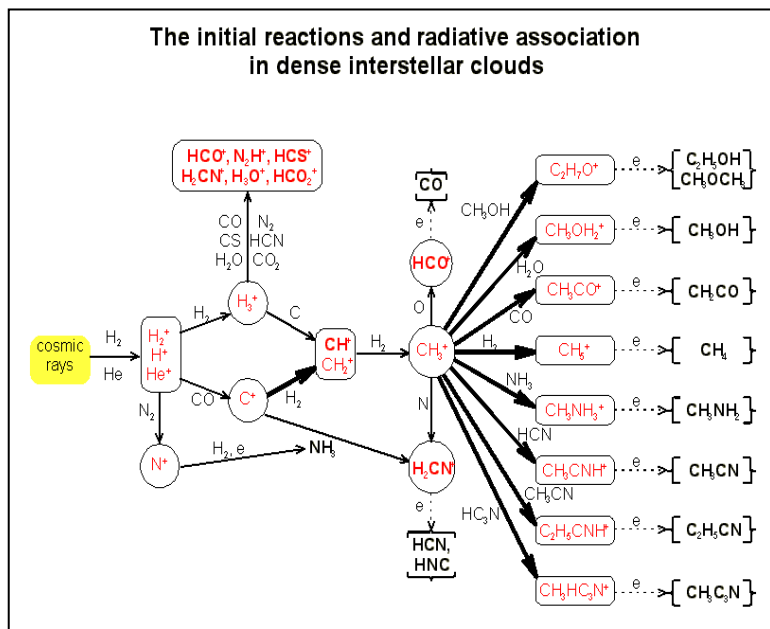
Once students are familiar with the basic terminology and computations, they can begin working the second tier of problems that cover the atmospheric carbon and carbon dioxide budgets. This discussion begins with students exploring how electricity consumption is measured in terms of kilowatt-hours (12, 13). This is then applied to having the student create an energy budget for their home and lifestyle, and calculating its carbon impact (14, 15, 16).

The terminology learned from these exercises (kWh and tons of carbon) are applied, first to the state-level (17), and then to the world electricity consumption (18). The last step is an investigation of the state of the global atmosphere (19 – 24) and the creation of a detailed description of sources and sinks of carbon, and carbon sequestration. In the later topic, students are encouraged to create Excel spreadsheets (22) to model and forecast future carbon dioxide levels in 2050 and 2100. Because even simple atmospheric models involve linear 'rates of change' one problem (24) is provided for advanced high school students who know calculus and can solve a simple first-order differential equations.

Additional problems (25 – 29) have been created so that students can study long-term historical trends in the carbon dioxide levels and solar insolation spanning up to 400,000 years. This issue is of considerable importance, not only as a test of sophisticated climate models, but also as a measure of whether current changes are caused by humans (anthropogenic) or simply part of natural trends spanning decades, centuries or even millennia. The later is a common argument, not supported by existing data, that is often cited by global warming nay-sayers. Students need to be exposed to the underlying data so that they can make educated decisions as future voters.

Needless to say, GCC science is sufficiently complex as a physical and chemical process that only opinions based upon a sound knowledge of the underlying system actually count for very much. This book will hopefully serve to supply some modest quantitative rigor to classroom discussions about GCC.

In terms of the mathematics to be encountered in this book, students will be exposed to many problems involving unit conversions. GCC science and reports involve terms such as kilowatt-hour, megawatt-hour and gigawatt-hours, as well as megatons and gigatons. Students will become versed in converting units where appropriate, and through the calculations, need to work with the concept of significant figures. Creating linear equations from graphical and tabular information will be covered as well as forecasting. Although each problem page contains from two to eight subsidiary math problems related to each of the 40 main topics, students need not complete all of the problems to have a meaningful understanding of GCC.



Because molecules and atoms come in 'integer' packages, the ratios of various molecules or atoms in a compound are often expressible in simple fractions. Adding compounds together can often lead to interesting mixtures in which the proportions of the various molecules involve mixed numbers.

The figure shows some of the ways in which molecules are synthesized in interstellar clouds. (Courtesy D. Smith and P. Spanel, "Ions in the Terrestrial Atmosphere and in Interstellar Clouds", *Mass Spectrometry Reviews*, v.14, pp. 255-278.)

In the problems below, do not use a calculator and state all answers as simple fractions or integers.

Problem 1 - What makes your car go: When 2 molecules of gasoline (ethane) are combined with 7 molecules of oxygen you get 4 molecules of carbon dioxide and 6 molecules of water.

- A) What is the ratio of ethane molecules to water molecules?
- B) What is the ratio of oxygen molecules to carbon dioxide molecules?
- C) If you wanted to 'burn' 50 molecules of ethane, how many molecules of water result?
- D) If you wanted to create 50 molecules of carbon dioxide, how many ethane molecules would you have to burn?

Problem 2 - How plants create glucose from air and water: Six molecules of carbon dioxide combine with 6 molecules of water to create one molecule of glucose and 6 molecules of oxygen.

- A) What is the ratio of glucose molecules to water molecules?
- B) What is the ratio of oxygen molecules to the total number of carbon dioxide and water molecules?
- C) If you wanted to create 120 glucose molecules, how many water molecules are needed?
- D) If you had 100 molecules of carbon dioxide, what is the largest number of glucose molecules you could produce?

Answer Key

Problem 1 - What makes your car go: When 2 molecules of gasoline (ethane) are combined with 7 molecules of oxygen you get 4 molecules of carbon dioxide and 6 molecules of water.

A) In this reaction, 2 molecules of ethane yield 6 molecules of water, so the ratio is 2/6 or 1/3.

B) 7 oxygen molecules and 4 carbon dioxide molecules yield the ratio 7/4

C) The reaction says that 2 molecules of ethane burn to make 6 molecules of water. If you start with 50 molecules of ethane, then you have the proportion:

$$\frac{50 \text{ ethane}}{2 \text{ ethane}} = \frac{x\text{-water}}{6 \text{-water}} \quad \text{so } X = 25 \times 6 = \mathbf{150 \text{ water molecules.}}$$

D) Use the proportion:

$$\frac{50 \text{ Carbon Dioxide}}{4 \text{ carbon dioxide}} = \frac{X \text{ ethane}}{2 \text{ ethane}} \quad \text{so } X = 2 \times (50/4) = \mathbf{25 \text{ molecules ethane}}$$

Problem 2 - How plants create glucose from air and water: Six molecules of carbon dioxide combine with 6 molecules of water to create one molecule of glucose and 6 molecules of oxygen.

A) What is the ratio of glucose molecules to water molecules?

B) What is the ratio of oxygen molecules to the total number of carbon dioxide and water molecules?

C) If you wanted to create 120 glucose molecules, how many water molecules are needed?

D) If you had 100 molecules of carbon dioxide, what is the largest number of glucose molecules you could produce?

A) Glucose molecules /water molecules = $\mathbf{1 / 6}$

B) Oxygen molecules / (carbon dioxide + water) = $6 / (6 + 6) = 6/12 = \mathbf{1/2}$

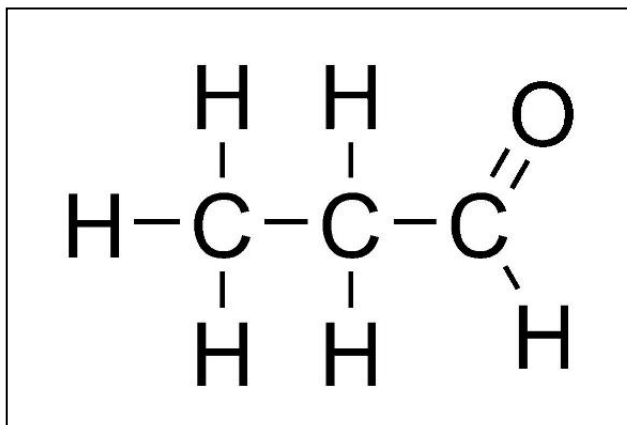
C)

$$\frac{120 \text{ glucose}}{1 \text{ glucose}} = \frac{X \text{ water}}{6 \text{ water}} \quad \text{so } X = 6 \times 120 = \mathbf{720 \text{ water molecules}}$$

D)

$$\frac{100 \text{ carbon dioxide}}{6 \text{ carbon dioxide}} = \frac{X \text{ glucose}}{1 \text{ glucose}} \quad \text{so } X = 100/6 \text{ molecules.}$$

The problem asks for the largest number that can be made, so we cannot include fractions in the answer. We need to find the largest multiple of '6' that does not exceed '100'. This is 96 so that $6 \times 16 = 96$. That means we can get no more than $\mathbf{16 \text{ glucose molecules}}$ by starting with 100 carbon dioxide molecules. (Note that $100/6 = 16.666$ so '16' is the largest integer).



The complex molecule Propanal was discovered in a dense interstellar cloud called Sagittarius B2(North) located near the center of the Milky Way galaxy about 26,000 light years from Earth. Astronomers used the giant radio telescope in Greenbank, West Virginia to detect the faint signals from a massive cloud containing this molecule. It is one of the most complex molecules detected in the 35 years that astronomers have searched for molecules in space. Over 140 different chemicals are now known.

Problem 1 - How many atoms of hydrogen (H), carbon (C) and oxygen (O) are contained in this molecule?

Problem 2 - What percentage of all atoms are hydrogen? Carbon? Oxygen?

Problem 3 - What is the ratio of carbon atoms to hydrogen atoms in propanal?

Problem 4 - If the mass of a hydrogen atom is defined as 1 AMU, and carbon and oxygen have masses of 12.0 and 16.0 AMUs, what is the total mass of a propanal molecule in AMUs?

Problem 5 - What is the complete chemical formula for propanal?



Problem 6 - If this molecule could be broken up, how many water molecules could it make if the formula for water is H_2O ?

Problem 1 - How many atoms of hydrogen (H), carbon (C) and oxygen (O) are contained in this molecule?

Answer; **There are 6 atoms of hydrogen, 3 atoms of carbon and 1 atom of oxygen.**

Problem 2 - What percentage of all atoms are hydrogen? Carbon? Oxygen?

Answer: There are a total of 10 atoms in propanal so it contains $100\% \times (6 \text{ atoms} / 10 \text{ atoms}) = 60\%$ **hydrogen**; $100\% \times (3/10) = 30\%$ **carbon** and $100\% \times (1/10) = 10\%$ **oxygen**.

Problem 3 - What is the ratio of carbon atoms to hydrogen atoms in propanal?

Answer: 3 carbon atoms / 6 hydrogen atoms = **1/2** or a proportion of '1 to 2'.

Problem 4 - If the mass of a hydrogen atom is defined as 1 AMU, and carbon and oxygen have masses of 12.0 and 16.0 AMUs, what is the total mass of a propanal molecule in AMUs?

Answer: 1 AMU x 6 hydrogen + 12 AMU x 3 carbon + 16 AMU x 1 oxygen = 6 AMU + 36 AMU + 16 AMU = **58 AMU for the full molecule mass.**

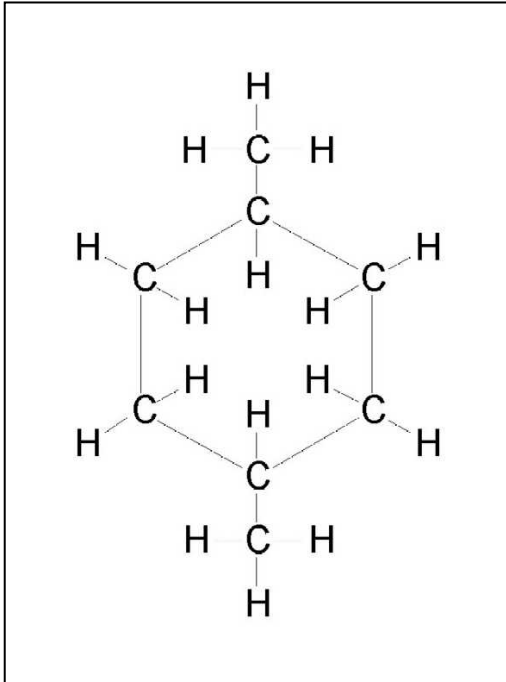
Problem 5 - What is the complete chemical formula for propanal?



Problem 6 - If this molecule could be broken up, how many water molecules could it make if the formula for water is H₂O?

Answer: A single water molecule requires exactly 1 oxygen atom, and since propanal only has 1 atom of oxygen per molecule, you can only get **1 molecule of water** by breaking up a single propanal molecule.

Parts Per Hundred (pph)



A common way of describing the various components of a population of objects is by the number of parts that they represent for every 100, 1000 or 1 million items that are sampled.

For instance, if there was a bag of 100 balls: 5 were red and 95 were white, you would say that the red balls represented 5 parts per hundred (5 pph) of the sample.

This also means that if you had a bag with 300 balls in the same proportions of red and white balls, the red balls would still be '5 parts per hundred' even though there are now 15 red balls in the sample ($15/300 = 5/100 = 5$ pph).

For each of the situations below, calculate the parts-per-hundred (pph) for each sample.

Problem 1 - Your current age compared to 1 century

Problem 2 - 10 cubic centimeter (10 cc) of food coloring blended into 1 liter (1000 cc) of water.

Problem 3 - The 4 brightest stars in the Pleiades star cluster, compared to the total population of the cluster consisting of 200 stars.

Problem 4 - One day compared to one month (30 days)

Problem 5 - Five percent of anything.

Problem 6 - The figure above shows the atoms of hydrogen (H) and carbon (C) in the molecule of dimethylcyclohexane. What is the pph of the carbon atoms in this molecule compared to the total number of atoms?

Answer Key

3

Problem 1 - Your current age compared to 1 century

Answer: If your current age is 14 years, then compared to 100 years, your lifetime is **14 pph of a century**.

Problem 2 - 1 cubic centimeter (10 cc) of food coloring blended into 1 liter (1000 cc) of water.

Answer: 10 cc / 1000 cc is the same as 1 / 100 so the food coloring is **1 pph of a liter**.

Problem 3 - The 4 brightest stars in the Pleiades star cluster, compared to the total population of the cluster consisting of 200 stars.

Answer: 4 stars / 200 members = 2 / 100 = **2 pph of the cluster stars**.

Problem 4 - One day compared to one month (30 days)

Answer: 1 day / 30 days = 0.0333 so 0.033 x 100 = **3.3 pph of a month**.

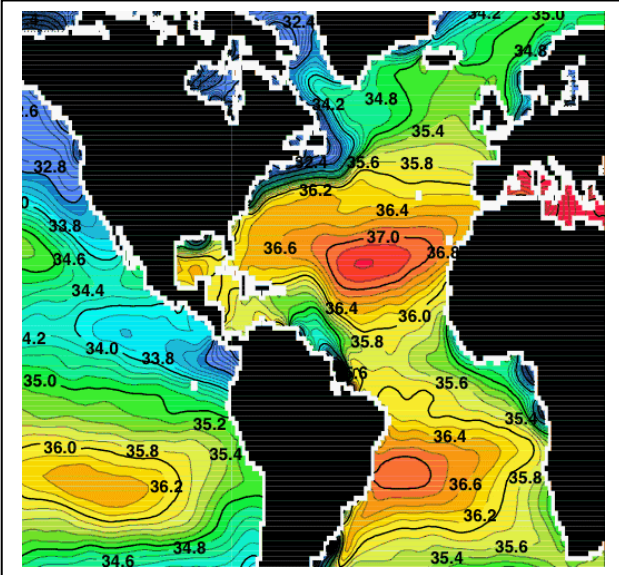
Problem 5 - Five percent of anything.

Answer: 5 % = 5 / 100 = **5 pph of anything**.

Problem 6 - The figure above shows the atoms of hydrogen (H) and carbon (C) in the molecule of dimethylcyclohexane. What is the pph of the carbon atoms in this molecule compared to the total number of atoms?

Answer: There are a total of 24 atoms in this molecule. There are a total of 8 carbon atoms, so the fraction is $8/24 = 1/3$ which equals $0.333 \times 100 =$ **33 pph of the total atoms**.

Parts per Thousand (PPT)



A convenient way to compare the number of items of one kind mixed into a population of other items is the part-per-thousand (ppt).

For example your age (say 15 years) compared to the entire span of recent history (2,000 years) is the ratio $15/2000$ or 0.0075 . This can be expressed as 75 parts per 10,000 or 7.5 parts per thousand (ppt).

The span of 30 minutes compared to a day (1440 minutes) is $30/1440 = 0.021$ which is 21 parts per thousand.

Problem 1 - One day compared to a year.

Problem 2 - You compared to the number of living relatives in your family.

Problem 3 - The moon compared to the 167 satellites orbiting all of the planets in our solar system (not including Pluto!).

Problem 4 - The figure above shows the amount of salt in ocean water determined by the NODC Ocean Climate Laboratory. The units are in parts per thousand where 1 gram of salt in 1 kilogram of water represents 1 ppt.

A) What is the difference in ppt between the saltiest and least-salty ocean water?

B) Suppose you have 10 kilograms of ocean water at 30 ppt of salt, if you removed all the water by evaporation, how many grams of salt would you collect?

C) A sea water distillation plant converts ocean water into freshwater. If it processes 14 million kilograms of ocean water, how much salt, in metric tons, has to be disposed of at a 34 ppt concentration if 1 metric ton equals 1,000 kilograms?

Problem 1 - One day compared to a year.

Answer; $1 / 365 = 0.0027$ or **2.7 ppt.**

Problem 2 - You compared to the number of living relatives in your family.

Answer: Students answers will vary, but for 37 living relatives, $1/37 = 0.027$ which is **27 ppt.**

Problem 3 - The moon compared to the 167 satellites orbiting all of the planets in our solar system (not including Pluto!).

Answer; Students answers may vary depending on the sources that they use, but current counts give a total of 167 planetary satellites so the moon represents $1/167 = 0.006$ or **6 ppt.**

Problem 4 - The figure above shows the amount of salt in ocean water determined by the NODC Ocean Climate Laboratory. The units are in parts per thousand where 1 gram of salt in 1 kilogram of water represents 1 ppt.

A) What is the difference in ppt between the saltiest and least-salty ocean water? Answer: The least-salty is 32.4 ppt in the water between Greenland and North America. The most salty is 37.0 in the mid-Atlantic. The difference is $37.0 - 32.4$ ppt = **4.6 ppt.**

B) If you have 10 kilograms of ocean water at 30 ppt of salt, how many grams of salt would you collect? Answer: $10 \text{ kilograms} \times 30/1000 =$ **0.3 kilograms or 300 grams of salt.**

C) A sea water distillation plant converts ocean water into freshwater. If it processes 14 million kilograms of ocean water, how much salt in metric tons has to be disposed of at a 34 ppt concentration? Answer: $14 \text{ million kg} \times 34/1000 = 0.47$ million kilograms or 470,000 kilograms. 1 metric ton = 1000 kg, so this equals **470 tons of salt.**

Kelvin Temperatures and Very Cold Things!

183 K	Vostok, Antarctica
160 K	Phobos
134 K	Superconductors
128 K	Europa summertime
120 K	Moon at night
95 K	Titan Noon
90 K	Liquid oxygen
88 K	Miranda Noon
81 K	Enceladus summertime
77 K	Liquid nitrogen
70 K	Mercury at night
63 K	Solid nitrogen
55 K	Pluto summertime
54 K	Solid oxygen
50 K	Quaoar summertime
45 K	Moon shadowed crater
40 K	Star-forming nebula
33 K	Pluto wintertime
20 K	Hydrogen liquifies
19 K	Bose-Einstein matter
4 K	Helium liquifies
3 K	Cosmic fireball light
1 K	Helium becomes solid
0 K	ABSOLUTE ZERO

To keep track of some of the coldest things in the universe, scientists use the Kelvin temperature scale which begins at 0 Kelvin, which is also called Absolute Zero. Nothing can ever be colder than Absolute Zero because at this temperature, all motion stops. The table to the left shows some typical temperatures of different systems in the universe.

You are probably already familiar with the Centigrade (C) and Fahrenheit (F) temperature scales. The two formulas below show how to switch from degrees-C to degrees-F.

$$C = \frac{5}{9} (F - 32) \quad F = \frac{9}{5} C + 32$$

Because the Kelvin scale is related to the Centigrade scale, we can also convert from Centigrade to Kelvin (K) using the equation:

$$K = 273 + C$$

Use these three equations to convert between the three temperature scales:

Problem 1: 212 F converted to K

Problem 2: 0 K converted to F

Problem 3: 100 C converted to K

Problem 4: -150 F converted to K

Problem 5: -150 C converted to K

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F. A) What are the equivalent temperatures on the Kelvin scale; B) What is the average daytime temperature on the Kelvin scale?

Answer Key

$$C = \frac{5}{9} (F - 32) \qquad F = \frac{9}{5} C + 32 \qquad K = 273 + C$$

Problem 1: 212 F converted to K:
First convert to C: $C = 5/9 (212 - 32) = +100$ C. Then convert from C to K:
 $K = 273 + 100 = 373$ Kelvin

Problem 2: 0 K converted to F: First convert to Centigrade:
 $C = K - 273$ so $C = -273$ degrees. Then convert from C to F:
 $F = 9/5 (-273) + 32 = -459$ Fahrenheit.

Problem 3: 100 C converted to K : $K = 273 + C = 373$ Kelvin.

Problem 4: -150 F converted to K : Convert to Centigrade
 $C = 5/9 (-150 - 32) = -101$ C. Then convert from Centigrade to Kelvin: $K = 273 - 101 = 172$ Kelvin.

Problem 5: -150 C converted to K : $K = 273 + (-150) = 123$ Kelvin

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F.

A) What are the equivalent temperatures on the Kelvin scale?;
107 C becomes $K = 273 + 107 = 380$ Kelvins.
221 F becomes $C = 5/9 (221 - 32) = 105$ C, and so $K = 273 + 105 = 378$ Kelvins.

B) What is the average daytime temperature on the Kelvin scale?
Answer: $(380 + 378)/2 = 379$ Kelvins.

Does Anybody Really Know What Time It Is?

Period	Age (years)	Days per year	Hours per day
Current	0	365	
Upper Cretaceous	70 million	370	
Upper Triassic	220 million	372	
Pennsylvanian	290 million	383	
Mississippian	340 million	398	
Upper Devonian	380 million	399	
Middle Devonian	395 million	405	21.6
Lower Devonian	410 million	410	
Upper Silurian	420 million	400	
Middle Silurian	430 million	413	
Lower Silurian	440 million	421	
Upper Ordovician	450 million	414	
Middle Cambrian	510 million	424	20.7
Ediacarin	600 million	417	
Cryogenian	900 million	486	

We learn that an 'Earth Day' is 24 hours long, and that more precisely it is 23 hours 56 minutes and 4 seconds long. But this hasn't always been the case. Detailed studies of fossil shells, and the banded deposits in certain sandstones, reveal a much different length of day in past eras! These bands in sedimentation and shell-growth follow the lunar month and have individual bands representing the number of days in a lunar month. By counting the number of bands, geologists can work out the number of days in a year, and from this the number of hours in a day when the shell was grown, or the deposits put down. The table above shows the results of one of these studies.

Problem 1 - Complete the table by calculating the number of hours in a day during the various geological eras. It is assumed that Earth orbits the sun at a fixed orbital period, based on astronomical models that support this assumption.

Problem 2 - Plot the number of hours lost compared to the modern '24 hours' value, versus the number of years before the current era.

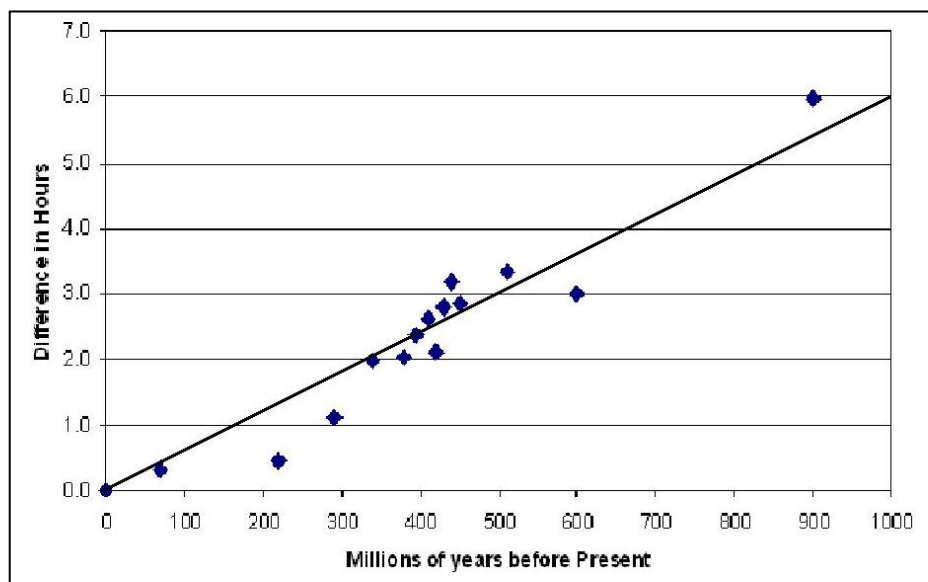
Problem 3 - By finding the slope of a straight line through the points can you estimate by how much the length of the day has increased in seconds per century?

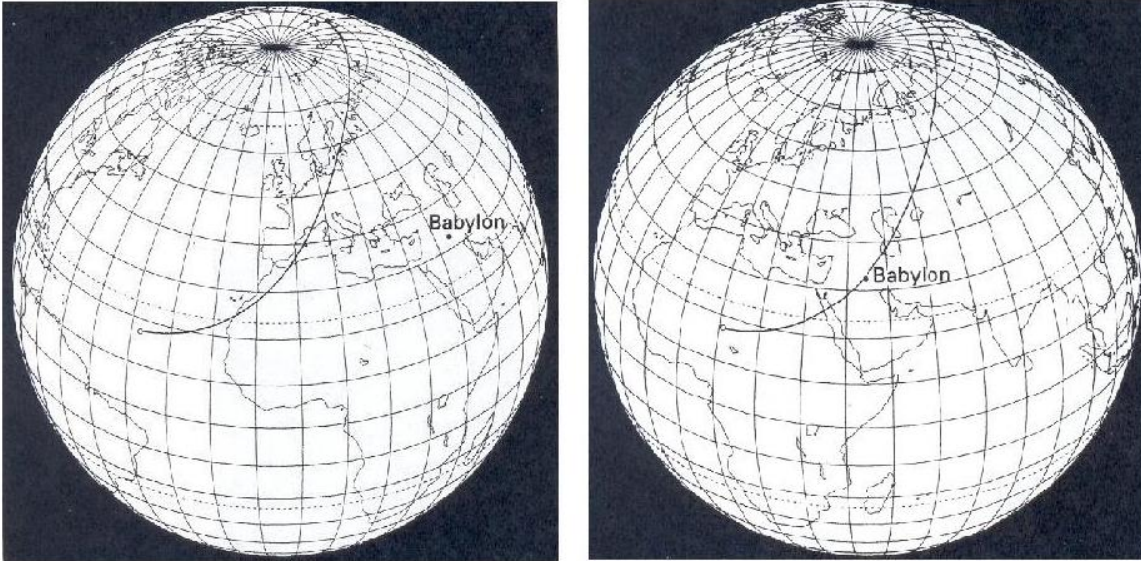
Period	Age (years)	Days per year	Hours per day
Current	0	365	24.0
Upper Cretaceous	70 million	370	23.7
Upper Triassic	220 million	372	23.5
Pennsylvanian	290 million	383	22.9
Mississippian	340 million	398	22.0
Upper Devonian	380 million	399	22.0
Middle Devonian	395 million	405	21.6
Lower Devonian	410 million	410	21.4
Upper Silurian	420 million	400	21.9
Middle Silurian	430 million	413	21.2
Lower Silurian	440 million	421	20.8
Upper Ordovician	450 million	414	21.2
Middle Cambrian	510 million	424	20.7
Ediacarin	600 million	417	21.0
Cryogenian	900 million	486	18.0

Problem 1 - Answer; See table above.

Problem 2 - Answer; See figure below

Problem 3 - Answer: From the line indicated in the figure below, the slope of this line is $m = (y_2 - y_1) / (x_2 - x_1) = 6 \text{ hours} / 900 \text{ million years}$ or $0.0067 \text{ hours/million years}$. Since there are 3,600 seconds/ hour and 10,000 centuries in 1 million years (Myr), this unit conversion yields $0.0067 \text{ hr/Myr} \times (3600 \text{ sec/hr}) \times (1 \text{ Myr} / 10,000 \text{ centuries}) = 0.0024 \text{ seconds/century}$. This is normally cited as 2.4 milliseconds per century.





Astronomers can use sophisticated mathematical models to accurately predict when eclipses occurred anywhere on Earth as long ago as 10,000 years. But to achieve this, the models must account for the length of the day being shorter than it is today! The above figure (right) shows the track of the total solar eclipse of April 15, 136 BC recorded on cuneiform tablets by Babylonian astronomers. The figure (left) also shows the predicted track without allowing for the different length of day back in 136 BC!

Problem 1 - At the latitude of Babylon, the computed track crosses at a longitude of +4.3 West. The actual longitude of Babylon is +44.5 East. By how many degrees in longitude does the computed track miss Babylon?

Problem 2 - How long does it take Earth, in seconds, to rotate through the longitude difference in Problem 1?

Problem 3 - To calculate the rate, R , at which the day is lengthening in milliseconds per century, geologists use the formula $T = 18 R t^2$. If T is your answer to Problem 2, and t is the number of centuries between 136 BC and today, what is R ?

Problem 4 - Based on your answer for R , what will be the length-of-day in 100 million years from now expressed in decimal hours?

Problem 1 - At the latitude of Babylon, the computed track crosses at a longitude of +4.3 West. The actual longitude of Babylon is +44.5 East. By how many degrees in longitude does the computed track miss Babylon? Answer; $44.3 + 4.3 = 48.8$ degrees.

Problem 2 - How long does it take Earth, in seconds, to rotate through the longitude difference in Problem 1? Answer: In 24 hours it rotates through 360 degrees in longitude, so to rotate 48.8 degrees it will take 24 hours $\times (48.8 / 360) = 3.25$ hours, and so $3.25 \text{ hrs} \times (3600 \text{ sec}/1 \text{ hour}) = 11,700$ seconds.

Problem 3 - To calculate the rate, R, at which the day is lengthening in milliseconds per century, geologists use the formula $T = 18 R t^2$. If T is your answer to Problem 2, and t is the number of centuries between 136 BC and today, what is R?

Answer: $T = 11,700$ seconds

$t = (2008 + 136)/100 = 21.4$ centuries (note there was no Year-Zero !)

solve the equation for R to get $R = T/18t^2$

and so $R = (11,700) / (18 \times 21.4^2)$

$R = 1.4$ milliseconds/century

Problem 4 - Based on your answer for R, what will be the length-of-day in 100 million years from now expressed in decimal hours?

Answer: In 100 million years there are 1 million centuries, so

$1 \text{ million centuries} \times 1.4 \text{ milliseconds/century} \times (0.001 \text{ seconds/millisecond}) = 1400$ seconds. Since there are 3600 seconds in 1 hour, the new 'day' will be 24 hours + $1400/3600$ hours = $24 + 0.39$ hours = 24.39 hours long.

Year	Month	X (meters)	Y (meters)
2008	1	-3	+8
2008	2	-4	+10
2008	3	-4	+13
2008	4	-2	+15
2008	5	+1	+16
2008	6	+4	+16
2008	7	+7	+15
2008	8	+9	+13
2008	9	+9	+10
2008	10	+8	+7
2008	11	+6	+5
2008	12	+3	+4
2009	1	0	+4
2009	2	-3	+6
2009	3	-4	+9
2009	4	-4	+12
2009	5	-2	+15
2009	6	+1	+16
2009	7	+4	+16
2009	8	+7	+15

The Earth rotates on its axis once every 24 hours (23 hours 56 minutes and 4 seconds more-accurately!), but like a bobbing, spinning top, the direction doesn't point exactly at an angle of $23 \frac{1}{5}$ degrees.

For over 200 years, careful measurements of the rotation axis have shown that it moves slightly. The data in the table to the left gives the axis location of the North Pole as it passes through Earth's surface. The X-axis runs East-West and the Y-axis runs North-South. The units are in meters measured on the ground.

Problem 1 - On a Cartesian graph, plot the location of the North Pole during the time span indicated by the table.

Problem 2 - About how long, in days, does it take the North Pole to return to its starting position based on this data?

Problem 3 - About what is the average speed of this Polar Wander in meters per day?

Problem 4 - Based on your plot, does it look like the motion will exactly repeat itself in space during the next cycle?

Problem 5 - About what is the X and Y location of the center of the movement pattern?

Problem 6 - What would you use as the best location of the North Pole?

Problem 1 - Answer; See below.

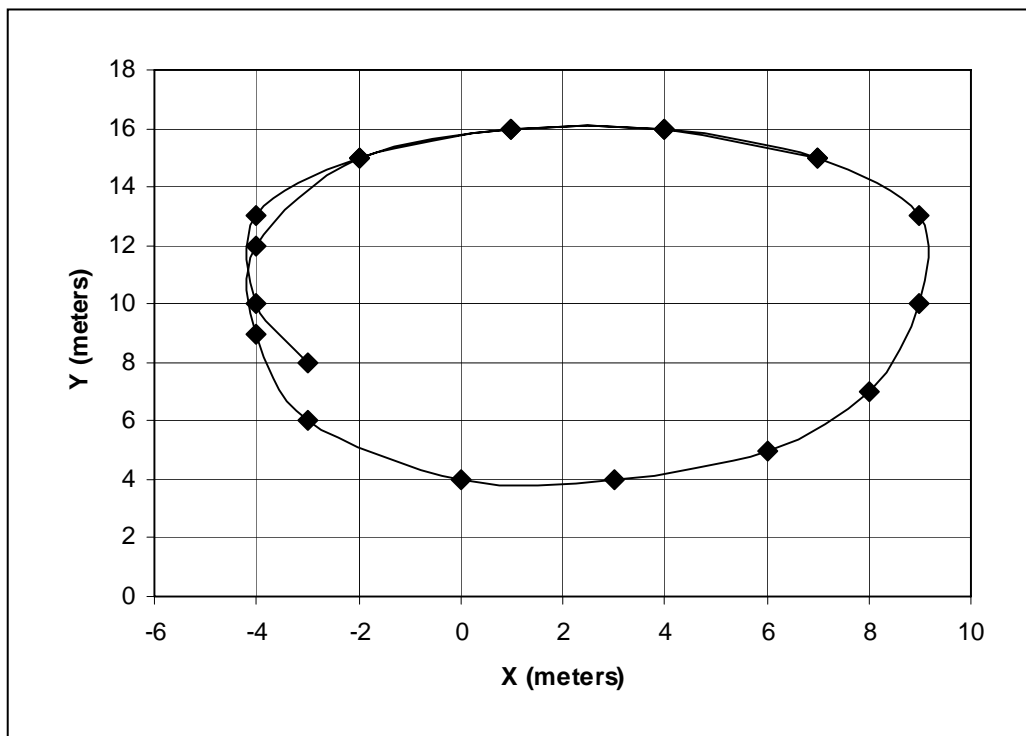
Problem 2 - Answer; The plotted track is nearly circular and one complete loop takes about 14 months or 420 days. This is called the Chandler Period.

Problem 3 - Answer: In one month the point moves on the circle about 3 meters, so the speed is about 3 meters/30 days or 0.1 meter/day.

Problem 4 - Answer; Not exactly. The new cycle points do not follow the circle of the older cycle. This motion is called the Chandler Wobble.

Problem 5 - Answer; $X = + 2$ meters, $Y = +10$ meters.

Problem 6 - Answer: Probably the point at the center of the circle, because it represents the average position of the North Pole over time.



50%	78%
3%	30%

Material	Reflectivity
Snow	80%
White Concrete	78%
Bare Aluminum	74%
Vegetation	50%
Bare Soil	30%
Wood Shingle	17%
Water	5%
Black Asphalt	3%

When light falls on a material, some of the light energy is absorbed while the rest is reflected. The absorbed energy usually contributes to heating the body. The reflected energy is what we use to actually see the material! Scientists measure reflectivity and absorption in terms of the percentage of energy that falls on the body. The combination must add up to 100%.

The table above shows the reflectivity of various common materials. For example, snow reflects 80% of the light that falls on it, which means that it absorbs 20% and so $80\% + 20\% = 100\%$. This also means that if there are 100 watts of light energy falling on the snow, 80 watts will be reflected and 20 watts will be absorbed.

Problem 1 - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

Problem 2 - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

Problem 3 - What is the average reflectivity of these four equal-area regions combined?

Problem 4 - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?

Problem 1 - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

Answer: The reflectivity is $100\% \times (300 \text{ watts}/1000 \text{ watts}) = 30\%$. From the table, Bare Soil has this same reflectivity and so is a likely composition.

Problem 2 - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

Answer: 50% = Vegetation
 78% = White Concrete
 30% = Bare Soil
 3% = Black Asphalt

Problem 3 - What is the average reflectivity of these four equal-area regions combined? Answer: Because each of the four materials cover the same area, we just add up their reflectivities and divide by 4 to get $(50\% + 78\% + 30\% + 3\%)/4 = 40\%$.

Problem 4 - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?

Answer: Each material covers 10 meters x 10 meters = 100 square meters:

A) Vegetation: $0.50 \times 1300 \text{ watts/m}^2 \times 100 \text{ m}^2 = 65,000 \text{ watts}$.
 Concrete: $0.78 \times 1300 \text{ watts/m}^2 \times 100 \text{ m}^2 = 101,400 \text{ watts}$.
 Bare Soil: $0.30 \times 1300 \text{ watts/m}^2 \times 100 \text{ m}^2 = 39,000 \text{ watts}$.
 Black Asphalt: $0.03 \times 1300 \text{ watts/m}^2 \times 100 \text{ m}^2 = 3,900 \text{ watts}$.

B) $65,000 + 101,400 + 39,000 + 3,900 = 209,300 \text{ watts}$.

C) The total wattage entering this area is $1,300 \text{ watts/m}^2 \times 100 \text{ m}^2 \times 4 = 520,000 \text{ watts}$. Since 209,300 watts are reflected, that means that $520,000 \text{ watts} - 209,300 \text{ watts} = 310,700 \text{ watts are being absorbed}$.

Note: If you work with significant figures the answers are to 2 SF: A) 65,000 ; 100,000; 39,000 and 3,900 B) = 210,000 C) $520,000 - 210,000 = 310,000 \text{ watts}$.

Material	R(UV)	R(Vis)	R(NIR)
Snow	90%	80%	70%
White Concrete	22%	80%	73%
Aluminum Roof	75%	74%	68%
Vegetation	15%	50%	40%
Bare Soil	15%	30%	50%
Wood Shingle	7%	17%	28%
Water	2%	5%	1%
Black Asphalt	4%	3%	3%

The amount of light a body reflects isn't the same for all of the different light wavelengths that fall on its surface. Because of this, each substance can have a unique fingerprint of reflectivity at different wavelengths that lets you identify it. The table above shows the reflectivity of various common materials. For example, snow reflects 80% of the light that falls on it at visible light wavelengths (Vis= 400 to 600 nm), but reflects quite a bit more at ultraviolet wavelengths (UV= 200 to 300 nm), and quite a bit less at near-infrared wavelengths (NIR= 700 to 1500 nm).

Problem 1 - If 1000 watts falls on a body in the ultraviolet band, and you measure 150 watts reflected, what is the reflectivity of the body, and from the Table, A) what might be its composition? B) What other reflectivity measurements can you make to tell the difference between your choices?

Problem 2 - You are given the reflectivity maps in each of the three wavelength bands, UV, VIS and NIR at the bottom of this page. What are the likely compositions of the areas in the map?

UV			VIS			NIR		
15	15	15	50	50	30	40	40	50
15	15	22	50	30	80	40	50	73
22	90	75	80	80	74	73	70	68

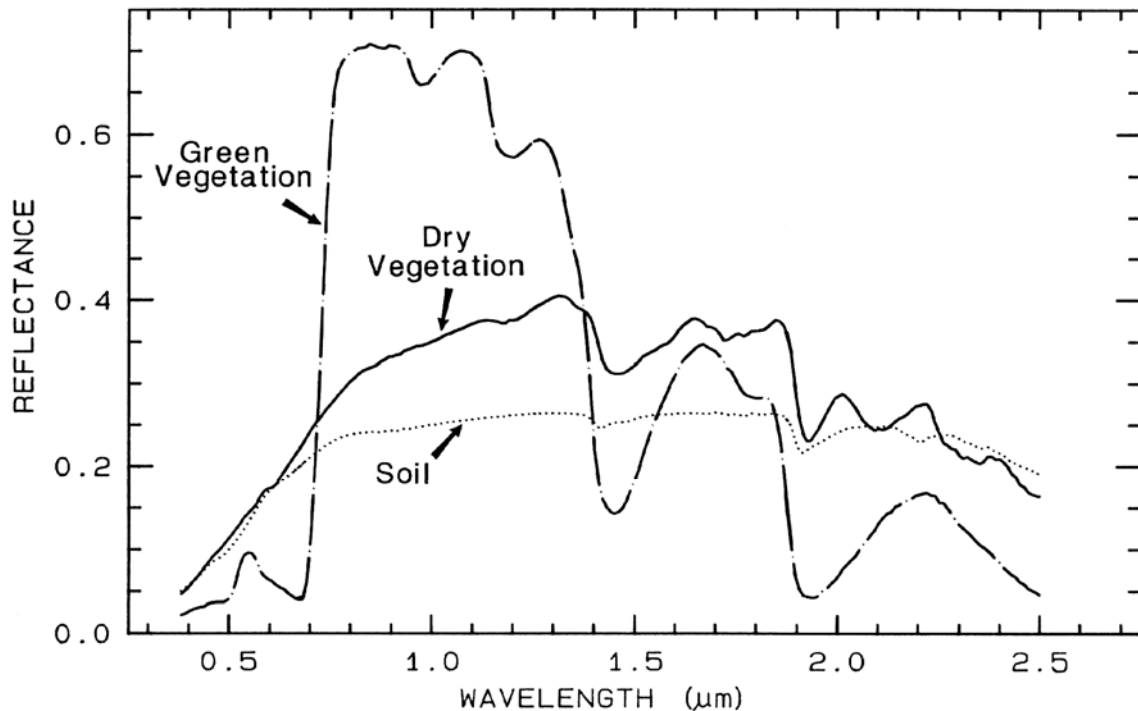
Problem 1 - If 1000 watts falls on a body in the ultraviolet band, and you measure 150 watts reflected, what is the reflectivity of the body, and from the Table, A) what might be its composition? B) What other reflectivity measurements can you make to tell the difference between your choices?

Answer; A) The reflectivity is $100\% \times (150 \text{ watts}/1000 \text{ watts}) = 15\%$ in the ultraviolet band. There are two candidates from the table: a surface covered with vegetation, and a surface covered with bare soil. B) By measuring the reflected power in the visible band (Vis) the difference in reflectivity is 50% for the vegetation and 30% for the bare soil, which is enough for you to be able to tell the difference.

Problem 2 - What are the likely compositions of the areas in the map? Answer: See map below. V = Vegetation; bS = Bare Soil; C = Concrete; S = Snow and A = Aluminum Roof.

Note: The problem can be made more challenging by only giving students two out of the three band measurements for a given map 'pixel', and have the student fill -in the missing reflectivity percent and then identify the material.

UV			VIS			NIR		
15 V	15 V	15 bS	50	50	30	40	40	50
15 V	15 bS	22 C	50	30	80	40	50	73
22 C	90 S	75 A	80	80	74	73	70	68



Very precise measurements can be made of the reflectivity of materials that more easily reveal their subtle differences. Above is a plot of the reflectivities of green vegetation, dry vegetation and soil between wavelengths of 0.4 and 2.5 micrometers (400 to 2500 nm). Scientists use graphs such as these to design instruments that help them discriminate between a variety of interesting materials and mineral deposits.

Problem 1 - An astronomer wants to map the surface of Mars with telescopes on Earth to search for plant life. What wavelength range would help her more easily discriminate between the martian soil and living vegetation?

Problem 2 - An earth scientist measures the intensity of light between two neighboring land areas at a wavelength of 2.0 microns and 0.7 microns. Spot A appears to be 5 times brighter than Spot B in the longer wavelength band, but nearly equal in brightness in the shorter-wavelength band. What may be the difference in substances between the two spots?

Problem 3 - The difference in the vegetation reflectivity between green vegetation and dry vegetation is that green vegetation still contains the molecule chlorophyll. What is the difference in absorption by chlorophyll molecules at a wavelength of 0.6 microns?

Problem 1 - An astronomer wants to map the surface of Mars with telescopes on Earth to search for plant life. What wavelength range would help her more easily discriminate between the martian soil and living vegetation?

Answer: The graph shows that the reflectivity of green vegetation is substantially brighter than soil between wavelengths of 0.7 to 1.4 microns. At 0.9 microns, the reflectivity of green vegetation is about 0.7 or 70%, while soil is only about 0.25 (25%) at the same wavelength, **so green vegetation is nearly three times as bright as bare soil at 0.7 microns.**

Problem 2 - An earth scientist measures the intensity of light between two neighboring land areas at a wavelength of 2.0 microns and 0.7 microns. Spot A appears to be 5 times brighter than Spot B in the longer wavelength band, but nearly equal in brightness in the shorter-wavelength band. What may be the difference in substances between the two spots?

Answer: The graph shows that at 0.7 microns, the reflectance curves for green vegetation and dry vegetation cross, which means they are of equal reflectivity at this wavelength. At 2.0 microns, the reflectivity of dry vegetation is about 0.25 or 25%, while green vegetation is much darker and only 0.05 or 5% reflective. **Spot B appears to be 5 times brighter than Spot A so that suggests that Spot A consists of green vegetation and Spot B consists of dry vegetation.**

Problem 3 - The difference in the vegetation reflectivity between green vegetation and dry vegetation is that green vegetation still contains the molecule chlorophyll. What is the difference in absorption by chlorophyll molecules at a wavelength of 0.6 microns?

Answer: From the graph, green vegetation has a reflectivity of about 5% while dry vegetation has a reflectivity of about 18%. Because %emission + %absorption = 100%, the absorption of green vegetation is 95% while dry vegetation absorbs only 82% of the light at this wavelength. **The difference in absorption is 13%.**

Note: Student's answers may vary in percentage depending on the graph estimation method used.

Electricity - Watts and kilowatt-Hours



Electricity is a complicated thing to keep track of and measure. You can't see it, but you sure can't live without it in the 21st Century!

One way is to measure the flow of electrical energy by using a unit called the Watt. A 100-watt bulb allows electrical energy to flow 10 times faster than a 10-watt bulb. This also means that a 100-watt bulb will be much brighter than a 10-watt bulb.

A second way to measure electrical energy is by the total energy that is used over a period of time. The unit we use is the kilowatt-Hour (kWh). If a 100-watt bulb is left on for 10 hours, it uses $100 \times 10 = 1000$ watt-hours or 1 kWh.

Problem 1 - A 350-watt computer is left on for 5 hours each day. How many kilowatt-hours of electrical energy does it consume each day?

Problem 2 - Six, 75-watt flood lights in a living room ceiling are left on for 5 hours each night for 1 month (30 days). How many kilowatt-hours are consumed?

Problem 3 - Your refrigerator uses 200-watts and runs continuously for 1 year. How many kilowatt-hours of electricity does it consume in a year if there are 8,760 hours in one year?

Problem 4 - To prepare your dinner, your 1000-watt stove is used for 1/2 hour. You bake a cake in your 500-watt oven for 30 minutes, and your 150-watt dishwasher runs for 45 minutes to clean the dishes. How many kWh of electricity are used?

Problem 5 - Your home has 15 'instant on' clocks, radios, computers, TVs and kitchen appliances. If each of them uses 5 watt in this mode how many kWh of electricity will be used each year?

Problem 6 - If electricity costs \$0.11 for each kWh consumed, how much will each of your answers to the previous 5 problems cost?

Problem 7 - Why is a kilowatt-hour a better electrical unit to use than a watt when you are comparing how energy is used in a home? Can you think of an analogy that describes this difference?

Problem 1 - A 350-watt computer is left on for 5 hours each day. How many kilowatt-hours of electrical energy does it consume each day? Answer: $350 \times 5 = 1,750$ watt-hours = **1.75 kWh**

Problem 2 - Six, 75-watt flood lights in a living room ceiling are left on for 5 hours each night for 1 month (30 days). How many kilowatt-hours are consumed? Answer: $6 \times 75 \times 5 \times 30 = 67,500$ watt-hours = **67.5 kWh**.

Problem 3 - Your refrigerator uses 200-watts and runs continuously for 1 year. How many kilowatt-hours of electricity does it consume in a year if there are 8,760 hours in one year? Answer: $200 \times 8,760 = 1,752,000$ watt-hours = **1,752 kWh**.

Problem 4 - To prepare your dinner, your 1000-watt stove is used for 1/2 hour. You bake a cake in your 500-watt oven for 30 minutes, and your 150-watt dishwasher runs for 45 minutes to clean the dishes. How many kWh of electricity are used? Answer: $1000 \times 0.5 + 500 \times 0.5 + 150 \times 0.75 = 862$ watt-hours = **0.862 kWh**.

Problem 5 - Your home has 15 'instant on' clocks, radios, computers, TVs and kitchen appliances. If each of them uses 5 watt in this mode how many kWh of electricity will be used each? Answer: $15 \times 5 \times 8760 = 657,000$ watt-hours = **657 kWh**.

Problem 6 - If electricity costs \$0.11 for each kWh consumed, how much will each of your answers to the previous 5 problems cost? Answer:

Problem 1 = $1.75 \text{ kWh} \times 0.11 = \text{\$0.19}$; Problem 2 = $67.5 \times 0.11 = \text{\$7.43}$; Problem 3 = $1,752 \times 0.11 = \text{\$192.72}$; Problem 4 = $0.862 \times 0.11 = \text{\$0.09}$; Problem 5 = $657 \times 0.11 = \text{\$72.27}$.

Problem 7 - Why is a kilowatt-hour a better electrical unit to use than a watt when you are comparing how energy is used in a home? Can you think of an analogy that describes this difference? Answer: Because the wattage only tells you how fast energy is being consumed, not the total amount being consumed. One student analogy can involve a car: A car travels a total of 250 miles on one tank of gasoline. At its fastest road speed, it consumes 0.1 gallons of gasoline each mile traveled. The wattage of the bulb is like the usage of gasoline (gallons per mile) and the kiloWatt-hour is like the total gasoline used to travel 250 miles. Alternatively, the total number of gallons that go over a waterfall in one hour is like the kilowatt-hour, while the rate of the water going over the fall, in gallons per minute, is like the wattage.

Energy in the Home



Every month, we get the Bad News from our local electrical company. A bill comes in the mail saying that you used 900 Kilowatt Hours (kWh) of electricity last month, and that will cost you \$100.00! *What is this all about?*

Definition: 1 kiloWatt hour is a unit of energy determined by multiplying the electrical power, in kilowatts, by the number of hours of use.

Example: A 100-watt lamp is left on all day. $E = 0.1 \text{ kilowatts} \times 24\text{-hours} = 2.4 \text{ kWh}$. Note: At 11-cents per kWh, this costs you $2.4 \times 11 = 26$ cents!

Problem 1 – You turned on your computer at 3:00 PM, and finished your homework at 9:00 PM, but you forgot to turn it off before going to bed. At 7:00 AM, it was finally shut off by an angry parent after being on all night. If this happened each school day in the month (25 days):

- How many hours was the computer operated in this way during a 25-day period?
- How many kilowatt hours were wasted by leaving the computers on but not used?
- If the computer runs at 350 watts, and if electricity costs 11-cents per kilowatt hour, how much did this waste cost each month?
- How many additional songs could you buy with iTunes for the wasted money each month if the cost of a song is \$0.99?

Problem 2 – The Tevatron ‘atom smasher’ at Fermilab in Batavia, Illinois collides particles together at nearly the speed of light to explore the innermost structure of matter. When operating, the accelerator requires 70 megaWatts of electricity – about the same as the power consumption of the entire town of Batavia (population: 27,000). If an experiment, from start to finish, lasts 24 hours:

- What is the Tevatron’s electricity consumption in kilowatt hours?
- At \$0.11 per kilowatt-hour, how much does one experiment cost to run?

Problem 1 –A) How many hours was the computer operated in this way during a 25-day period? Answer: The computer was turned off between 7:00 AM and 3:00 PM which is 8-hours, so the total time it stayed on each day was $24 - 8 = 16$ hours. Over 25 days, this comes to $25 \times 16 = 400$ hours.

B) How many kilowatt hours were wasted? Answer: The computer was used between 3:00 PM and 9:00 PM, which is 6-hours out of the 16, so the wasted time was 10 hours each day, or $10 \times 25 = 250$ -hours of wasted 'ON' time each month.

C) If the computer runs at 350 watts, and if electricity costs 11-cents per kilowatt hour, how much did this waste cost each month? Answer: The wasted time = 250 hours, so the number of kilowatt hours is $0.35 \text{ kilowatts} \times 250 \text{ hours} = 87.5$ kilowatt hours. At \$0.11 per kilowatt hours, this becomes $87.5 \times 0.11 = \$9.63$ wasted each month.

D) How many additional songs can you buy with iTunes for the wasted money each month? Answer: At \$0.99 per song, you can buy $\$9.63 / .99 = 9$ songs.

Problem 2 – The Tevatron 'Atom Smasher' at Fermilab in Batavia, Illinois collides particles together at nearly the speed of light to explore the innermost structure of matter. When operating, the accelerator requires 70 megaWatts of electricity – about the same as the power consumption of the entire town of Batavia (population: 27,000). If an experiment, from start-up to finish, lasts 24 hours,

A) What is the Tevatron's electricity consumption in kilowatt hours?

Answer: $70 \text{ megaWatts} \times (1,000 \text{ kiloWatts} / 1 \text{ megaWatt}) = 70,000 \text{ kiloWatts}$.

Then $70,000 \text{ kiloWatts} \times 24 \text{ hours} = 1,680,000 \text{ kiloWatt hours}$.

B) At \$0.11 per kilowatt-hour, how much does one experiment cost to run? Answer: $1,680,000 \text{ kiloWatt-hrs} \times \$0.11 / \text{kilowatt-hr} = \$184,800.00$

Energy Consumption in an Empty House!

Device	Number	Total Watts
Aquarium	1	5
Telephone	4	12
iPod Home	2	6
Radio	2	6
Night Light	1	5
Oven clock	1	5
Microwave clock	1	5
Kitchen lights	2	80
TV	3	12
Stereo	1	4
Computer	3	40

Total Watts = Number x watts/device

It makes sense that, when no one is at home, there should be no electricity consumption, but that isn't usually the case. Many appliances have 'instant on' or 'standby' features, and an energy audit shows that this can add up over time.

The table to the left shows an audit of a typical suburban home in Maryland using a power measuring device called a 'kill-a-watt' meter.

Electrical energy consumption is measured in units called kilowatt-Hours. For example, a 100-watt bulb that is left on for 10 hours will consume 100 watts x 10 hours = 1000 watt-hours. Since 'kilo' means 1000, this energy is equal to 1 kiloWatt-hour, which we abbreviate as 1 kWh. When you are charged each month for electrical energy use, you are charged, not by how fast you use the energy (called a watt) but how much electrical energy you use (the kWh). This is like charging for the total number of gallons of water that you used to fill your swimming pool, but not how many gallons per hour you are using in order to fill it. Let's apply the idea of the kiloWatt-hour to a very practical problem!

Problem 1 - What is the total wattage of this house for 'instant-on' appliances when no one is at home?

Problem 2 - What is the total number of kilowatt-hours of electricity consumed by this home during one day (24 hours)?

Problem 3 - What is the total number of kilowatt-hours of electricity consumed by this home during one year?

Problem 4 - If electricity costs \$0.11 per kilowatt-hour (kWh), how much does it cost to keep this house running at this minimum 'instant-on' level each year?

Problem 5 - How could this house substantially reduce its electricity consumption cost below \$80.00 each year?

Problem 6 - Using the data in the table, conduct your own daytime electricity audit and estimate what your own dwelling costs to run in a 'stand by' mode!

Problem 1 - What is the total wattage of this house for 'instant-on' appliances when no one is at home?

Answer: Add up the number in the right-hand column of the table to get **180 watts**.

Problem 2 - What is the total number of kilowatt-hours of electricity consumed by this home during one day (24 hours)? Answer: In 24 hours, you will use 180 watts x 24 hours = 4,320 watt-hours or **4.32 kWh**.

Problem 3 - What is the total number of kilowatt-hours of electricity consumed by this home during one year? Answer: In 365 days you will use 180 x 24 x 365 = **1,577 kWh**.

Problem 4 - If electricity costs \$0.11 per kilowatt-hour (kWh), how much does it cost to keep this house running at this minimum 'instant-on' level each year? Answer: 1,577 kWh x \$0.11 = **\$173.47 per year**.

Problem 5 - How could this house substantially reduce its electricity consumption cost below \$80.00 each year? Answer: **If you shut off the kitchen lights and unplug the computers**, you will save 120 watts each hour or 120 watts x 24 hrs = 2.88 kWh each day, or 2.88 kWh x 365 days/year = 1,051 kWh each year. This is a savings of 1,051 kWh x \$0.11 = \$115.61, which will bring your annual electricity consumption down to 1,577 - 1,051 = 526 kWh or just \$57.86.

Problem 6 - Using the data in the table, conduct your own daytime electricity audit and estimate what your own dwelling costs to run in a 'stand by' mode!

Answer: Students may include hot water heaters, washer and drier, air conditioning, TV, attic fans, and other electricity-consuming devices. They may also create a timeline that shows when devices are on or idle, and convert this into a daily, monthly, annual electricity profile for their family. This can also be compared to electricity bills so students can track down hidden waste.

Annual Electricity Consumption in a Home



A typical single-family home in suburban Maryland has the monthly energy consumption shown in the table below. The table shows the number of kilowatt-hours (kWh) of electricity used each month for the years 2006, 2007 and 2008. For example, in 2007 during the month of August the house used 1,270 kWh of electricity.

	2008	2007	2006		2008	2007	2006
January	830	930	1260	July	1330	1620	1440
February	820	1090	1040	August	1110	1270	1630
March	840	950	830	September	1040	1310	1940
April	770	770	790	October	740	920	1520
May	750	810	640	November	650	830	1150
June	1160	1360	680	December	1100	1200	950

Problem 1 - What was the total amount of electrical energy used each year in kilowatt-hours (kWh)?

Problem 2 - What was the average monthly kWh used each year?

Problem 3 - Did the annual electricity use increase, decrease or stay about the same between 2006 - 2008?

Problem 4 - What was the rate of change in electricity consumption between 2006 and 2008?

Problem 5 - In most locations, electricity is generated from the burning of fossil fuels such as oil, coal or natural gas. The burning of fossil fuels produces water and carbon dioxide, which can both enter the atmosphere and contribute to global warming. For every kWh of electrical energy that is consumed, 700 grams of carbon dioxide are also produced. For 2006, 2007 and 2008, what was the total amount of carbon dioxide, in tons, that was produced in order to supply the electrical energy for this house? (Note: 1 metric ton equals 1 million grams)

Problem 6: For the same home, the electrical energy use during one year is proportioned as follows: Air Conditioning = 10%, Refrigerator = 20%, Lights = 40%, and Miscellaneous Electronics (computers, radios etc) = 30%. From your answer to Problem 5 for 2008, how many tons of carbon dioxide were produced by each of these electrical systems?

Problem 1 - What was the total amount of electrical energy used each year in kilowatt-hours (kWh)? Answer: 2006 = **13,870 kWh**; 2007 = **13,060 kWh** and 2008 = **11,140 kWh**.

Problem 2 - What was the average monthly kWh used each year? Answer: Divide your answers to Problem 1 by 12 months to get 2006 = **1,156 kWh/month**; 2007 = **1,088 kWh/month** and 2008 = **928 kWh/month**.

Problem 3 - Did the annual electricity use increase, decrease or stay about the same between 2006 - 2008? Answer: It **decreased** from 13,870 kWh in 2006 to 11,140 kWh in 2008.

Problem 4 - The rate of change is the slope defined by $(y_2 - y_1)/(x_2 - x_1) = (11140 - 13870)/(2008 - 2006) = -1.37$ kWh/year. A negative slope means that the consumption has decreased between 2006 and 2008. For a comparison, if you ran a 100-watt bulb for 13 hours each year, it would equal 1.3 kWh/year, so the decrease in electrical energy usage for this house is about equal to NOT using a 100-watt bulb for 10 hours each year starting in 2006!

Problem 5 - In most locations, electricity is generated from the burning of fossil fuels such as oil, coal or natural gas. The burning of fossil fuels produces water and carbon dioxide, which can both enter the atmosphere and contribute to global warming. For every kWh of electrical energy that is consumed, 700 grams of carbon dioxide are also produced. For 2006, 2007 and 2008, what was the total amount of carbon dioxide, in tons, that was produced in order to supply the electrical energy for this house? (Note: 1 metric ton equals 1 million grams)

Answer: For 2006: $13,870 \text{ kWh} \times 700 \text{ grams/kWh} = \mathbf{9.7 \text{ metric tons}}$.

For 2007: $13,060 \text{ kWh} \times 700 \text{ grams/kWh} = \mathbf{9.1 \text{ metric tons}}$.

For 2008: $11,140 \text{ kWh} \times 700 \text{ grams/kWh} = \mathbf{7.8 \text{ metric tons}}$

Problem 6: For the same home, the electrical energy use during one year is proportioned as follows: Air Conditioning = 10%, Refrigerator = 20%, Lights = 40%, and Miscellaneous Electronics (computers, radios etc) = 30% . From your answer to Problem 5 for 2008, how many tons of carbon dioxide were produced by each of these electrical systems?

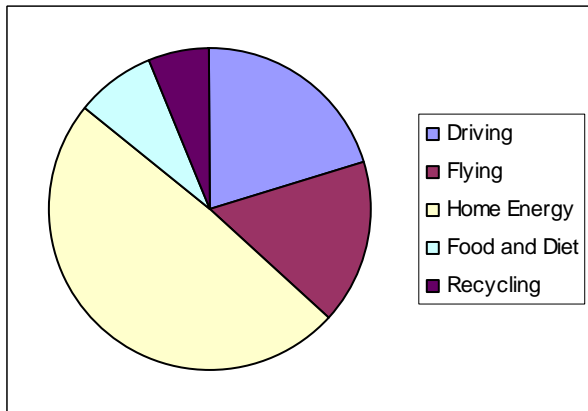
Answer:

Air Conditioning = $0.10 \times 7.8 \text{ tons} = \mathbf{0.8 \text{ tons of carbon dioxide}}$.

Refrigerator = $0.20 \times 7.8 \text{ tons} = \mathbf{1.6 \text{ tons of carbon dioxide}}$

Lights = $0.4 \times 7.8 \text{ tons} = \mathbf{3.1 \text{ tons of carbon dioxide}}$

Electronics = $0.3 \times 7.8 \text{ tons} = \mathbf{2.3 \text{ tons of carbon dioxide}}$.



A 'Carbon Footprint' calculator found on the Internet calculates the number of tons of carbon dioxide generated by a family of 4 living in suburban Maryland, driving one car (26 mpg) for 14,000 miles each year, and taking 3 long flights and 4 short flights each year. The family recycles, uses energy-efficient appliances and lights, and organically grown food. The results are shown in the figure to the left.

The household produced 50 tons of carbon dioxide each year. The US average for such a household is 110 tons, and the world average is 22 tons/year.

In the figure, Home Energy = 24 tons; Food and Diet = 4 tons; Flying = 8 tons, Driving = 10 tons, and Recycling = 3 tons. The per-capita (per person) carbon dioxide production for this household is $50 \text{ tons} / 4 \text{ people} = 12 \text{ tons/year}$. We can compare this with the national average of 1,600 gigatons of carbon dioxide in a population of about 300 million people and get 53 tons/person for the average American.

Problem 1 - Find a Carbon Footprint calculator on the Internet, and fill out the required information for your own home. How does your per-capita carbon production compare with the 12 tons per person each year produced by the Maryland family in the example?

Problem 2 - A human exhales about 0.7 kilogram of carbon dioxide each day. How much, in percentage, does the respiration from the family of four in Maryland change the total household carbon dioxide production?

Problem 3 – What would be the easiest life-style change that the Maryland family could make to reduce their carbon dioxide production?

Problem 4 - What is the largest contribution to your home carbon dioxide production, and how might you consider reducing it?

Problem 5 - What would you have to do to reduce your carbon production to the world average of 4 tons per person per year?

Problem 1 - Find a Carbon Footprint calculator on the Internet, and fill out the required information for your own home. How does your per-capita carbon production compare with the 12 tons/year/person produced by the Maryland family in the example?

Answer: Students answers will vary depending on the calculator they used and the detailed information they provide. Some calculators ask for much more detailed information than others. Students might compare the answers they get from several different calculators, and discuss which answers they had the greatest confidence in. They might also experiment by filling in only partial information to see just how different lifestyle issues change the results. Students may be asked to specify whether they want the answer in terms of the tons per individual, or the tons per household. The 'per-capita' answer would be for the individual.

Problem 2 - A human exhales about 0.7 kilogram of carbon dioxide each day. How much, in percentage, does the respiration from the family of four in Maryland change the total household carbon dioxide production? Answer: $0.7 \text{ kg} \times 365 = 0.25 \text{ tons/person/year} \times 4 \text{ people} = 1.0 \text{ tons/year for the house}$. This only changes our estimate of 50 tons for the entire household by about 2 %.

Problem 3 - What would be the easiest life-style change that the Maryland family could make to reduce their carbon dioxide production? Answer: **By not taking any jet flights, or doing so rarely, they could reduce their production by 8 tons/year or 16%.**

Problem 4 - What is the largest contribution to your home carbon dioxide production, and how might you consider reducing it? Answer: **Home energy consumption, especially if you get your electricity and heating from the combustion of fossil fuels at a distant power plant. You can reduce it by 1) not leaving computers, TVs or lights on when not in use, 2) By using solar panels to generate electricity; 3) By switching to an energy supplier that uses nuclear power, wind, hydroelectric or solar generation.**

Problem 5 - What would you have to do to reduce your carbon production to the world average of 4 tons per person per year? Answer: The world average for a family of four is $4 \times 4 \text{ tons/year/person} = 16 \text{ tons/year per family}$. For example, the Maryland household would have to reduce its 50 tons/year to 16 tons/year by reducing their output by 34 tons/year. **They would have to stop flying and driving to save 18 tons, and would have to cut their home energy consumption from 24 tons to only 8 tons/year!**

State	gWh	State	gWh
Alabama	143,826	Montana	28,931
Alaska	6,821	Nebraska	32,442
Arizona	113,340	Nevada	32,669
Arkansas	54,596	New Hamps.	23,277
California	210,847	New Jersey	62,671
Colorado	53,907	New Mexico	35,985
Connecticut	33,171	New York	145,878
Delaware	8,534	North Carolina	130,115
Florida	225,416	North Dakota	31,224
Georgia	145,155	Ohio	155,155
Hawaii	11,533	Oklahoma	72,819
Idaho	11,484	Oregon	55,077
Illinois	200,260	Pennsylvania	226,088
Indiana	130,637	Rhode Island	7,049
Iowa	49,789	South Carolina	103,402
Kansas	50,122	South Dakota	5,991
Kentucky	97,225	Tennessee	95,113
Louisiana	92,578	Texas	405,492
Maine	16,128	Utah	45,372
Maryland	50,197	Vermont	5,823
Massachusetts	47,075	Virginia	78,360
Michigan	119,309	Washington	106,990
Minnesota	54,477	West Virginia	93,933
Mississippi	50,043	Wisconsin	63,390
Missouri	91,153	Wyoming	43,144

Source:
http://www.eia.doe.gov/cneaf/electricity/epa/generation_state.xls

Although a typical single-family home may use about 13,000 kiloWatt-hours (kWh) in one year, when you multiply this by the number of homes in the US, and all of the industries that use electricity, the electrical energy consumption for a country like the United States is simply staggering!

According to the Department of Energy, the total United States consumption of electrical energy during 2007 was 4,154,013 gigaWatt-hours. The state-by-state breakdown is shown in the table to the left.

Because these energy units are so enormous, we need to re-write them into a simpler form by using the prefix multipliers 'mega;' and 'giga'. There are 1,000 kWh in one megaWatt-hr (mWh), and 1,000 mWh in one gigaWatt-hour (gWh).

The example of the average, single-family home consumes 13,000 kWh which equals 13 mWh, or 0.013 gWh.

Problem 1 - Which state consumes the least amount of electrical energy? Why do you think this is so?

Problem 2 - Which state consumes the most electrical energy? Why do you think this is so?

Problem 3 - Approximately 71% of the total US electrical energy comes from burning fossil fuels, which produces the greenhouse gas carbon dioxide. If 700 grams of carbon dioxide are produced for each kilowatt-hour of electricity production from fossil fuels, how much carbon dioxide is produced in units of A) kilograms, B) tons, C) megatons, D) gigatons?

Problem 4 - For your state, and from the table above, how many megatons of carbon dioxide are produced each year to generate your electricity?

Problem 1 - Which state consumes the least amount of electrical energy? Why do you think this is so?

Answer: **Vermont** consumes only 5,823 gigaWatt-hours of electricity. It is a sparsely populated state with no heavy industry to require large amounts of electricity.

Problem 2 - Which state consumes the most electrical energy? Why do you think this is so?

Answer: **Texas** consumes 405,498 gigaWatt-hours of electricity each year. It is a heavily populated state with many large industries.

Problem 3 - Approximately 71 % of the total US electrical energy comes from burning fossil fuels, which produces the greenhouse gas carbon dioxide. If 700 grams of carbon dioxide are produced for each kilowatt-hour of electricity production from fossil fuels, how much carbon dioxide is produced in units of A) kilograms, B) tons, C) megatons, D) gigatons?

A) The US gets $0.71 \times 4,154,013$ gigaWatt-hours = 2,900,000 gigaWatt-hours from fossil fuels. Note that since the percentage is only known to 2 significant figures, our answer can be rounded to this accuracy as well.

$2,900,000 \text{ gWh} \times (1,000,000 \text{ kWh}/1 \text{ gWh}) = 2.9 \text{ trillion kWh.}$

$2.9 \text{ trillion kWh} \times (0.7 \text{ kg} / \text{kWh}) = 2.0 \text{ trillion kg}$

B) $2.0 \text{ trillion kg} \times (1 \text{ ton} / 1,000 \text{ kg}) = 2.0 \text{ billion tons.}$

C) $2.0 \text{ billion tons} \times (1 \text{ megaton}/1,000,000 \text{ tons}) = 2,000 \text{ megatons.}$

D) $2.0 \text{ billion tons} \times (1 \text{ gigaton}/1,000,000,000 \text{ tons}) = 2.0 \text{ gigatons.}$

Problem 4 - For your state, and from the table above, how many megatons of carbon dioxide are produced each year to generate your electricity?

Example, for Massachusetts, $47,075 \text{ gWh} = 47,075,000,000 \text{ kWh.}$ Then $47,075,000,000 \text{ kWh} \times (0.7 \text{ kg}/\text{kWh}) \times (1 \text{ ton}/1,000 \text{ kg}) = 33,000,000 \text{ tons}$ or **33 megatons.**



A single-family home consumes about 13,000 kWh of electrical energy each year, and since most of this energy is produced by burning fossil fuels, this generates about 8 tons of carbon dioxide each year.

On the national scale, the United States consumes about 4.2 million gigaWatt-hours (gWh) of electricity each year, and this generates about 2 gigatons of carbon dioxide.

On the global scale, the table to the left shows how a selection of other countries consumed electrical energy in 2007. The world electrical consumption was 19 million gWh from 213 countries.

Country	Energy (gWh)
USA	4,167,000
China	3,256,000
European Union	3,256,000
Japan	1,195,000
Russia	1,016,000
India	665,000
Canada	612,000
Germany	594,700
France	570,000
United Kingdom	371,000
Spain	294,000
South Africa	264,000
Australia	244,000
Mexico	243,300
Taiwan	225,300
Iran	193,000
Saudi Arabia	179,000
Sweden	143,000
Finland	74,000
Philippines	53,600
Iraq	37,000
Puerto Rico	24,000
Cuba	18,000
Zimbabwe	9,500
Guatemala	7,600
Namibia	1,600
Afghanistan	840
Haiti	550
Greenland	300
Somalia	280
Rwanda	130
Chad	95
Tonga	40
Kiribati	10
Niue	4

Source: CIA World Fact Book

Problem 1 - The world population in 2007 was about 6.67 billion people. If each person used about the same amount of electrical energy, about how many kWh per person would this be? This is called the 'per capita' world electrical energy usage.

Problem 2 - The population of the US in 2007 was 303 million people. What was the per-capita electrical consumption for the US in kilowatt-hours?

Problem 3 - The population of Rwanda in 2007 was 66 million people. What was the per-capita electrical consumption for Rwanda in kilowatt-hours?

Problem 4 - The amount of carbon dioxide produced by electricity generation is about 0.7 kilograms per kilowatt-hour. How many tons per year were produced by A) An average US citizen? B) A citizen of Rwanda?

Problem 5 - What is the total number of tons of carbon dioxide generated by the world-wide consumption of electrical energy, assuming that 70% of all electrical power is produced by fossil fuel burning? Give your answer in gigatons.

Problem 1 - The world population in 2007 was about 6.67 billion people. If each person used about the same amount of electrical energy, about how many kWh would this be? This is called the 'per capita' electrical energy usage. Answer: Dividing the total world consumption of 19 million gWh by 6.67 billion people we get $19 \text{ million gWh} / 6.67 \text{ billion people} = 2.8 \text{ million Wh} \times (1 \text{ kWh}/1,000 \text{ Wh})$, or **28,000 kWh per person each year.**

Problem 2 - The population of the US in 2007 was 303 million people. What was the per-capita electrical consumption for the US? Answer: From the table, we see that the US electrical usage was 4,167,000 gWh, so dividing this by 303 million people we get an average consumption of

$4,167,000 \text{ gWh} \times (1 \text{ billion watts}/1 \text{ gWh}) / 303 \text{ million people}$
 $= (4.167 \text{ million} / 303 \text{ million}) \times 1 \text{ billion Watt-hrs}$
 $= 13.7 \text{ megaWatt-hours per person.}$
 $= 13,700,000 \text{ WattHours per person}$
 But since $1 \text{ kWh} = 1,000 \text{ Wh}$, we get $13.7 \text{ mWh} \times (1,000 \text{ kWh}/1 \text{ mWh}) = \mathbf{13,700 \text{ kWh per person.}}$

Problem 3 - The population of the Rwanda in 2007 was 66 million people. What was the per-capita electrical consumption for Rwanda in kilowatt-hours per person? Answer: From the table, Rwanda used 130 gigaWatt-hours each year, so the per-capita consumption is $130,000,000,000 \text{ Wh} \times (1 \text{ kWh}/ 1000 \text{ Wh}) / 66 \text{ million people} = 130,000,000 \text{ kWh} / 66,000,000 \text{ people} = \mathbf{2,000 \text{ kWh per person each year, rounded to 2 significant figures.}}$

Problem 4 - The amount of carbon dioxide produced by electricity generation is about 0.7 kilograms per kilowatt-hour. How many tons per year were produced by A) An average US citizen? B) A citizen of Rwanda? Answer; US = $13,700 \text{ kWh} \times (0.7 \text{ kg}/1\text{kWh}) = \mathbf{9.6 \text{ tons of carbon dioxide.}}$ B) $2,000 \text{ kWh} \times (0.7 \text{ kg}/\text{kWh}) = \mathbf{1.4 \text{ tons per person.}}$

Problem 5 - What is the total number of tons of carbon dioxide generated by the world-wide consumption of electrical energy assuming that 70% of all electrical power is produced by fossil fuel burning? Give your answer in gigatons. Answer: The grand total was 19 million gWh. Now convert this to gigatons of carbon dioxide:

$19 \text{ million billion Wh} \times (1 \text{ kWh}/1,000 \text{ Wh}) = 19 \text{ thousand billion kWh.}$
 $19 \text{ thousand billion kWh} \times (0.7 \text{ kg}/1 \text{ kWh}) = 13,300 \text{ billion kg}$
 $13,300 \text{ billion kg} \times (1 \text{ ton}/1,000 \text{ kg}) = 13.3 \text{ billion tons}$
 or **13.3 gigatons of carbon dioxide.**

Earth's Atmosphere



The International Space Station orbits earth at an altitude of 350 kilometers, but even at this great distance, it still flies through a low-density atmosphere. At 120 km, the atmosphere is dense enough to have an effect upon re-entering space shuttles, yet by an altitude of 11 kilometers, nearly 3/4 of the atmospheric mass is below you!

The atmosphere consists of a large number of gas components, only a few of the more common ones are shown in the table. The total mass of the atmosphere is estimated to be about 5.1×10^{18} kilograms. Scientists compare the abundances of each element in several ways.

Gas Component	Fraction by Mass	Parts Per Million
Nitrogen	0.755	781,000
Oxygen	0.231	209,000
Argon	0.013	9,340
Carbon Dioxide	0.000582	383
Neon	0.000013	18
Helium	0.0000072	5.2
Methane	0.0000094	1.7
Krypton	0.0000033	1.1
Hydrogen	0.00000038	0.55

Method 1 - In the table, neon is listed as '18 ppm' by mass, which means that for every million particles in the gas there are 18 neon atoms.

Method 2 - Another way is by stating the gas's mass fraction. For example, if a sample of the atmosphere has a mass of 100 kilograms, and nitrogen has a mass fraction of 0.755, that means nitrogen constitutes $100 \times 0.755 = 75.5$ kilograms of the sample.

Problem 1 - Suppose you have 100 kilograms of air. How many grams of carbon dioxide will the sample contain?

Problem 2 - What is the mass of Earth's atmosphere in gigatons?

Problem 3 - What is the total mass of carbon dioxide gas in Earth's atmosphere in units of A) kilograms? B) tons? C) gigatons?

Problem 4 - What is the total mass of methane gas in Earth's atmosphere in units of gigatons?

Problem 1 - Suppose you have 100 kilograms of air. How many grams of carbon dioxide will the sample contain?

Answer: $100 \text{ kg} \times 0.000582 = 0.058 \text{ kilograms}$ or **58 grams**.

Problem 2 - What is the mass of earth's atmosphere in gigatons?

Answer: $5.1 \times 10^{18} \text{ kilograms} \times (1 \text{ ton}/1,000 \text{ kilograms}) \times (1 \text{ gigaton}/10^9 \text{ tons}) =$
5,100,000 gigatons.

Problem 3 - What is the total mass of carbon dioxide gas in the atmosphere in units of A) kilograms? B) tons? C) gigatons?

Answer; A) $5.1 \times 10^{18} \text{ kilograms} \times 0.000582 = 3.0 \times 10^{15} \text{ kilograms}$ B) $1 \text{ ton} = 1,000 \text{ kg}$
so $3.0 \times 10^{15} \text{ kilograms}/1,000 = 3.0 \times 10^{12} \text{ tons}$. C) $1 \text{ gigaton} = 1.0 \times 10^9 \text{ tons}$ so $3.0 \times 10^{12} \text{ tons}/10^9 = 3,000 \text{ gigatons}$

Problem 4 - What is the total mass of methane gas in Earth's atmosphere in units of gigatons?

Answer: $5,100,000 \text{ gigatons} \times 0.0000094 = 48 \text{ gigatons}$.

Note to Teacher: Carbon dioxide and methane, along with water vapor, are the three most important greenhouse gases in the atmosphere. Although humans have little direct control over the water vapor content, we do have measurable impacts on the methane and carbon dioxide amounts. An important calculation for the chemistry of the atmosphere is to convert from the volume abundance of a gas, normally cited in parts-per-million to an equivalent number of gigatons for the gas. Here is a step-by-step method. Let's take the very important gas, carbon dioxide.

It is usually stated as having an abundance of 383 parts per million (383 ppm) by volume. This means that for a given volume of the atmosphere, every 1 million particles sampled will have 383 molecules of carbon dioxide. We can also think of this as if the original volume represented 100%, then carbon dioxide comprises $100\% \times 383/1 \text{ million} = 0.0383\%$ of the volume. To convert this into a mass, we have to use the fact that one-mole (6.02×10^{23} particles) of atmospheric particles has a total mass of 28.97 grams (also called 28.97 AMU). Also, the mass of a carbon dioxide molecule, which has 1 carbon and 2 oxygen atoms is $12 \text{ AMU} + 2 \times 16 \text{ AMU} = 44 \text{ AMU}$. This means that 1 mole of carbon dioxide will have a mass of 44 grams. So, for the atmosphere, there are 383 carbon dioxide molecules for every 1 million air particles, so the mass will be $(44 \text{ AMU}/28.97 \text{ AMU}) \times 383 \text{ ppm} = 582 \text{ ppm}$ of carbon dioxide by mass. This means that for a given MASS of atmosphere $582/1 \text{ million} =$ the fraction by mass that is carbon dioxide. Since the mass of the atmosphere is 5,100,000 gigatons, we have $5,100,000 \times 582/1 \text{ million} = 2,968 \text{ gigatons}$ of carbon dioxide, which we round to 2 significant figures to get 3,000 gigatons.



The concentration of carbon dioxide in the atmosphere is a balance between processes that produce it such as burning fossil fuels, human respiration and forest fires, and processes that remove it (called sequestration) such as burying dead leaves and plant respiration.

The picture shows a plot of landscape measuring 1 kilometer on a side. The green trees sequester carbon dioxide at a rate of 1 ton per acre per year. The bare land sequesters it at a rate of 0.2 tons per acre per year.

Problem 1 - Estimate the size of the forested (dark green) area of the picture in square kilometers. If 1 acre is equal to 0.004 square kilometers. How many acres are forested in this picture?

Problem 2 - Estimate the size of the de-forested, bare area of the picture in square kilometers. How many acres have been de-forested in this picture?

Problem 3 - What is the total rate of carbon dioxide sequestration in this particular area in terms of tons per year?

Problem 4 - A typical American home produces about 10 tons of carbon dioxide per year. What is the net production of carbon dioxide from the area shown in this photograph including the impact of the one house?

Problem 5 - The home owner who owns the above property, and the single house shown in the photograph, decides to sell the de-forested area to a developer who builds 50 houses. What is the net carbon dioxide rate?

Problem 1 - Estimate the size of the green area of the picture in square kilometers .If 1 acre is equal to 0.004 square kilometers. How many acres are forested in this picture?

Answer: For an accurate value, students may grid the picture into smaller squares, count the squares, and multiply by the area of a grid square to determine the total area of the irregular region in green. They may also estimate that $\frac{2}{3}$ of the area is covered in green so the forested area is about $\frac{2}{3}$ square km. This equals $0.66/0.004 = 165$ acres.

Problem 2 - Estimate the size of the de-forested, bare area of the picture in square kilometers. How many acres have been de-forested in this picture? Answer: Students may estimate that about $\frac{1}{3}$ of the picture is de-forested, so this equals 0.33 square kilometers or $0.33/0.004 = 83$ acres.

Problem 3 - What is the total rate of carbon dioxide sequestration in this particular area in terms of tons per year?

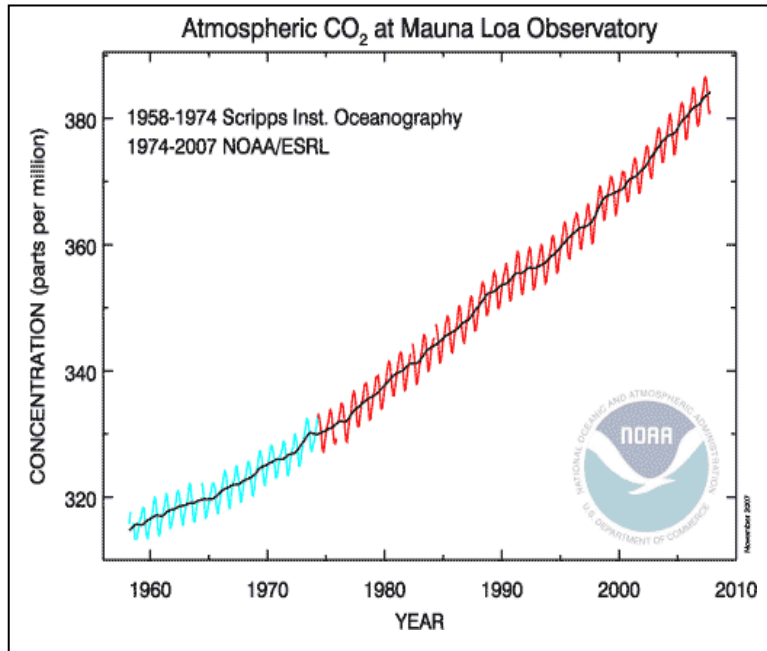
Answer: The forest removes 1 ton per acre per year x 165 acres = 165 tons per year. The de-forested area sequesters 0.2 tons/acre/year x 80 acres = 16 tons/year, so the total rate is $165+16 = 181$ tons/year.

Problem 4 - A typical American home produces about 10 tons of carbon dioxide per year. What is the net production of carbon dioxide from the area shown in this photograph including the impact of the one house? Answer: In this case, with only one house per square kilometer, the net rate is 181 tons/year - 10 tons/year = 171 tons/year sequestered.

Problem 5 - The home owner who owns the above property, and the single house shown in the photograph, decides to sell the de-forested area to a developer who builds 50 houses. What is the net carbon dioxide rate?

Answer: The deforested land no longer sequesters any carbon dioxide because it has been paved over, so the net sequestration rate is from the forested area only and is 181 tons/year. The 51 houses and families that now occupy this land each produce 10 tons of carbon dioxide each year, so the total production rate from the houses is $51 \times 10 = 510$ tons/year. The net carbon dioxide rate is 510 tons/year - 181 tons/year = 329 tons per year, which goes into the atmosphere.

Carbon Dioxide Increases



The Keeling Curve to the left, shows the variation in concentration of atmospheric carbon dioxide since 1958. It is based on continuous measurements taken at the Mauna Loa Observatory in Hawaii under the supervision of Dr. Charles Keeling of the Scripps Institution of Oceanography in San Diego.

Keeling's measurements beginning in 1958, showed the first significant evidence of rapidly increasing carbon dioxide levels in the atmosphere. Additional measurements by scientists working at NOAA have extended the Keeling Curve from 1974-2006.

Atmospheric scientists measure the concentration of gases in terms of parts-per-million (ppm). One ppm = 1 particle out of 1 million particles in a sample. It also represents a percentage: $\text{ppm}/1\text{million} \times 100\%$.

Problem 1 - In the graph, what was the concentration of carbon dioxide in 2005?

Problem 2 - What percentage of Earth's atmosphere, by volume, was carbon dioxide gas in 2005?

Problem 3 - If a concentration of 127 ppm of carbon dioxide in the atmosphere equals a total of 1,000 gigatons of carbon dioxide (1,000 billion tons), about what was the total mass of carbon dioxide gas in 2005?

Problem 4 - How many additional gigatons of carbon dioxide were added to the atmosphere between 1958 and 2005?

Problem 5 - What was the average rate of increase of carbon dioxide gas in gigatons per year between 1958 and 2005?

Problem 6 - The seasonal change in carbon dioxide is shown by the 'wavey' shape of the line. What is the width of this wave (range from maximum to minimum) in ppm, and about how many gigatons does this natural change correspond to?

Problem 1 - In the graph, what was the concentration of carbon dioxide in 2005?
Answer: About 379 ppm.

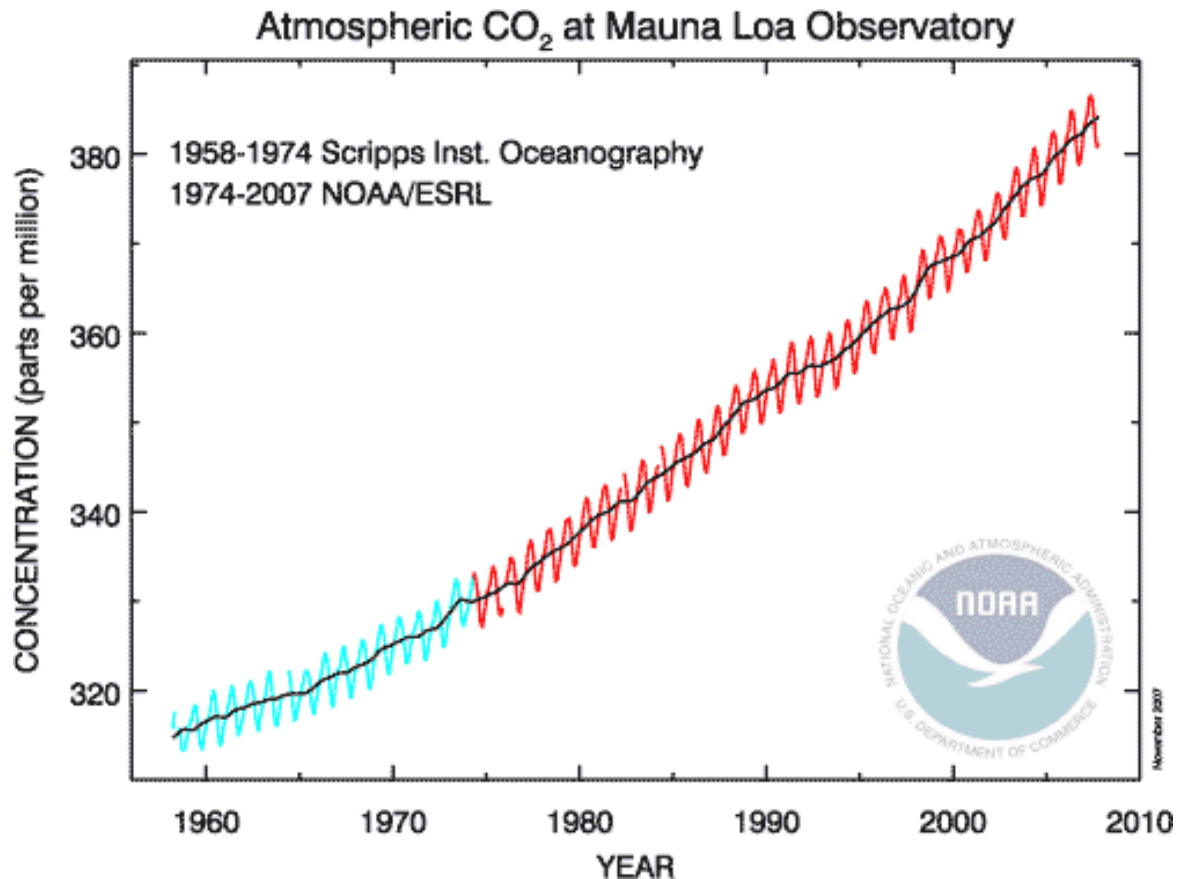
Problem 2 - What percentage of Earth's atmosphere, by volume, was carbon dioxide gas in 2005? Answer: 379 ppm is equal to $100\% \times 379/1 \text{ million} = 0.0379\%$

Problem 3 - If a concentration of 127 ppm of carbon dioxide in the atmosphere equals a total of 1,000 gigatons of carbon dioxide (1,000 billion tons), about what was the total mass of carbon dioxide gas in 2005? Answer: $(379/127) \times 1,000 \text{ gigatons} = 2,984 \text{ gigatons}$, or to 3 significant figures, 2,980 gigatons.

Problem 4 - How many additional gigatons of carbon dioxide were added to the atmosphere between 1958 and 2005?
Answer: In 1958 the concentration was about 315 ppm, so it gained $379 - 315 = 64$ ppm of carbon dioxide. Since 1,000 gigatons corresponds to 127 ppm, we have $(64/127) \times 1,000 \text{ gigatons} = 500 \text{ gigatons were added}$.

Problem 5 - What was the average rate of increase of carbon dioxide gas in gigatons per year between 1958 and 2005? Answer: For the 47 years that span the time interval, there was a 500 gigaton increase, so the rate was $500 \text{ gigatons}/47 \text{ years} = 11 \text{ gigatons/year}$.

Problem 6 - The seasonal change in carbon dioxide is shown by the 'wavey' shape of the line. What is the width (range from maximum to minimum) of this wave in ppm, and about how many gigatons does this natural change correspond to? Answer: Using a metric ruler and estimating the peak to peak variation of the amplitude, students should get about 5 ppm as the range. This equals $(5 \text{ ppm}/127 \text{ ppm}) \times 1,000 \text{ gigatons} = 39 \text{ gigatons}$.



This is the Keeling Curve, derived by researchers at the Mauna Kea observatory from atmospheric carbon dioxide measurements made between 1958 and 2005. The accompanying data, in Excel spreadsheet form, for the period between 1982 and 2008 is provided at

<http://spacemath.gsfc.nasa.gov/data/KeelingData.xls>

Problem 1 - Based on the tabulated data, create a single mathematical model that accounts for, both the periodic seasonal changes, and the long-term trend.

Problem 2 - Convert your function, which describes the carbon dioxide volume concentration in parts per million (ppm), into an equivalent function that predicts the mass of atmospheric carbon dioxide if 383 ppm (by volume) of carbon dioxide corresponds to 3,000 gigatons.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm) and mass for the years: A) 2020? B)2050, C)2100?

Data from: C. D. Keeling, S. C. Piper, R. B. Bacastow, M. Wahlen, T. P. Whorf, M. Heimann, and H. A. Meijer, Exchanges of atmospheric CO₂ and ¹³CO₂ with the terrestrial biosphere and oceans from 1978 to 2000. I. Global aspects, SIO Reference Series, No. 01-06, Scripps Institution of Oceanography, San Diego, 88 pages, 2001. Excel data obtained from the Scripps CO₂ Program website at http://scrippsco2.ucsd.edu/data/atmospheric_co2.html

Problem 1 - Answer: The general shape of the curve suggests a polynomial function of low-order, whose amplitude is modulated by the addition of a sinusoid. The two simplest functions that satisfy this constraint are a 'quadratic' and a 'cubic'... where 't' is the elapsed time in years since 1982

$$F1 = A \sin(Bt + C) + (Dt^2 + Et + F) \text{ and } F2 = a \sin(bt + c) + (dt^3 + et^2 + ft + g)$$

We have to solve for the two sets of constants A, B, C, D, E, F and a,b,c,d,e,f,g. Using *Excel* and some iterations, as an example, the constants that produce the best fits appear to be: F1: (3.5, 6.24, -0.5, +0.0158, +1.27, 342.0) and F2: (3.5, 6.24, -0.5, +0.0012, -0.031, +1.75, +341.0). Hint: Compute the yearly averages and fit these, then subtract this polynomial from the actual data, and fit what is left over (the residual) with a sine function.) The plots of these two fits are virtually identical. We will choose $F_{ppm} = F1$ as the best candidate model because it is of lowest-order. The comparison with the data is shown in the graph below: red=model, black=monthly data. Students should be encouraged to obtain better fits.

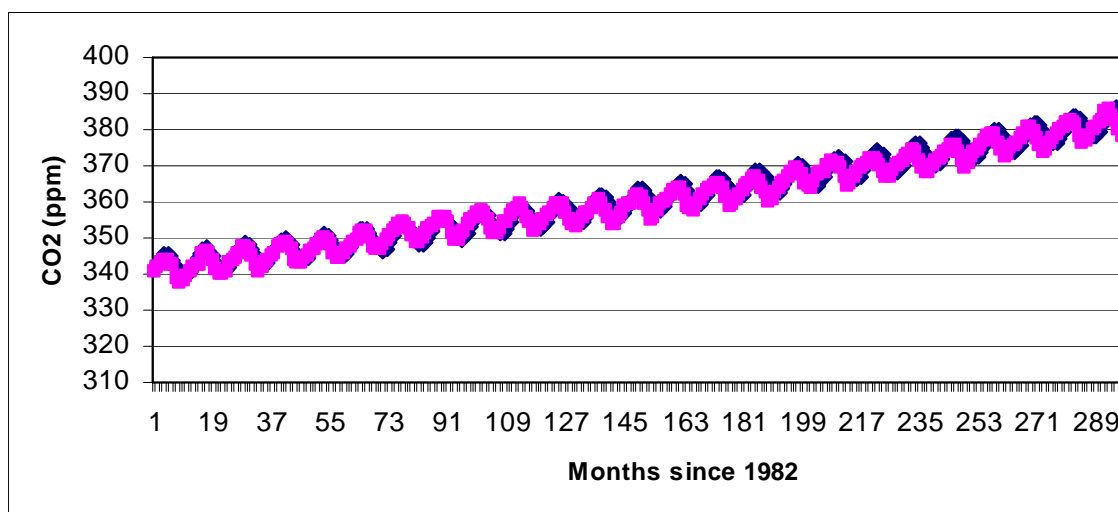
Problem 2 - Answer: The model function gives the atmospheric carbon dioxide in ppm by volume. So take F_{ppm} and multiply it by the conversion factor $(3,000/383) = 7.83$ gigatons/ppm to get the desired function, F_{co2} for the carbon mass.

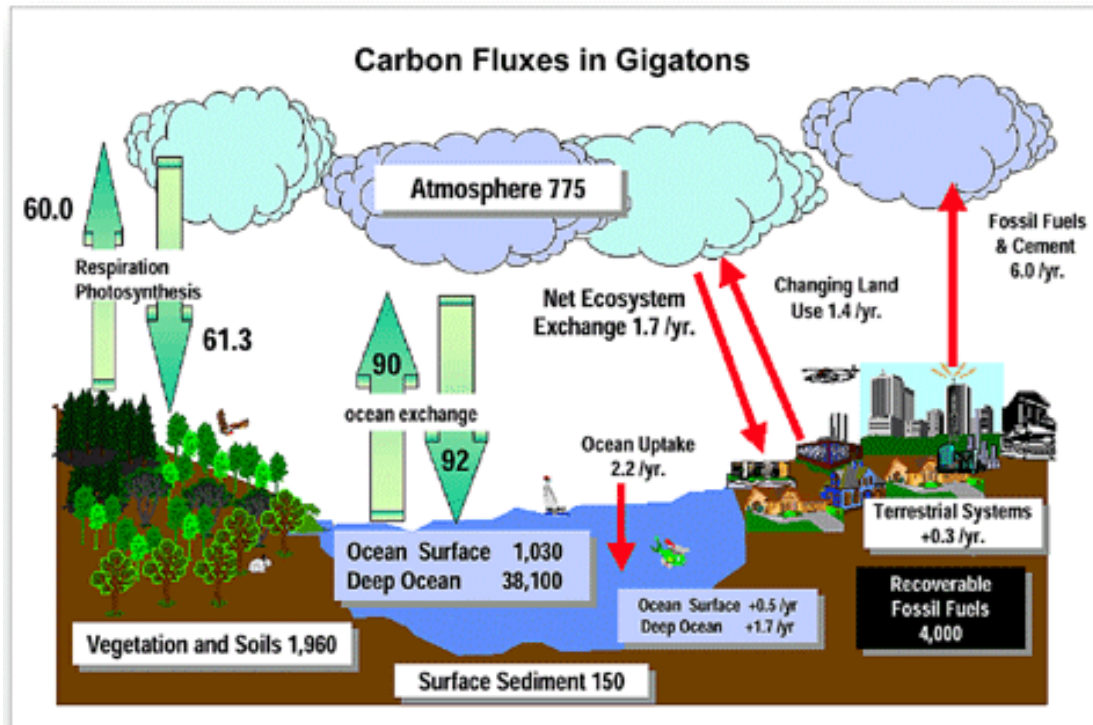
Problem 3 - What would you predict as the carbon dioxide concentration (ppm), and mass for the years: A) 2020? B)2050, C)2100? Answer:

A) $t = 2020-1982 = 38$, so $F_{co2}(38) = 7.83 \times 410 \text{ ppm} = 3,200 \text{ gigatons}$

B) $t = 2050-1982 = 68$, so $F_{co2}(68) = 7.83 \times 502 \text{ ppm} = 3,900 \text{ gigatons}$

C) $t = 2100-1982 = 118$, so $F_{co2}(118) = 7.83 \times 718 \text{ ppm} = 5,600 \text{ gigatons}$





This diagram, provided by the US Department of Energy, shows the many pathways that contribute to the current atmospheric carbon dioxide levels. Currently, at 380 ppm, there are 3,000 gigatons of carbon dioxide in the atmosphere. Scientists consider carbon dioxide a part of the global 'Carbon Cycle'. Since carbon dioxide, CO_2 , has a mass of 44 AMU and C has a mass of 12 AMU, to get the equivalent carbon mass, we divide the CO_2 mass by $(44/12) = 3.67$ so that 3670 gigatons of carbon dioxide will equal 1000 gigatons of carbon. In the above diagram, the basis for the current atmospheric level of carbon dioxide is $775 \times 3.67 = 2,800$ gigatons (for a carbon dioxide concentration of 350 ppm). The arrows give the direction of flow of the carbon in each part of this system. The boxed numbers give the size of each reservoir of carbon. For example, plant respiration and photosynthesis emit 60 gigatons of carbon into the atmosphere, but removes 61.3 gigatons, so plants remove (sequester) a net of 1.3 gigatons of carbon (or 4.8 gigatons of carbon dioxide) from the atmosphere.

Problem 1 - What is the net direction of change, and magnitude, of carbon exchange between the atmosphere and ocean?

Problem 2 - Combining all of the arrows and man-made impacts, what is the net amount and direction of these carbon sources and sinks?

Problem 3 - Since 1850, the average rate of increase of carbon dioxide in the atmosphere has been about 11 gigatons/year. What is this in terms of carbon increase, and what does this indicate about the flow diagram above?

The diagram was obtained from

http://www.netl.doe.gov/technologies/carbon_seq/overview/what_is_CO2.html

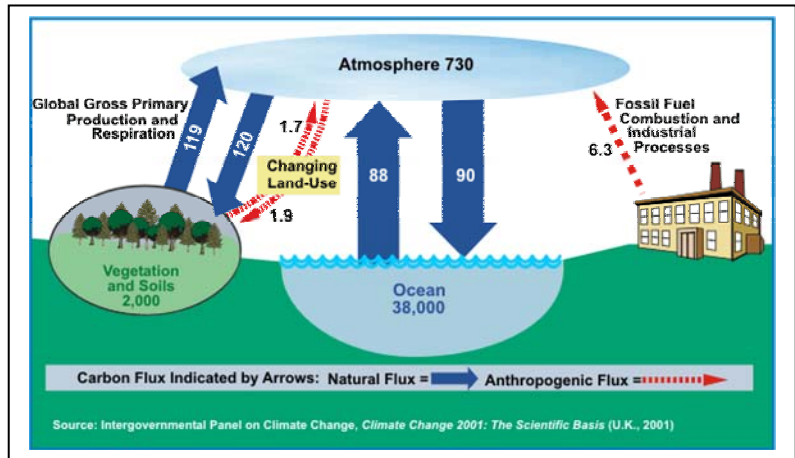
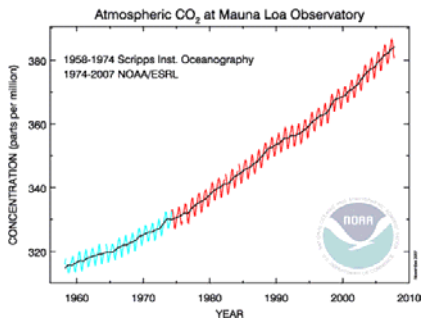
Problem 1 - What is the net direction of change, and magnitude, of carbon exchange between the atmosphere and ocean? Answer: $+90 - 92 = -2$ gigatons/year, so **2 gigatons/year of carbon are removed from the atmosphere. This is called sequestering.**

Problem 2 - Combining all of the arrows and man-made impacts, what is the net amount and direction of these carbon sources and sinks? Answer: $60 - 61.3 + 90 - 92 - 2.2 - 1.7 + 1.4 + 6.0 = +0.2$ gigatons/year of carbon are added to the atmosphere each year.

Problem 3 - Since 1850, the average rate of increase of carbon dioxide in the atmosphere has been about 11 gigatons/year. What is this in terms of carbon increase, and what does this indicate about the flow diagram above?

Answer; $11/3.67 = 3$ gigatons/year of carbon. From Problem 2, the net change was an increase of 0.2 gigatons/year of carbon, and $0.2 \times 3.67 = 0.7$ gigatons/year of carbon dioxide. This means that the diagram has not completely accounted for all of the carbon increase since 1850, because an additional 2.8 gigatons/year is needed to completely account for the rise in carbon dioxide levels during this time.

Note to Teacher: When reading about global climate change, it is very important to keep straight the difference between a gigaton of carbon and a gigaton of carbon dioxide. As we have seen, there is nearly a factor of four (actually 3.67) difference between these amounts. When newspapers or the news media choose to cite such figures, they may gloss over the difference between 'carbon' and 'carbon dioxide', which can be confusing. Also, when graphs are published showing 'carbon' changes, it is always helpful to check that the Author actually meant 'carbon' and not 'carbon dioxide'. You might challenge your students to find examples of graphs 'on line' where this distinction is not clear.



The graph to the left shows the 'Keeling Curve' which plots the increase in atmospheric carbon dioxide between 1958 and 2005. The average net annual rate implied by this curve is about 11 gigatons of carbon dioxide per year. The figure to the right shows a simplified view of the sources and sinks of carbon on Earth. Note that, for every 44 gigatons of the carbon dioxide molecule, there are 12 gigatons of the element carbon.

Problem 1 - What are the sources of carbon increases to the atmosphere in the above diagram? What are the sinks of carbon?

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon in gigatons.

Problem 3 - Integrate the equation in Problem 2, assuming that $C(2005) = 730$ gigatons, and derive A) the simple equation describing the total amount of carbon in the atmosphere as a function of time; B) the equation for the total amount of carbon dioxide.

Problem 4 - What does your model predict for the amount of carbon in the atmosphere in 2050 if the above source and sink rates remain the same?

Problem 5 - If the current carbon dioxide abundance is 384 ppm, what does your model predict for the carbon dioxide abundance in 2050?

Problem 6 - Does your answer for the net change in Problem 2 match up with the Keeling Curve data that indicates a net annual increase of carbon dioxide of +11 gigatons/year?

Problem 1 - Answer: The sources of the carbon (arrows pointed into the atmosphere in the figure) are Vegetation (+119.6 gigatons/yr), oceans (+88 gigatons/yr), human activity (+6.3 gigatons/yr) and changing land use (+1.7 gigatons/yr). The sinks remove carbon (the arrows pointed down in the figure) and include vegetation (-120 gigatons/yr), oceans (-90 gigatons/yr), and changing land use (-1.9 gigatons/yr).

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon dioxide, $C(t)$, in gigatons.

Answer:
$$\frac{dC(t)}{dt} = +119 + 88 + 6.3 + 1.7 - 120 - 90 - 1.9$$

so
$$\frac{dC(t)}{dt} = +3.1$$

Problem 3 - Answer: $C(t) = 3.1 t + a$ where a is the constant of integration.
Since $C(2005) = 730$

$$730 = 3.1 (2005) + a \quad \text{and so } a = -5500$$

$$C(t) = 3.1 t - 5500. \quad \text{for the total element carbon.}$$

Since 44 gigatons of carbon dioxide contain 12 gigatons of carbon, the equation for the CO₂ increase is $44/12 = 3.7x C(t)$ so that $CO_2(t) = 11.5 t - 20,300$

Problem 4 - What does your model predict for the amount of carbon dioxide in the atmosphere in 2050 if the above source and sink rates remain the same?

Answer: $C(2050) = 3.1 (2050) - 5500$
 $C(2050) = 860 \text{ gigatons of carbon.}$

Problem 5 - If the current carbon dioxide abundance is 384 ppm, what does your model predict for the abundance in 2050? Answer: Since we are interested in the carbon dioxide increase we use the equation

$$CO_2(t) = 11.4 t - 20,300$$

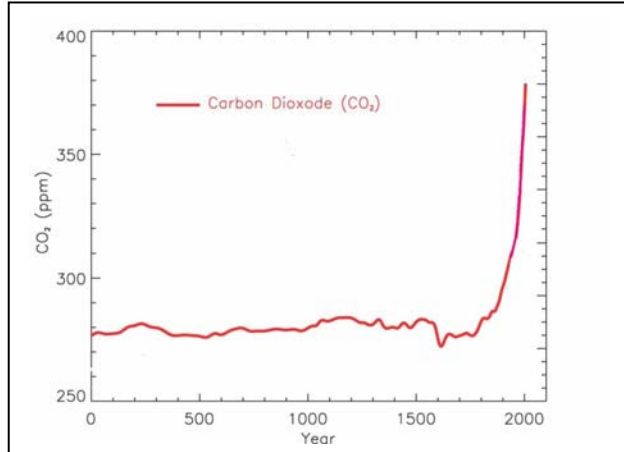
For $t = 2050$ we get $CO_2(2050) = +3070$ gigatons of CO₂.

Since 384 ppm corresponds to 730 gigatons of CO₂, by a simple proportion, 3275 gigatons corresponds to $384 \text{ ppm} \times (3070/730) = 1,600 \text{ ppm of CO}_2$.

Problem 5 - Does your answer for the net change in Problem 2 match up with the Keeling Curve data that indicates a net annual increase of carbon dioxide 11 gigatons/year?

Answer: Yes. The net change in Problem 2 was +3.1 gigatons/year of the element carbon. Since for every 44 gigatons of carbon dioxide there are 12 gigatons of the element carbon, we have $+3.1 \times (44/12) = +11.4$ gigatons of carbon dioxide increase, which is similar to the annual average of the increase implied by the Keeling Curve. This means that our simple model, based on the specified rates, provides a consistent picture of the atmospheric changes at the present time. Note: This simple, linear, model is only valid over the time span of the Keeling Curve.

Carbon Dioxide Increases During the Last 2000 Years.



This graph shows the changes in the carbon dioxide concentration in the atmosphere during the last 2000 years. It was prepared by the International Committee on Climate Change in 2008.

Atmospheric scientists measure gas concentrations in parts-per-million (ppm), which means there is 1 particle of a substance for every 1 million particles of the gas that contains it. For carbon dioxide in the atmosphere, a value of 127 ppm would correspond to 1,000 gigatons of carbon dioxide in the atmosphere.

Problem 1 - What was the average concentration of carbon dioxide, in ppm, in the atmosphere before the year 1800?

Problem 2 - How many gigatons did this pre-1800 concentration correspond to?

Problem 3 - About what was the rate at which the pre-1800 concentration was changing during the period from 0 to 1800 according to the graph? Give your estimate in ppm per year, and gigatons per year.

Problem 4 - By what amount, compared to the pre-1800 average, did the concentration increase between 1800-2005 in ppm and gigatons?

Problem 5 - What was the annual rate of increase in atmospheric carbon dioxide between 1800 and 2005, in ppm/year and in gigatons/year?

Problem 6 - The world population in 1800 was 1 billion while in 2005 it was 6 billion. From your answer to Problem 3 and 5, what was the average carbon dioxide production per person before 1800, and in 2005 in tons per year per person?

Problem 7 - Can you explain why scientists think human activity is to blame for the recent rise in atmospheric carbon dioxide concentrations since 1800?

Problem 1 - What was the average concentration of carbon dioxide, in ppm, in the atmosphere before the year 1800?

Answer: Students may use a ruler to draw a horizontal line that follows the average values during this period of time. The value would be about **278 ppm**.

Problem 2 - How many gigatons did this pre-1800 concentration correspond to?

Answer: $(278 \text{ ppm}/127 \text{ ppm}) \times 1,000 \text{ gigatons} = \mathbf{2,190 \text{ gigatons}}$ to three significant figures.

Problem 3 - About what was the rate at which the pre-1800 concentration was changing during the period from 0 to 1800 according to the graph? Give your estimate in ppm per year, and gigatons per year. Answer: Students estimates will vary, especially since the graph does not show an obvious slope. One possibility is that the initial value is 277 ppm and the final value is 280 ppm, so the change is +3 ppm over 1800 years or **0.0017 ppm/year**, or $(0.0017/127) \times 1,000 = \mathbf{0.013 \text{ gigatons/year}}$.

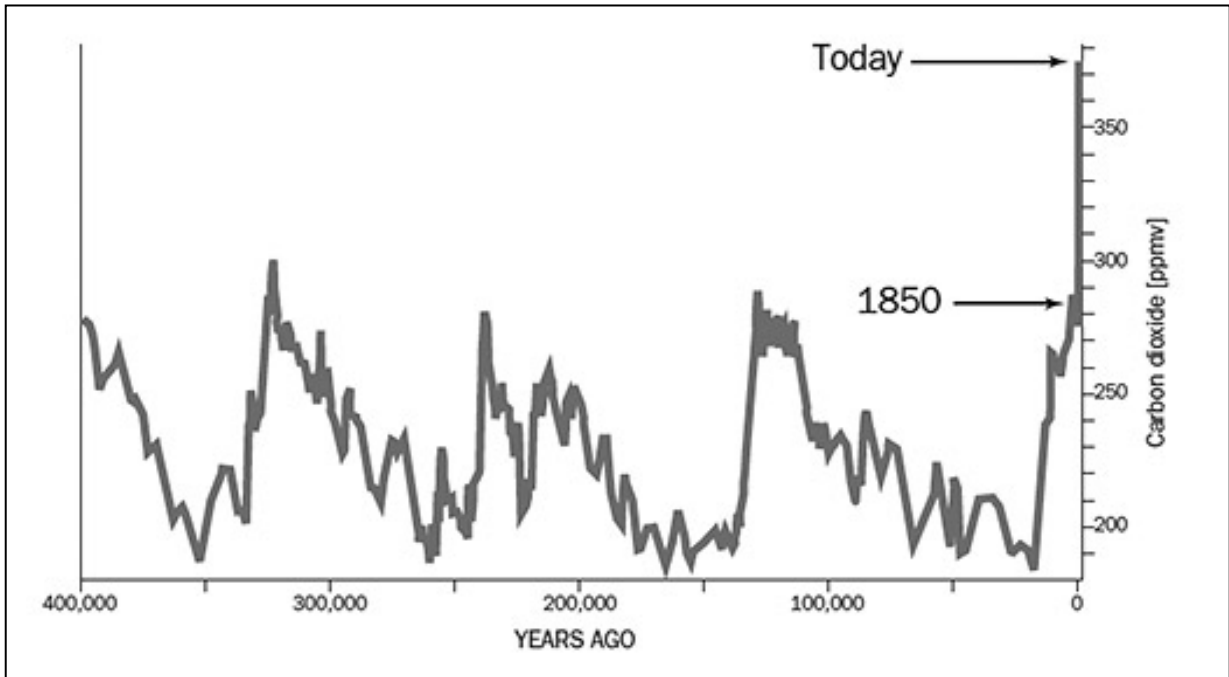
Problem 4 - By what amount, compared to the pre-1800 average, did the concentration increase between 1800-2005 in ppm and gigatons? Answer: by 2005, the concentration had increased to 380 ppm, for a change of $380 - 278 = \mathbf{102 \text{ ppm}}$ or $(102/127) \times 1000 = \mathbf{800 \text{ gigatons}}$.

Problem 5 - What was the annual rate of increase in atmospheric carbon dioxide between 1800 and 2005, in ppm/year and in gigatons/year? Answer: for a rate of $102 \text{ ppm}/205 \text{ years} = \mathbf{0.5 \text{ ppm/year}}$, and $(0.5/127) \times 1000 = \mathbf{4.0 \text{ gigatons/year}}$.

Note, this rate of increase is $0.5/0.0017 = 294$ times faster than the pre-1800 rates we can estimate.

Problem 6 - The world population in 1800 was 1 billion while in 2005 it was 6 billion. From your answers to Problem 3 and 5, what was the average carbon dioxide production per person before 1800, and in 2005, in tons per year per person? Answer: In 1800, $0.01 \text{ gigatons}/1 \text{ billion people} = \mathbf{0.01 \text{ tons}}$ per person per year. In 2005 it was $4.0 \text{ gigatons}/6 \text{ billion people} = \mathbf{0.7 \text{ tons/person/year}}$.

Problem 7 - Can you explain why scientists think human activity is to blame for the recent rise in atmospheric carbon dioxide concentrations since 1800? Answer; Students may use the result from Problem 6 to explain that before 1800 the average person was adding only 0.01 tons/year to the atmosphere, but after the Industrial Revolution this increased to 0.7 tons per person per year, and that the amount being added is keeping step with the population increase on a 'per person' basis.



The graph above shows the changes in carbon dioxide concentration during the last 400,000 years. It was produced by measuring ice samples in Antarctica and extracting atmospheric gases from trapped bubbles of air. The 100,000-year ice age cycle is clearly recognizable. Also shown on the graph is the last 150 years of carbon dioxide concentration data since 1850. (Data sources: Petit et al. 1999; Keeling and Whorf, 2004).

Atmospheric scientists measure gas concentrations in parts per million (ppm). For every 1 million atoms or molecules of Earth's atmosphere, there are currently about 375 molecules of carbon dioxide, so the current concentration is 375 ppm. In terms of mass, 127 ppm equals 1,000 gigatons of carbon dioxide in the entire atmosphere of Earth.

Problem 1 - If it were not for the sharp rise in carbon dioxide after 1850, what would you predict would have been the carbon dioxide changes during the next 100,000 years?

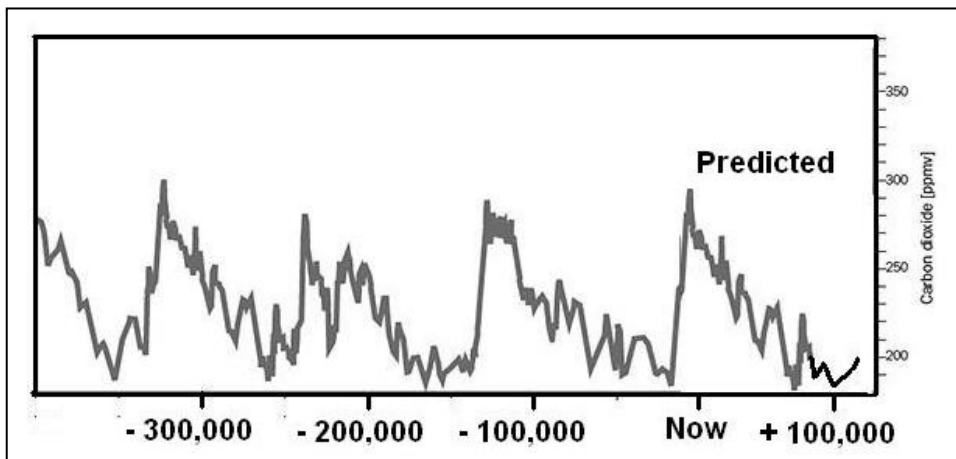
Problem 2 - Some people believe that the current rise in carbon dioxide is part of a natural cycle of changes. Based on this graph, what would your reply be to this belief?

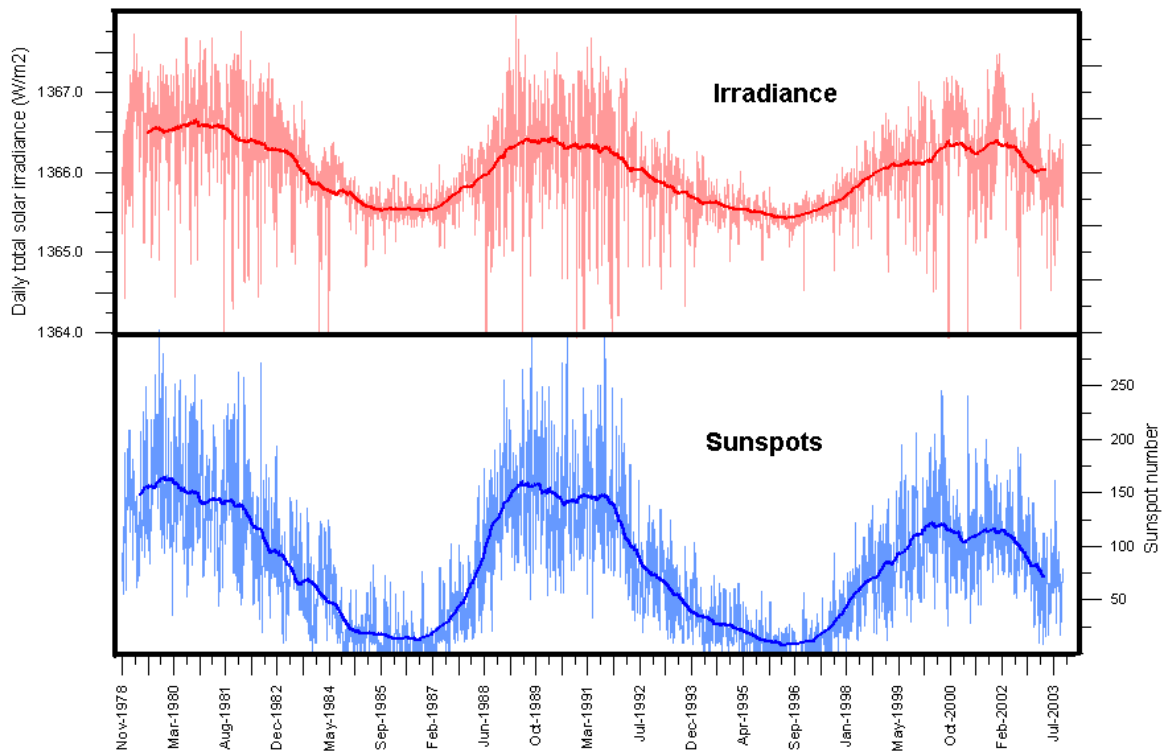
Problem 1 - If it were not for the sharp rise in carbon dioxide after 1850, what would you predict would have been the carbon dioxide changes during the next 100,000 years?

Answer: Students may draw various extrapolations using the previous rise and fall of Ice Age carbon dioxide concentrations as a guide. See the figure below as an example.

Problem 2 - Some people believe that the current rise in carbon dioxide is part of a natural cycle of changes. Based on this graph, what would your reply be to this belief?

Answer: Students may provide various answers, but from the graph of the data, there is no evidence that in the last 400,000 years there have ever been carbon dioxide concentrations as high as the 'spike' corresponding to the most recent 150 years. There is no evidence that the current changes are part of any 'periodic' cycles in atmospheric concentrations, even though such cycles for previous Ice Ages and warming episodes are clearly seen in the data. The current CO₂ concentration is 380 ppm and continues to increase every year, but the peaks of the previous 400,000 years of Ice Age cycles do not exceed 300 ppm! In the last 150 years, we have modified the atmosphere's CO₂ in a way that has not been seen in the previous 400,000 years.





Insolation is a measure of the amount of solar energy that falls upon one square meter of exposed surface, usually measured at the 'top' of Earth's atmosphere. This energy increases and decreases with the season and with your latitude on Earth, being lower in the winter and higher in the summer, and also lower at the poles and higher at the equator. But the sun's energy output also changes during the sunspot cycle!

The figure above shows the solar power (called irradiance) and sunspot number since January 1979 according to NOAA's National Geophysical Data Center (NGDC). The thin lines indicate the daily irradiance (red) and sunspot number (blue), while the thick lines indicate the running annual average for these two parameters. The total variation in solar irradiance is about 1.3 watts per square meter during one sunspot cycle. This is a small change compared to the 100s of watts we experience during seasonal and latitude differences, but it may have an impact on our climate. The solar irradiance data obtained by the ACRIM satellite, measures the total number of watts of sunlight that strike Earth's upper atmosphere before being absorbed by the atmosphere and ground.

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003?

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance?

Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989?

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003?

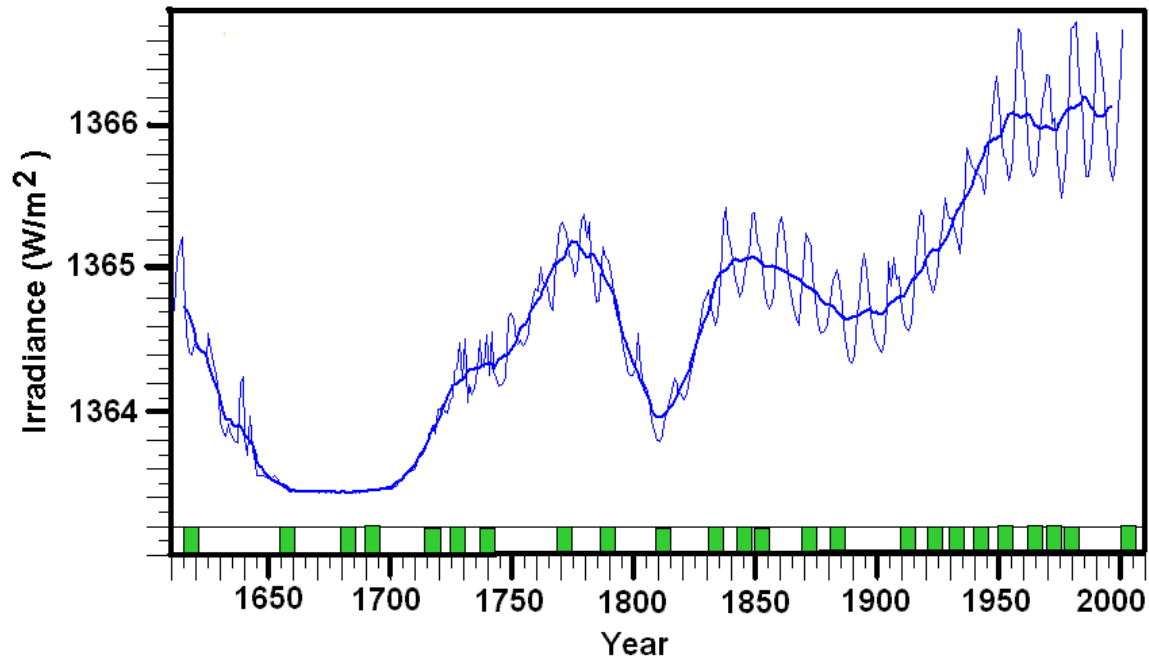
Answer: Draw a horizontal line across the upper graph that is mid-way between the highest and lowest points on the curve. An approximate answer would be **1366.3 watts per square meter**.

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance? Answer: When there are a lot of sunspots on the sun (called sunspot maximum) the amount of solar radiation is higher than when there are fewer sunspots. The solar irradiance changes follow the 11-year sunspot cycle.

Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989?

Answer: In 1985, the amount of insolation was about 1365.5 watts per square meter when the photovoltaic system was built. Because of changes in the sunspot cycle, in 1989 the insolation increased to about 1366.5 watts per square meter. This insolation change was a factor of $1366.5/1365.5 = 1.0007$. That means that by scaling, if the system was generating 3,000 kilowatt-hours of electricity in 1985, it will have generated $1.0007 \times 3,000 \text{ kWh} = 2 \text{ kWh more}$ in 1989 during sunspot maximum! That is equal to running one 60-watt bulb for about 1 day (actually 33 hours).

The Solar Constant Since 1600



The graph above shows the solar irradiance (also called the solar constant) since 1610. Although sensitive instruments were not available back in 1600 to make these measurements, scientists have developed many techniques to 'reconstruct' the irradiance values using other astronomical and geophysical data. The thin line in the graph indicates the annual solar irradiance, while the thick line shows the running average. It is plain to see that today's values are significantly higher than in centuries past. Some scientists feel this accounts for global warming which has also increased, mostly since 1800. The majority of scientists have concluded that solar increases only account for 1/2 of the global warming since the 1970's, and significantly less than 1/2 in the 2000s.

Problem 1 - Between 1645 and 1715, a period known as the Maunder Minimum, there were few sunspots observed, and no 11-year sunspot cycle. It was a time of extremely cold winters in Europe. What was the average solar insolation (Solar Constant) during this time?

Problem 2 - What has been the average insolation during the period after 1950?

Problem 3 - What is the percent change in solar insolation between the Maunder Minimum period and the time period after 1950?

Problem 4 - Although some global warming has been measured since 1800, the most dramatic change in global temperatures has occurred since 1970. Based on the historical insolation derived from solar activity cycles, why do scientists who study climate change claim that solar changes do not account for the most recent temperature increases?

Problem 1 - Between 1645 and 1715, a period known as the Maunder Minimum, there were few sunspots observed, and no 11-year sunspot cycle. It was a time of extremely cold winters in Europe. What was the average solar insolation (Solar Constant) during this time? Answer: The graph shows a pronounced minimum to the insolation with a value **near 1,363.4 watts per square meter**.

Problem 2 - What has been the average insolation during the period after 1950? Answer: **Estimates near 1,366 watts per square meter are acceptable** based on the trend in the data.

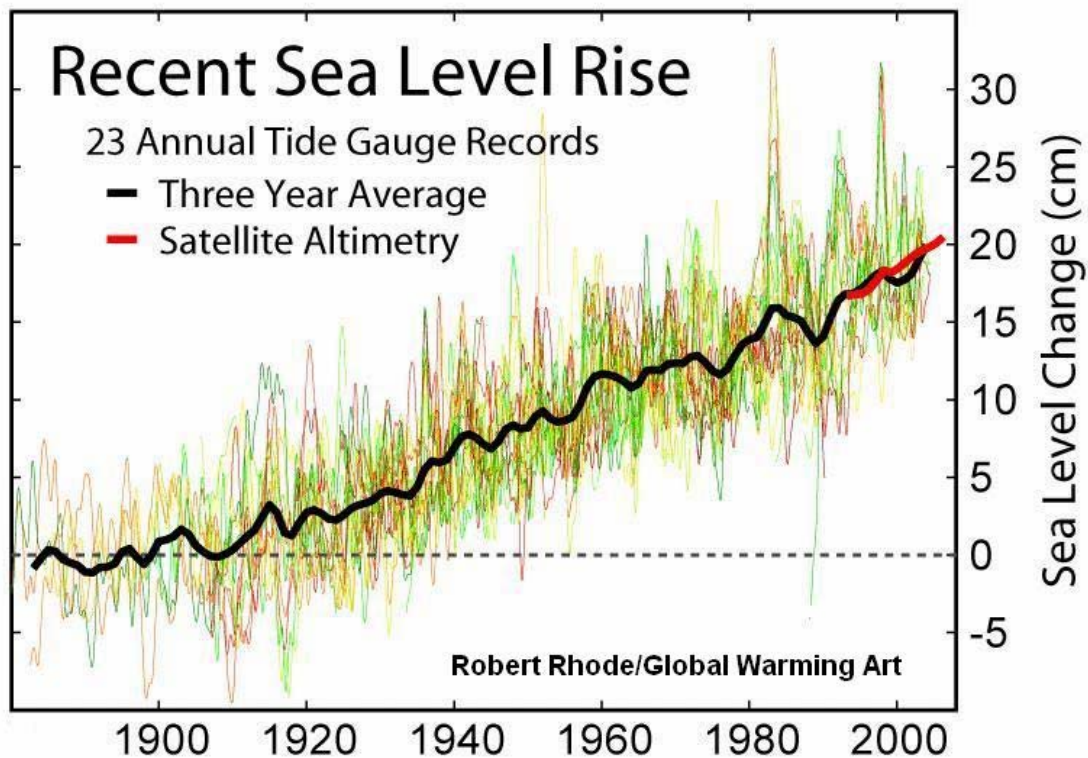
Problem 3 - What is the percent change in solar insolation between the Maunder Minimum period and the time period after 1950? Answer: $100\% \times (1,366 - 1,363.4)/1,363.4 = 0.2\%$ **increase since the 1600s**.

Problem 4 - Although some global warming has been measured since 1800, the most dramatic change in global temperatures has occurred since 1970. Based on the historical insolation derived from solar activity cycles, why do scientists who study climate change claim that solar changes do not account for the most recent temperature increases?

Answer: When the historical changes in solar insolation since 1600 are compared with global temperature changes, the significant increases measured since 1970 do not match up with the essentially 'flat' average insolation changes since the 1950's. This suggests to scientists that at least since 1970, solar changes have been less responsible for global temperature rise than other factors. The most significant one that has been measured is the rise in carbon dioxide levels due to human activity.

Teacher Note: Satellites were not available before the 1970's to accurately measure the solar constant every year, so scientists have devised a number of 'proxies' that they think correlate with solar insolation changes. Once these proxies are verified using satellite data, they can be used to extrapolate how the solar constant has changed in previous decades and centuries. The more of these proxy data bases scientists can correlate with solar insolation, the more accurate are the extrapolations back in time for the particular quantity they are historically studying.

For more about how scientists reconstruct solar insolation from past years, see the article by J. Lean, "Evolution of the Sun's Spectral Irradiance Since the Maunder Minimum" published in the Geophysical Research Letters, Vol. 27, No. 16, pp. 2425-2428 on Aug. 15, 2000.



The graph, produced by scientists at the University of Colorado and published in the IPCC Report-2001, shows the most recent global change in sea level since 1880 based on a variety of tide records and satellite data. The many colored curves show the individual tide gauge trends. The black line represents an average of the data in each year.

Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line?

Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form?

Problem 3 - If the causes for the rise remained the same, what would you predict for the seal level rise in A) 2050? B) 2100? C) 2150?

Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line? Answer; See figure below. First, select any two convenient points on this line, for example X= 1910 and Y = 0 cm (1910, +0) and X = 1980 Y= +15 cm (1980, +15). The slope is given by $m = (y_2 - y_1) / (x_2 - x_1) = 15 \text{ cm} / 70 \text{ years} = 0.21 \text{ cm/year}$.

Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form? Answer:

$$\text{A) } y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1) \quad \text{so } y - 0 = \frac{(15 - 0)}{(1980 - 1910)} (x - 1910)$$

$$\text{B) } y - y_1 = m (x - x_1) \quad \text{so } y - 0 = 0.21 (x - 1910)$$

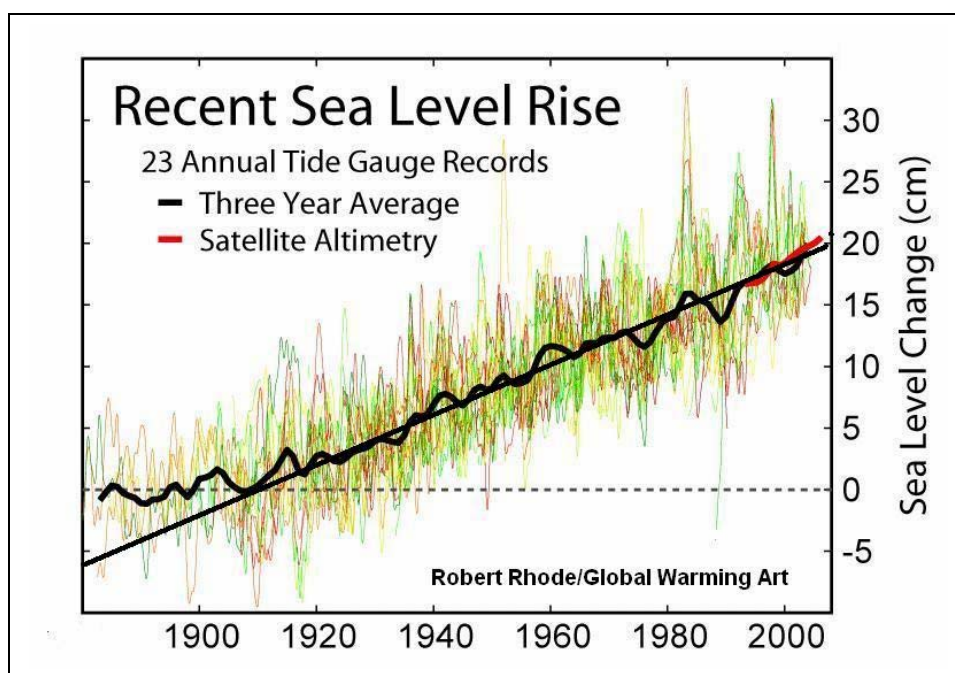
$$\text{C) } y = 0.21 x - 0.21(1910) \quad \text{so } y = 0.21x - 401.1$$

Problem 3 - If the causes for the rise remained the same, what would you predict for the seal level rise in A) 2050? B) 2100? C) 2150? Answer:

$$\text{A) } y = 0.21 (2050) - 401.1 = 29.4 \text{ centimeters.} \quad (\text{Note; this equals } 12 \text{ inches})$$

$$\text{B) } y = 0.21 (2100) - 401.1 = 39.9 \text{ centimeters} \quad (\text{Note: this equals } 16 \text{ inches})$$

$$\text{C) } y = 0.21 (2150) - 401.1 = 50.4 \text{ centimeters.} \quad (\text{Note: this equals } 20 \text{ inches})$$





This QuickBird Satellite image was taken of downtown Las Vegas Nevada from an altitude of 450 kilometers. Private companies such as Digital Globe (<http://www.digitalglobe.com>) provide images such as this to many different customers around the world. The large building shaped like an upside-down 'Y' is the Bellagio Hotel at the corner of Las Vegas Boulevard and Flamingo Road. The width of the image is 700 meters.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 700 meters wide.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter. Report your answer to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: How long is each of the three wings of the Bellagio Hotel in meters?

Question 2: What is the length of a car on the street in meters?

Question 3: How wide are the streets entering the main intersection?

Question 4: What is the smallest feature you can see, in meters?

Question 5: What kinds of familiar objects can you identify in this image?

Answer Key:

This QuickBird Satellite image was taken of downtown Las Vegas Nevada on October 14, 2005 from an altitude of 450 kilometers. Private companies such as Digital Globe (<http://www.digitalglobe.com>) provide images such as this to many different customers around the world. The large building shaped like an upside-down 'Y' is the Bellagio Hotel at the corner of Las Vegas Boulevard and Flamingo Road. The width of the image is 700 meters.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 700 meters wide.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?

Answer: 150 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Answer: The information in the introduction says that the image is 700 meters long.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter.

Answer: $700 \text{ meters} / 150 \text{ millimeters} = 4.66 \text{ meters} / \text{millimeter}$. To two sig.fig this becomes 4.7 meters/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: How long is each of the three wings of the Bellagio Hotel in meters?

Answer: About 25 millimeters on the image or $25 \text{ mm} \times (4.7 \text{ meters/mm}) = 120 \text{ meters}$.

Question 2: What is the length of a car on the street in meters?

Answer: About 1 millimeter on the image or $1 \text{ mm} \times 4.7 \text{ meters/mm} = 4.7 \text{ meters}$.

Question 3: How wide are the streets entering the main intersection?

Answer: About 8 millimeters on the image or $8 \text{ mm} \times 4.7 \text{ meters/mm} = 37 \text{ meters}$.

Question 4: What is the smallest feature you can see, in meters?

Answer: Some of the small dots on the roof tops are about 0.2 millimeters across which equals 1 meter.

Question 5: What kinds of familiar objects can you identify in this image?

Answer: Will vary depending on student.

1. Cars, busses
2. Swimming pools and reflecting ponds
3. Trees
4. Lane dividers
5. Shadows of people walking across the plaza to the Hotel.

Note: Ask the students to use image clues to determine the time of day (morning, afternoon, noon); Whether it is rush-hour or not; Time of year, etc.



This is a picture taken by International Space Station astronauts of Washington, DC, and can be found among many other pictures at <http://eol.jsc.nasa.gov/Coll/EarthObservatory/PostedSort.htm>. The bridge at the bottom-center of the image is the George Mason Bridge (1) and it is 0.75 kilometers from end to end across the main part of the Potomac River (2).

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. It is the most important number to determine because without it, you don't know how big the objects in the image are!

Step 1: Measure the length of the George Mason Bridge with a metric ruler. How many millimeters long is the image of the bridge?

Step 2: The information in the introduction says that the bridge is actually 0.75 kilometers long. Convert this number into meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: About what is the distance between the US Capitol Building (3) and the Washington Monument (4)?

Question 2: About how wide are the major boulevards and roadways?

Question 3: About how wide is the Potomac River?

Question 4: How big is the smallest feature you can measure, and what do you think it is?

Question 5: How big is the area covered by this image in kilometers rounded to the nearest tenth?

Question 6: What other features can you recognize in this image?

You can use GOOGLE-Earth to help find other interesting landmarks in the image!

Answer Key:

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. It is the most important number to determine because without it, you don't know how big the objects in the image are!

It is highly recommended that students use GOOGLE-Earth and dial-in 'Washington DC' to zoom-in on this area in higher resolution. They can use the various tools to bring up the labels for roads, buildings and geographic features.

Step 1: Measure the length of the Mason Bridge with a metric ruler. How many millimeters long is the image of the bridge? **Answer: 15 millimeters**

Step 2: The information in the introduction says that the bridge is actually 0.75 kilometers long. Convert this number into meters. **Answer: 750 meters**

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: The image scale is 50 meters/mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: About what is the distance between the US Capitol Building and the Washington Monument?
Answer: 72 millimeters on the image x 50 meters/mm = 3,600 meters or 3.6 kilometers.

Question 2: About how wide are the major boulevards and roadways?
Answer: The thick black lines are about 1.0 millimeter wide or 1.0mm x 50 meters/mm = 50 meters.

Question 3: About how wide is the Potomac River?
Answer: The river banks are about 12 millimeters apart along most of the river, so their true width is 12 mm x 50 meters/mm = 600 meters or 0.6 kilometers.

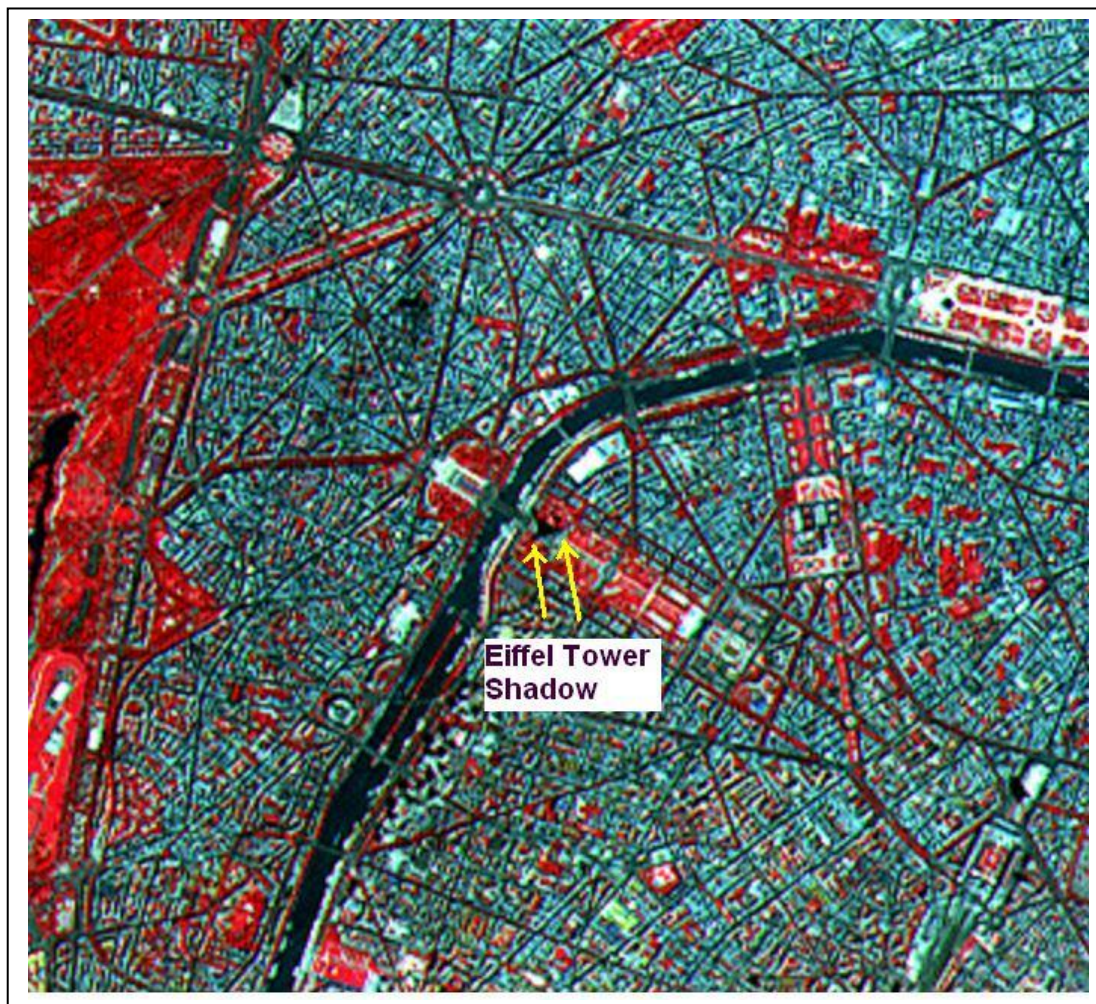
Question 4: How big is the smallest feature you can measure...and what do you think it is?
Answer: Students should be able to find many buildings that look like white spots with barely a square shape. These would be about 1 millimeter wide or 50 meters in true physical size.

Question 5: How big is the area covered by this image in kilometers rounded to the nearest tenth?
Answer; The field is 169 millimeters by 97 millimeters which is 8.5 kilometers x 4.9 kilometers in true size.

Question 6: What other features can you recognize in this image?
Answer: Students should be able to figure out the following features without using GOOGLE:

1. Rivers and waterways
2. Large and small buildings
3. Major boulevards
4. Minor streets
5. Bridges
6. Areas with trees and plant life

The Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) is one of five Earth-observing instruments launched December 18, 1999, on NASA's Terra satellite (<http://modis.gsfc.nasa.gov/gallery/>). The infrared image below is of Paris taken on July 23, 2000. The famous Seine River snakes through the image from left to right, and appears black because water is colder than the surrounding landscape. Vegetation appears red and buildings appear blue-gray because of the color-coding that was used to represent warm (red) and cold (blue) areas.



Problem 1 - This image is 5.6 kilometers wide. Using a metric ruler, what is the scale of the image in meters/mm?

Problem 2 - What is the width, in meters, of; A) The Seine River? B) The traffic circle (Arc de Triomphe)

Problem 3 - The shadow of the Eiffel Tower can be seen as the triangular spot between the two arrows. Measure the length of the shadow indicated by the arrows, and from the solar elevation angle of 60 degrees at the time of the photograph. Create a scaled drawing of the tower, the shadow and the solar angle. A) From your drawing, what is the height of the Eiffel Tower in meters? B) At what sun angle would the shadow be 311 meters long?

Answer Key

Problem 1 - This image is 5.6 kilometers wide. Using a metric ruler, what is the scale of the image in meters/mm? **Answer: The width is 140 mm, so the scale is $5600/140 = 40$ meters/mm.**

Problem 2 - What is the width, in meters, of

A) the Seine River? **Answer: About 5 mm or 200 meters.**

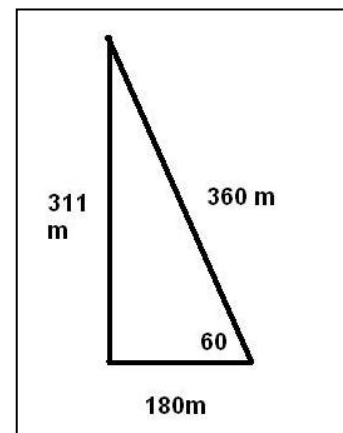
B) The traffic circle (Arc de Triomphe) **Answer: About 7mm or 280 meters.**

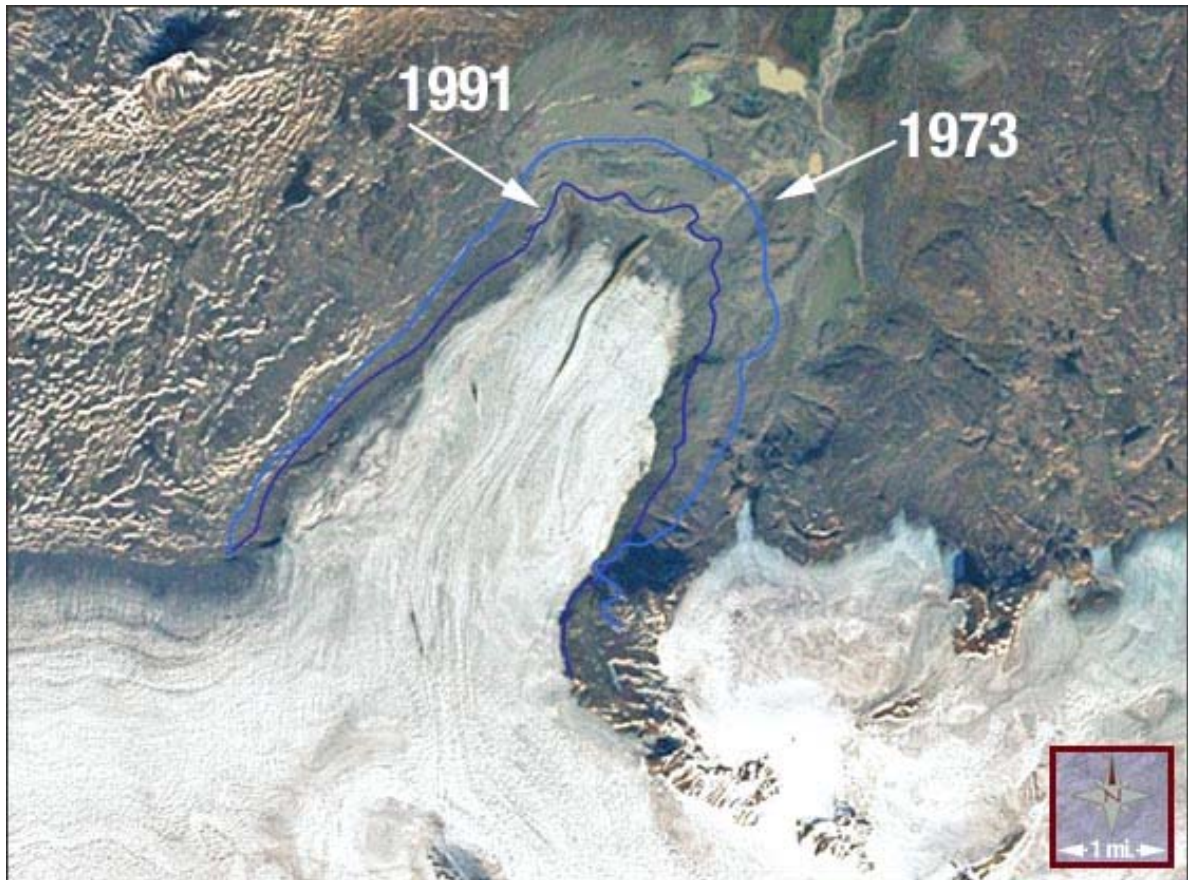
Problem 3 - The shadow of the Eiffel Tower can be seen in this ASTER image as the triangular spot between the two arrows. Measure the length of the shadow indicated by the arrows, and from the solar elevation angle of 60 degrees at the time of the photograph. Create a scaled drawing of the tower, the shadow and the solar angle. A) From your drawing, what is the height of the Eiffel Tower in meters? B) At what sun angle would the shadow be 311 meters long?

Answer: A) The sketch below gives this information. The length of the shadow is about 4.5 mm or 180 meters. Students may carefully draw a scaled triangle using $1\text{cm} = 10$ meters, so that the shadow (base of triangle) is 18cm long. The hypotenuse will then be $180\text{m} \times (2) = 360$ meters or 36 cm. The vertical side, which is the height of the Eiffel Tower will be $180\text{m} \times (3)^{1/2} = 180\text{m} \times 1.732 = 311$ meters. The actual height of the Eiffel Tower is 300 meters to the roof and 320 meters including the radio spire at its top, so the student's estimate from the shadow length is reasonably accurate, given the resolution of the photo.

B) Answer: If the height of the tower is the same as the shadow length, the angle must be 45 degrees.

The photograph below shows a more familiar view of this historic structure. (Courtesy FreeDigitalPhotos.net)





The Eyjabakkajökull Glacier is an outlet glacier of the Vatnajökull ice cap in Iceland. It has been retreating since a major surge occurred in 1973. This true-color Landsat-7 image shows the glacier terminus in September 2000. The light- and dark-blue outlines show the terminus extent in 1973 and 1991, respectively.

Problem 1 - Using a metric ruler, and the conversion 1 mile = 1.61 kilometers, what is the scale of the image in meters per millimeter?

Problem 2 - How many kilometers did the glacier retreat between A) 1973 and 1991? B) 1991 and 2000?

Problem 3 - From your answers to Problem 2, what is the average rate of retreat in kilometers per year between A) 1973-1991, and B) 1991 to 2000? C) Is the retreat of the glacier speeding up or slowing down?

Problem 4 - Assume that the height of the glacier is 1000 meters. About what volume of ice has been lost between 1973 and 1991 in cubic kilometers, assuming that the missing ice is shaped like a wall?

Answer Key

Problem 1 - Using a metric ruler, and the conversion 1 kilometer = 0.62 miles, what is the scale of the image in meters per millimeter? **Answer; The 1-mile legend on the lower right measures 14mm wide, and since 1 mile = 1.61 km, the scale is $1610 \text{ meters}/14\text{mm} = 115 \text{ meters}/\text{mm}$.**

Problem 2 - How many kilometers did the glacier retreat between
 A) 1973 and 1991? **Answer; At the head of the glacier (top end) the distance is 8 mm or $8 \times 115 = 920 \text{ meters}$.**
 B) 1991 and 2000? **Answer: the distance traveled is about 10mm or $10 \times 115\text{m} = 1,150 \text{ meters}$.**

Problem 3 - From your answers to Problem 2 ,what is the average rate of retreat in kilometers per year between A) 1973-1991: **$920 \text{ meters}/18 \text{ years} = 51 \text{ meters}/\text{year}$.** and B) 1991 to 2000? **$1150 \text{ meters}/9 \text{ years} = 128 \text{ meters}/\text{year}$.**
 C) Is the retreat accelerating (speeding up or slowing down?) **Answer: the retreat is definitely speeding up ($51 \text{ m}/\text{yr}$ compared to $128 \text{ m}/\text{yr}$).**

Problem 4 - Assume that the height of the glacier is 1000 meters. About what volume of ice has been lost between 1973 and 1991 in cubic kilometers?

Answer: The height of the wall is 1000 meters. The width of the wall is estimated by using the average width of the retreated ice between 1973 and 2000, which from the photo is about $(5\text{mm} + 8\text{mm} + 20 \text{ mm})/3 = 11 \text{ mm}$ or $11 \times 115 \text{ m}/\text{mm} = 1,300 \text{ meters}$. The length of the wall is the perimeter of the retreating ice which is about $140 \text{ mm} \times 115 \text{ m}/\text{mm} = 16,000 \text{ meters}$. The volume in cubic kilometers is then $1.0\text{km} \times 1.3\text{km} \times 16 \text{ km} = 21 \text{ cubic kilometers}$!

Estimating Biomass Loss from a Large Fire



The fires in Greece during the summer of 2007 devastated large tracks of forest and ground cover in this Mediterranean region. These before (left) and after (right) images were taken on July 18 and September 4 by Landsat-7. The red areas show the extent of the biomass loss from the fires.

Problem 1 - Using a metric ruler, and the conversion 1 mile = 1.61 kilometers, what is the scale of the image in meters per millimeter?

Problem 2 - About what is the total area, in square-kilometers, of this photo of Greece and its surroundings?

Problem 3 - About what was the land area, in square-kilometers, that was burned?

Problem 4 - What percentage of the total area was lost to the fires?

Problem 5 - Suppose that a typical forest in this region contains about 5.0 kilograms of biomass per square meter. How many metric tons of biomass were lost during the fires?

Answer Key

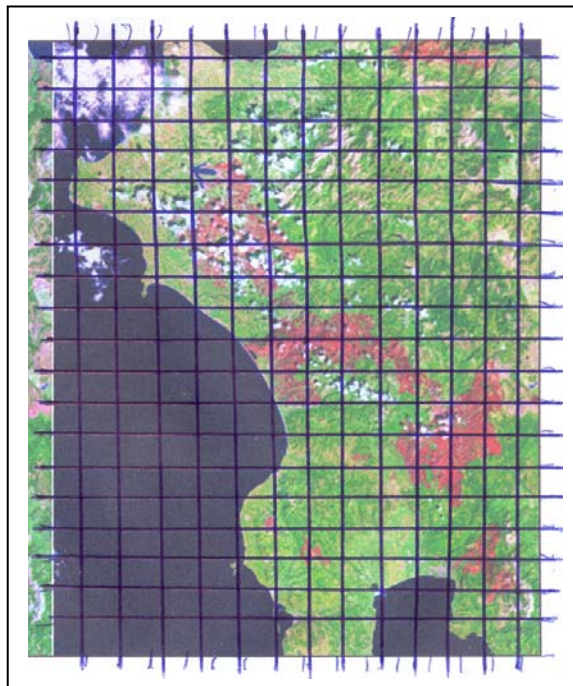
Problem 1 - Answer: The legend on the lower right indicates that 12 miles = 12 millimeters, so in kilometers this becomes $19.4 \text{ km} / 12 \text{ mm} = 1.6 \text{ km/mm}$

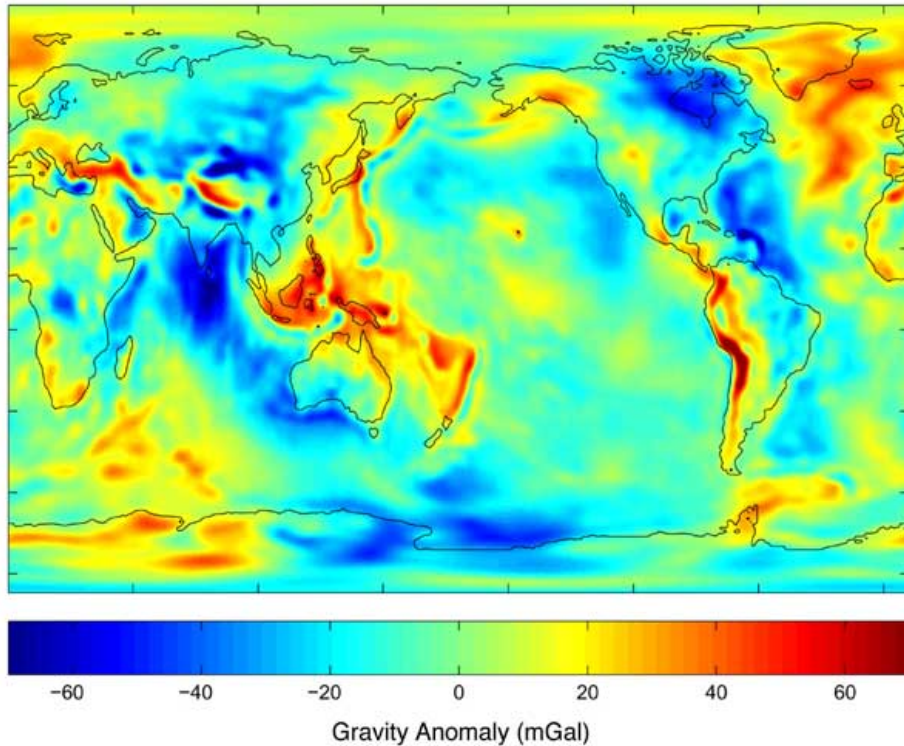
Problem 2 - Answer: The field on the right measures $78 \text{ mm} \times 98 \text{ mm} = 125 \text{ km} \times 157 \text{ km} = 19,700 \text{ km}^2$.

Problem 3 - Answer. To estimate the area of irregular regions, divide the image into a suitable number of smaller squares, for example, 5mm on a side (= 8 km on a side or an area of 64 km^2) as shown in the figure below. The full area has 13 squares across and 19 squares vertically, for a total of 247 cells and a total area of $16,000 \text{ km}^2$. Because the drawn cells are slightly irregular, we can re-calculate their average area as $19,700 \text{ km}^2 / 247 \text{ cells} = 80 \text{ km}^2$. The land area is covered by 173 cells for a total area of $173 \times 80 = 13,800 \text{ km}^2$. The red areas that were burned total about 30 cells or $2,400 \text{ km}^2$. Students answers will vary depending on how they counted the cells. Students may combine their counts and average them to get a more accurate estimate.

Problem 4 - Answer: $100\% \times 2400 / 13800 = 17\%$

Problem 5 - Answer: $5.0 \text{ kg/m}^2 \times (1,000,000 \text{ m}^2/\text{km}^2) \times 2,400 \text{ km}^2 = 12 \text{ billion kg}$ or 12 million metric tons.





The joint NASA-German Aerospace Center Gravity Recovery and Climate Experiment (GRACE) mission has created the most accurate map yet of Earth's gravity field. The map shows how the acceleration of gravity at Earth's surface varies from the standard $g=9.8067$ meters/sec² (32 feet/sec²) in units of milliGals. One thousand milliGals equals 9.8067 meters/sec², so 1 milliGal = 0.0098067 meters/sec². In the map, for example, an orange color means the acceleration of gravity, g , is +40 mGals larger than 9.8067 or $g= (9.8067 + 20(0.0098)) = +10.0027$ meters/sec². Regions where the crust is dense, or rich in iron deposits, will tend to have higher than average strengths.

Problem 1 - About what is the average acceleration of gravity across the continental United States?

Problem 2 - About where is the red 'gravity anomaly' located in the continental United States?

Problem 3 - The period of a 1-meter pendulum in seconds, T , is given by the formula $T^2 = 4\pi^2 L / g$ where g is the acceleration of gravity in meters/sec². From the map value for g and $L = 1.0$, what is T for a pendulum in: A) California? B) Hawaii? C) The middle of the Indian Ocean?

Problem 1 - About what is the average acceleration of gravity across the continental United States?

Answer: The average color is light-blue which corresponds to about -20 milliGals or -0.020 g which equals a difference of $-0.020 \times 9.8067 \text{ m/sec}^2$ or -0.196 m/sec^2 . The total acceleration is then $9.8067 - 0.196 = 9.611 \text{ m/sec}^2$.

Problem 2 - About where is the red 'gravity anomaly' located in the continental United States?

Answer: **The gravity anomaly is in Utah.** Students may attempt a more exact answer from scaled measurements and 'triangulation' with other geographic features in the GRACE map.

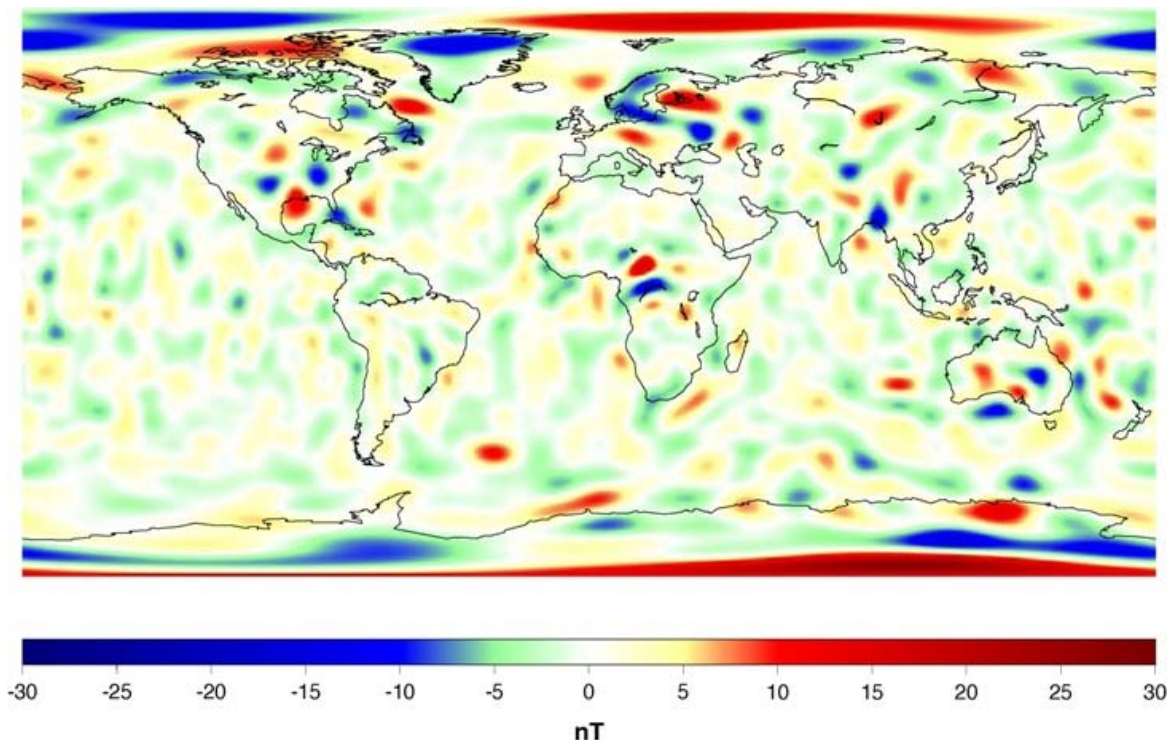
Problem 3 - The period of a 1-meter pendulum in seconds, T , is given by the formula

$T^2 = 4\pi^2 L / g$ where g is the acceleration of gravity in meters/sec². From the map value for g and $L = 1.0$, what is T for a pendulum in: A) California? B) Hawaii? C) The middle of the Indian Ocean?

Answer: A) In California on the map, the average color is light-blue which corresponds to about -20 milliGals or -0.020 g which equals a difference of $-0.020 \times 9.8067 \text{ m/sec}^2$ or -0.196 m/sec^2 . The total acceleration is then $9.8067 - 0.196 = 9.611 \text{ m/sec}^2$. From the formula $T^2 = 39.463 / g$ we get $T^2 = 39.463/9.611 = 4.106$ and so taking the square-root we get **$T = 2.026$ seconds.**

B) Hawaii is the small red dot on the map in the middle of the Pacific Ocean. The average color is red which corresponds to about +60 milliGals or +0.060 g which equals a difference of $+0.060 \times 9.8067 \text{ m/sec}^2$ or $+0.588 \text{ m/sec}^2$. The total acceleration is then $9.8067 + 0.588 = 10.395 \text{ m/sec}^2$. From the formula $T^2 = 39.463 / g$ so $T^2 = 39.463/10.395 = 3.796$ and so taking the square-root we get **$T = 1.948$ seconds.**

C) The average color is dark-blue which corresponds to about -60 milliGals or -0.060 g which equals a difference of $-0.060 \times 9.8067 \text{ m/sec}^2$ or -0.588 m/sec^2 . The total acceleration is then $9.8067 - 0.588 = 9.219 \text{ m/sec}^2$. From the formula $T^2 = 39.463 / g$ so $T^2 = 39.463/9.219 = 4.281$ and so taking the square-root we get **$T = 2.069$ seconds.**



The map was constructed using data collected from a variety of different spacecraft orbiting about 400 km above the Earth, including NASA's Magsat mission and Polar Orbiting Geophysical Observatory, the German CHAMP satellite, and the Danish Oersted satellite. The average magnetic field of Earth's surface has a strength of 70,000 nanoTeslas and is shown as a white color in the map scaling. The map shows variations in Earth's surface magnetism so that a variation of +30 nanoTeslas (dark red) means an actual surface strength of $70,000 + 30 = 70,030$ nanoTeslas. The variations are related to deposits of iron-rich ores in the lithosphere.

For the following problems, use a Mercator map of the Earth to determine latitude, longitude coordinates and distances.

Problem 1 - About how large, in kilometers, are the magnetic anomalies that can be detected in this map?

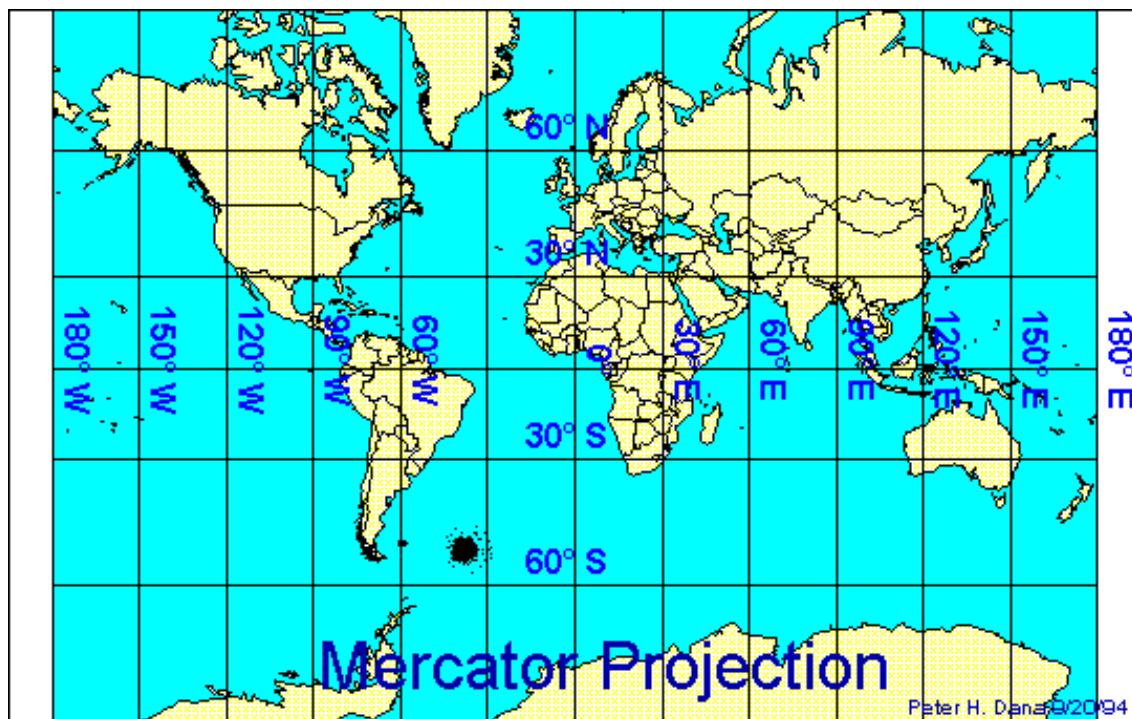
Problem 2 - What European country has the largest magnetic anomaly compared to the area of the country?

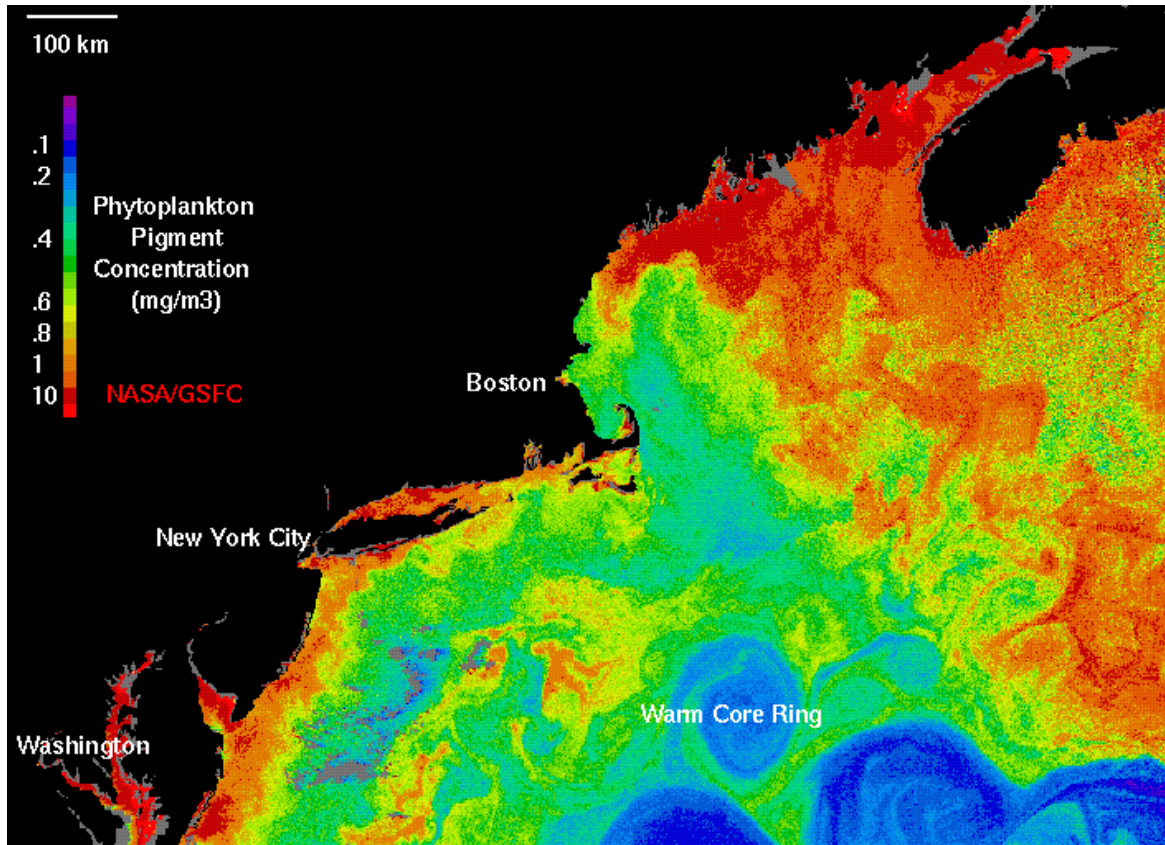
Problem 3 - At about what latitude and longitude is the South Atlantic magnetic anomaly located?

Problem 1 - About how large, in kilometers, are the magnetic anomalies that can be detected in this map? Answer: The red and blue spots are about **300 to 500 kilometers across**

Problem 2 - What European country has the largest magnetic anomaly compared to the area of the country? Answer: **Norway** is nearly completely covered by a magnetic anomaly with a strength of -30 nanoTeslas. This mountainous country is known for its iron mining.

Problem 3 - At about what latitude and longitude is the South Atlantic magnetic anomaly located? Answer: Students may estimate the latitude and longitude by comparing the anomaly map with a conventional map that includes a coordinate grid as shown below. The approximate location of the magnetic anomaly is indicated by the dark spot. The coordinates are approximately Latitude = -50 degrees and Longitude = 40 degrees West. This is a relatively barren part of the Atlantic Ocean.





The purpose of the NASA Sea-viewing Wide Field-of-view Sensor (SeaWiFS) Project is to provide quantitative data on global ocean bio-optical properties to the Earth science community. Subtle changes in ocean color signify various types and quantities of marine phytoplankton (microscopic marine plants), the knowledge of which has both scientific and practical applications. Since an orbiting sensor can view every square kilometer of cloud-free ocean every 48 hours, satellite-acquired ocean color data constitute a valuable tool for determining the abundance of ocean biota on a global scale and can be used to assess the ocean's role in the global carbon cycle.

Problem 1 - The above map gives the concentration of phytoplankton in units of milligrams per cubic meter of water. Near the coastline of the eastern United States, the concentration is about 10 milligrams per cubic meter. How much plankton, in kilograms, could you harvest by processing 1 billion gallons of seawater every day? (1 gallon equals 3.78 liters).

Problem 2 - In which areas would you most expect to find whales and other aquatic life?

Problem 3 - How far to the east of Cape Cod do fishing boats have to travel before they encounter areas where fishing might be economically profitable?

Problem 1 - The above map gives the concentration of phytoplankton in units of milligrams per cubic meter of water. Near the coastline of the eastern United States, the concentration is about 10 milligrams per cubic meter. How much plankton, in metric tons, could you harvest by processing 1 billion gallons of seawater every day? (1 gallon equals 3.78 liters).

Answer: There are several approaches to this problem. First convert cubic meters to gallons, then convert the plankton concentration to grams per gallon, then multiply by the number of gallons and convert to metric tons.

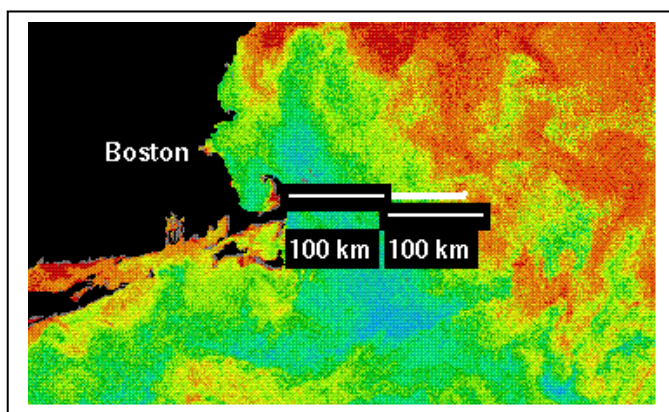
$1 \text{ m}^3 \times (1,000,000 \text{ cm}^3/\text{m}^3) \times (1 \text{ liter}/1000 \text{ cm}^3) \times (1 \text{ gallon}/3.78 \text{ liters}) = 265 \text{ gallons}$. Then $10 \text{ milligrams}/\text{m}^3 \times (1 \text{ m}^3/265 \text{ gallons}) = 0.000038 \text{ grams}/\text{gallon}$. Then $0.000038 \text{ grams}/\text{gallon} \times 1 \text{ billion gallons} = 38,000 \text{ grams}$ or **38 kilograms**.

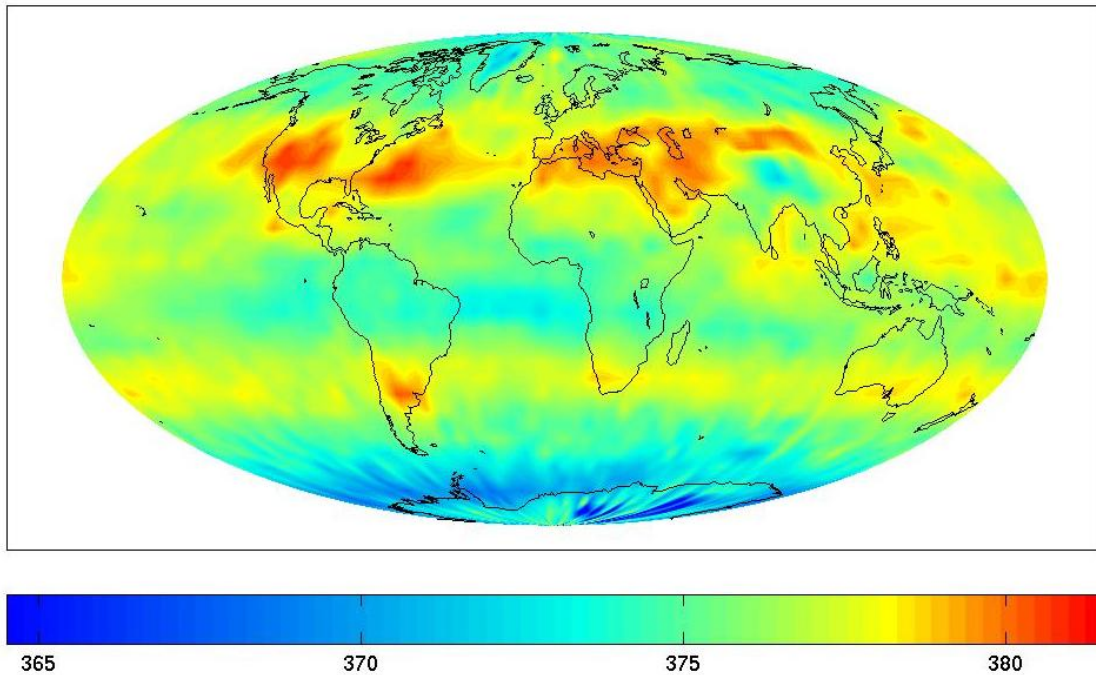
Note: One average-sized swimming pool contains about 700,000 gallons of water!

Problem 2 - In which areas would you most expect to find whales and other aquatic life?

Answer: The areas of the highest plankton concentration are coded in red. Plankton consists of embryonic and young life forms, and whose adult forms are often co-located with the plankton. Also, plankton is on the lowest rung of the food chain and is a major food source for aquatic animals, so **we would expect that the distribution of plankton in the ocean closely matches the distribution pattern of living things that feed on the plankton**. By measuring plankton concentration from space, we can also measure, and keep track of, the location of various aquatic biomes.

Problem 3 - How far to the east of Cape Cod do fishing boats have to travel before they encounter areas where fishing might be economically profitable? Answer: From the map below, and the scale bar that indicates '100 kilometers', we see that they have to travel about 150 to 200 kilometers to enter the densest plankton concentrations. This region is called Georges Bank, and fishermen have to consume quite a lot of diesel fuel to get their boats out to where they can start fishing, usually under hazardous conditions.





The Atmospheric Infrared Sounder (AIRS) instrument on NASA's Aqua spacecraft has been used by scientists to observe atmospheric carbon dioxide. The above map shows the concentrations of atmospheric carbon dioxide in units of 'parts per million', and range from 363 ppm (dark blue) to 380 ppm (red). The data was obtained in July 2003, and the gas is at an altitude of 8 kilometers. The map shows that carbon dioxide is not evenly mixed in the atmosphere, but there are regional differences that change in time. For example, the red 'clouds' move in time and change size and shape.

Problem 1 - From the color bar, about what is the average concentration of carbon dioxide across the globe, in ppm, not including the orange or red areas?

Problem 2 - What is the difference in ppm between your answer to Problem 1, and the highest levels of concentration?

Problem 3 - At these altitudes, atmospheric winds generally blow from west to east (left to right on the map). What geographic regions are nearest the highest concentrations of carbon dioxide in this map?

Problem 4 - The average mass of carbon dioxide in the atmosphere, at a concentration of 1 ppm equals 15 tons per square kilometer. How many tons/km² are represented by the: A) Red color? B) Yellow color? and C) The difference between red and yellow?

Problem 1 - From the color bar, about what is the average concentration of carbon dioxide across the globe, in ppm, not including the orange or red areas?

Answer: Most of the areas are yellowish, but the rest is light blue, so according to the color bar, the concentrations is about **375 ppm**...very roughly.

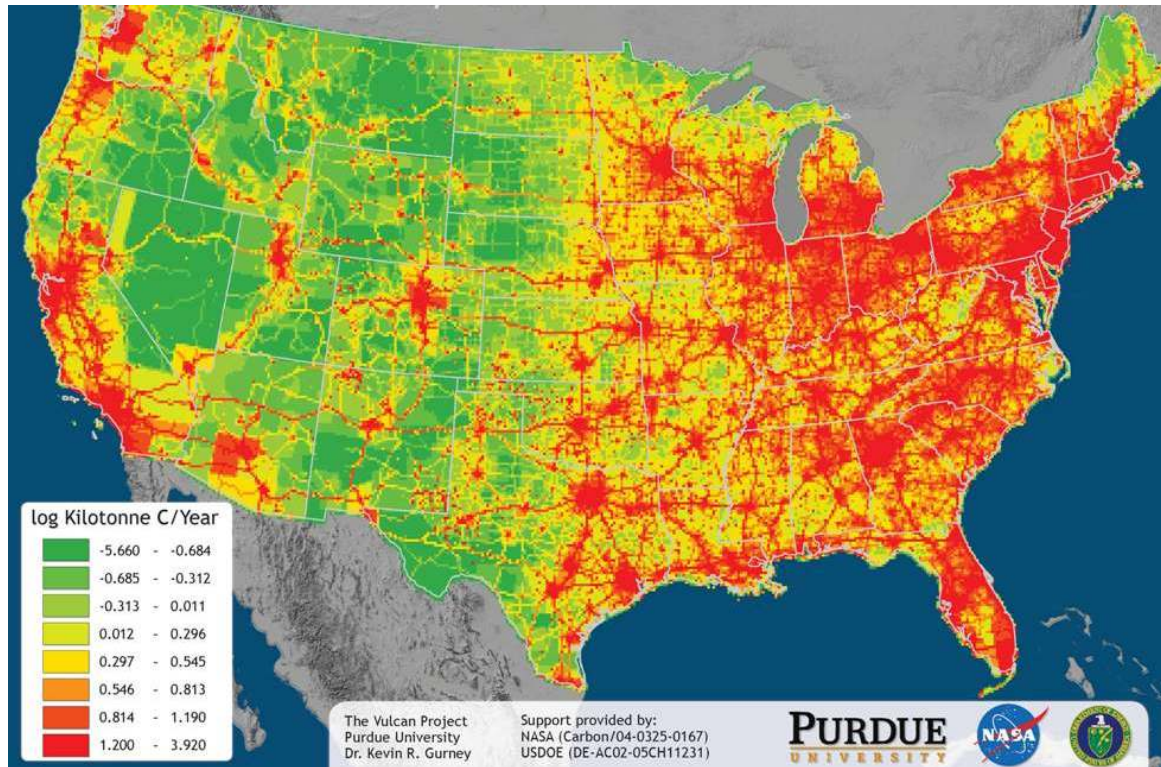
Problem 2 - What is the difference in ppm between your answer to Problem 1, and the highest levels of concentration? Answer: The darkest reds are near 382 ppm, so the difference is **about 7 ppm**.

Problem 3 - At these altitudes, atmospheric winds generally blow from west to east (left to right on the map) in the Northern Hemisphere. What geographic regions are nearest the highest concentrations of carbon dioxide in this map? Answer: **Western United States, the East Coast of the US, and regions in the Mediterranean and mid-East, some of which are 'downwind' from Europe.**

Problem 4 - The average mass of carbon dioxide in the atmosphere, at a concentration of 1 ppm equals 15 tons per square kilometer. How many tons/km² are represented by the: A) Red color? B) Yellow color? and C) The difference between red and yellow?

Answer: A) The color bar says that red = 382 ppm so the concentration equals $382 \times 15 = 5730 \text{ tons/km}^2$. B) yellow = 378 ppm so the concentration equals $378 \times 15 = 5670 \text{ tons/km}^2$. C) the difference is $5730 - 5670 = 60 \text{ tons/km}^2$ of additional CO₂.

Teacher Note: Most studies suggest that human activity add about 25 gigatons per year. Since the NASA map represents an average over a month, we see that our very crude estimate of $2.3 \text{ gigatons/month} \times 12 \text{ months} = 28 \text{ gigatons/year}$ which is close to more detailed estimates. This similarity may, however, be accidental since certain approximations had to be used in deriving our estimate that would not be made in the more careful studies. Also, the satellite only observed the carbon dioxide at an altitude of 8 kilometers, not all of the additional carbon dioxide down to sea-level.



The image above shows the total carbon dioxide emissions in the 48 States of the United States. The green areas represent an average annual production rate of 6 tons/km². The yellow areas produce about 50 tons/km² each year, and the red areas correspond to 450 tons/km² produced each year. The total US fossil fuel CO₂ emissions for 2002 come to 1.6 gigatons of carbon each year.

Problem 1 - The population, in 2002, was 280 million people. About what was the average carbon dioxide production per person in 2002?

Problem 2 - The total area of the 48-states is about 9.8 million km². From the map, estimate the fraction of the US area covered in green, yellow and red. Based on the color scaling and the total area of the 48-states, what would be your estimate for the total carbon dioxide production by the 48-states during 2002?

Problem 3 - From the published data for the total production, what was the average carbon dioxide production rate per square kilometer for the 48-states?

Problem 4 - Based on the features in the map, explain how the map shows that human activity produces most of the excess carbon dioxide generated by the United States.

Problem 1 - The population, in 2002, was 280 million people. About what was the average carbon dioxide production per person in 2002? Answer; 1,600 million tons/280 million people = **5.7 tons per person per year**.

Problem 2 - The total area of the 48-states is about 9.8 million km². From the map, estimate the fraction of the US area covered in green, yellow and red. Based on the color scaling and the total area of the 48-states, what would be your estimate for the total carbon dioxide production by the 48-states during 2002?

Answer: If the students estimate the each color occupies 1/3 of the land area, they will get

Green = $1/3 \times 9.8 \text{ million km}^2 \times 6 \text{ tons/km}^2 = 20 \text{ million tons}$;

Yellow = $1/3 \times 9.8 \text{ million km}^2 \times 50 \text{ tons/km}^2 = 163 \text{ million tons}$; and

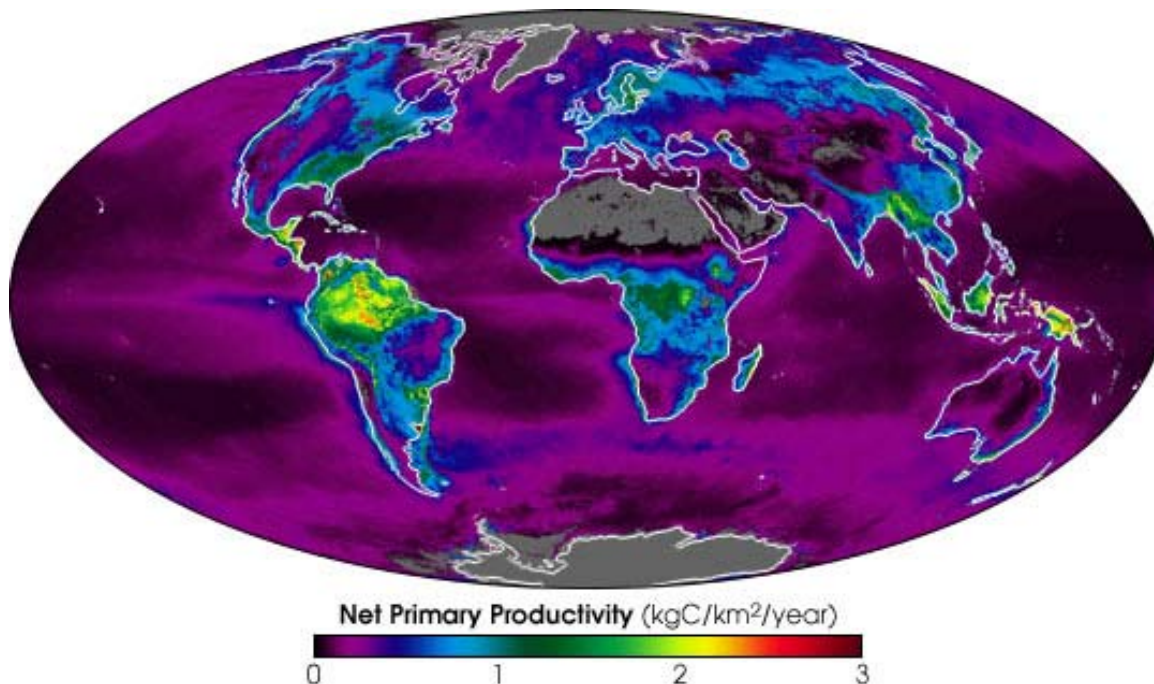
Red = $1/3 \times 9.8 \text{ million km}^2 \times 450 \text{ tons/km}^2 = 1,470 \text{ million tons}$.

The total would be 1,653 megatons or **1.7 gigatons**, which is about the same as the published value for 2002. Students estimates from the map areas may vary.

Problem 3 - From the published data for the total production, what was the average carbon dioxide production rate per square kilometer for the 48-states? Answer: 1,600 million tons/9.8 million km² = **163 tons/km²**.

Problem 4 - Based on the features in the map, explain how the map shows that human activity produces most of the excess carbon dioxide generated by the United States.

Answer: Students should note that the highest-production regions (red) show clustering near major cities, heavily populated states, and linear features that indicate major highways and communities adjacent to these highways.



NASA scientists unveiled the first consistent and continuous global measurements of Earth's "metabolism" based on data from the Terra and Aqua satellites. This new measurement is called Net Primary Production because it indicates how much carbon dioxide is taken in by vegetation during photosynthesis minus how much is given off during respiration. The false-color map shows the rate at which plants absorbed carbon out of the atmosphere during the years 2001 and 2002. The yellow and red areas show the highest rates of absorption, ranging from 2 to 3 kilograms of carbon taken out of the atmosphere per square kilometer per year. The green areas are intermediate rates, while blue and purple shades show progressively lower productivity.

Problem 1 - According to the map, which regions are the most productive in removing carbon from the atmosphere?

Problem 2 - Assume that the Amazon Basin has an area of 7 million square kilometers. How many tons of carbon does it remove from the atmosphere each year?

Problem 3 - The oceans cover an area of 335 million square kilometers. What is the average rate of carbon removal according to the map color, and how many tons of carbon is removed by plant life on the oceans?

Problem 4 - The mass of carbon dioxide is 3.7 times more than pure carbon. How many tons of carbon dioxide do your answers to Problem 2 and 3 represent?

Problem 1 - According to the map, which regions are the most productive in removing carbon from the atmosphere? Answer: The most productive places on Earth show up as a yellow color on the map. These are found, in large quantities in the Amazon Basin and in Indonesia.

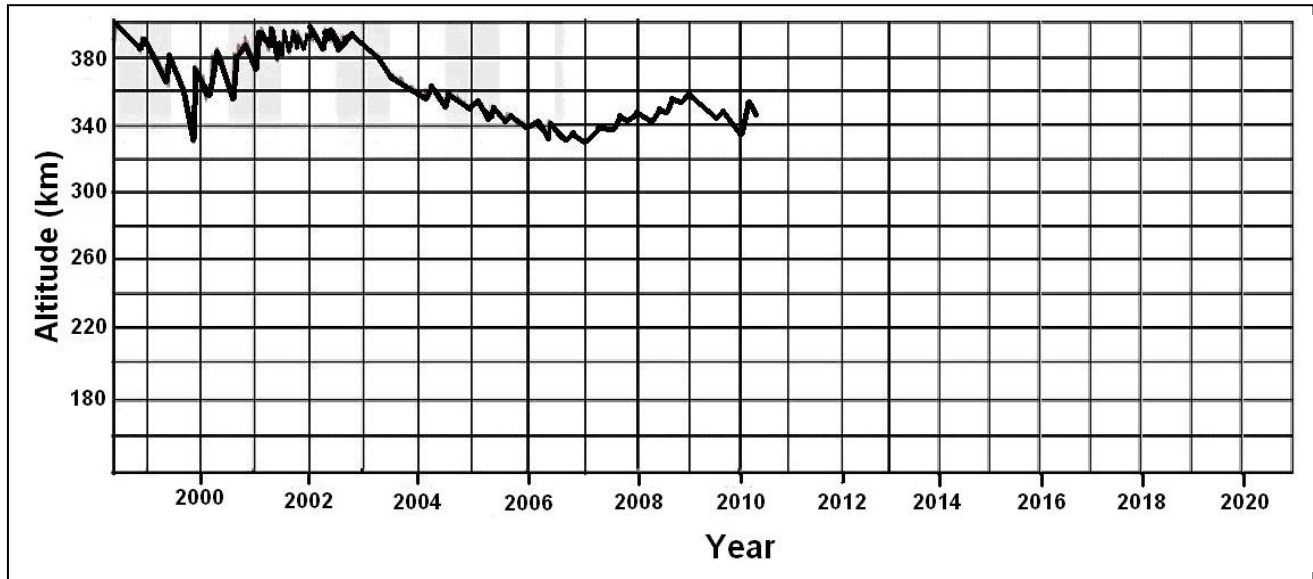
Problem 2 - Assume that the Amazon Basin has an area of 7 million square kilometers. How many tons of carbon does it remove from the atmosphere each year? Answer: Its yellow color indicates a carbon removal rate of about 2 kg per square kilometer per year, so the total removal is 14 million kg per year. This equals **14,000 tons of carbon per year**.

Problem 3 - The oceans cover an area of 335 million square kilometers. What is the average rate of carbon removal according to the map color, and how many tons of carbon is removed by plant life on the oceans? Answer: The average color is magenta, which corresponds to a rate of about 0.3 kg per square kilometer per year, so the removal rate from the world's oceans is about 0.3×335 million = 100 million kilograms/year or 100,000 tons/year of carbon.

Problem 4 - The mass of carbon dioxide is 3.7 times more than pure carbon. How many tons of carbon dioxide do your answers to Problem 2 and 3 represent?

Answer: $14,000$ tons of carbon/year $\times 3.7 =$ **52,000 tons/year** from the Amazon Basin, and $100,000$ tons/year $\times 3.7 =$ **370,000 tons/year** of carbon dioxide from the world's oceans.

At the present time, the International Space Station is losing about 300 feet (90 meters) of altitude every day. Its current altitude is about 345 km after a 7.0-km re-boost by the Automated Transfer Vehicle, Jules Vern spacecraft on June 20, 2008. The graph below shows the ISS altitude since 1999.



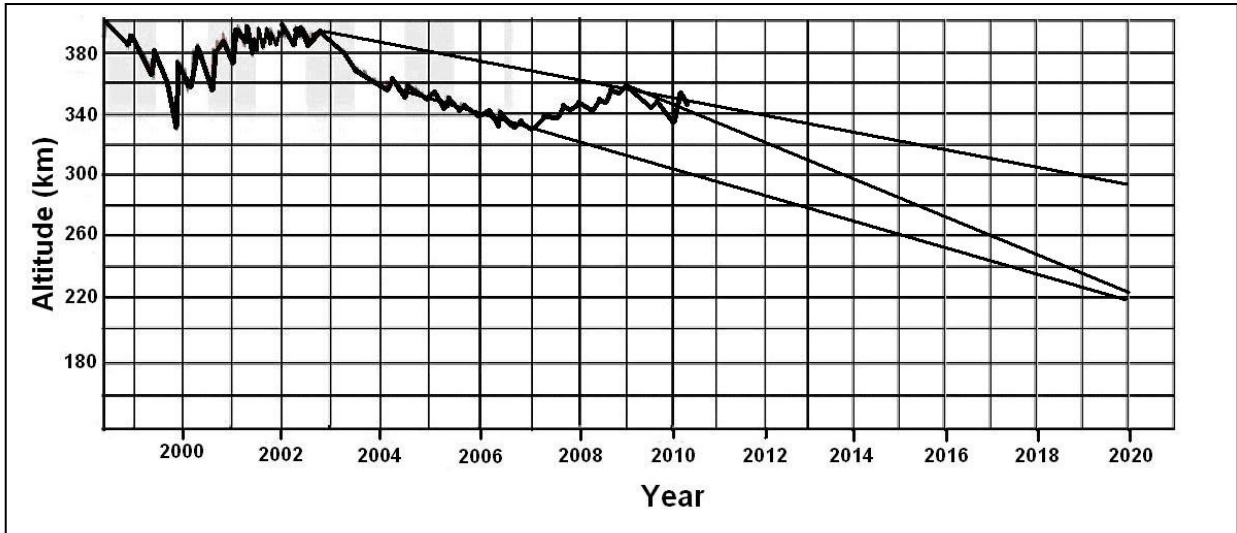
The drag of Earth's atmosphere causes the ISS altitude to decrease each day, and this is accelerated during sunspot maximum (between 2000-2001) when the dense atmosphere extends to a much higher altitude. At altitudes below about 200 km, spacecraft orbits decay and burn up within a week.

Problem 1 - From the present trends, what do you expect the altitude of the ISS to be between 2010 until its retirement year around 2020?

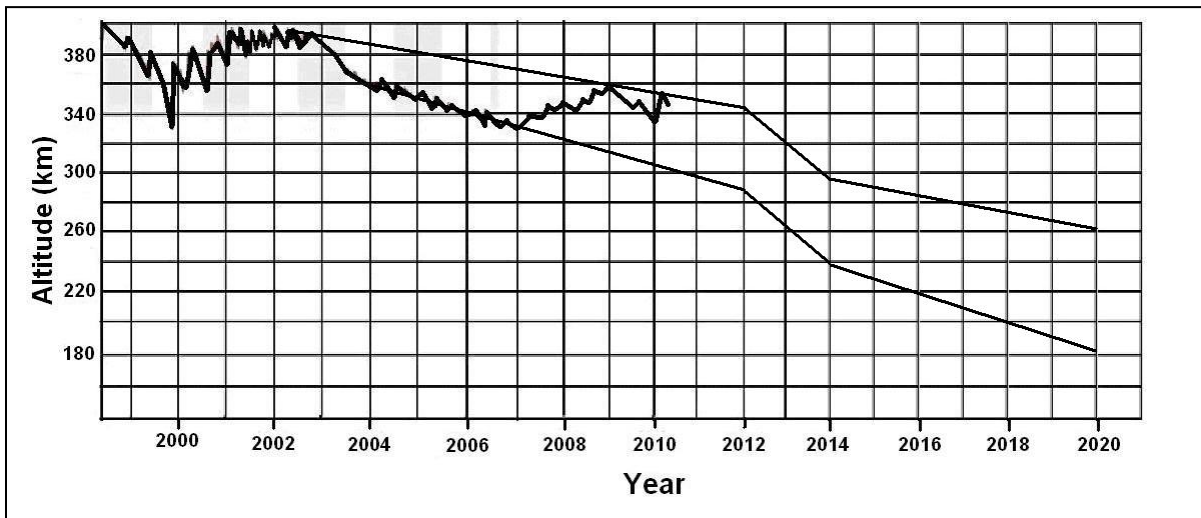
Problem 2 - Sunspot maximum will occur between 2012-2014, and we might expect a 50-km decline in altitude during this period if the solar activity weaker than the peak in 2000, which is currently forecasted. Including this effect, what might be the altitude of the ISS in 2020? Is the ISS in danger of atmospheric burn-up?

Problem 3 - What are the uncertainties in predicting ISS re-entry, and what strategy would you use if you were the Program Manager for the ISS?

Problem 1 – Answer: The graph below shows several plausible linear trends depending on which features you use as a model for the slope. The predicted altitude would be between 220 and 300 km .



Problem 2 - The graph below shows, for example, two forecasts that follow the extremes of the general decline trend, but then assume all of the altitude loss occurred between 2012-2014 at 50 km. Note that the range of altitudes in the graph in either case is 180-260 km. This places the ISS in danger of burn-up before its retirement year in 2014.

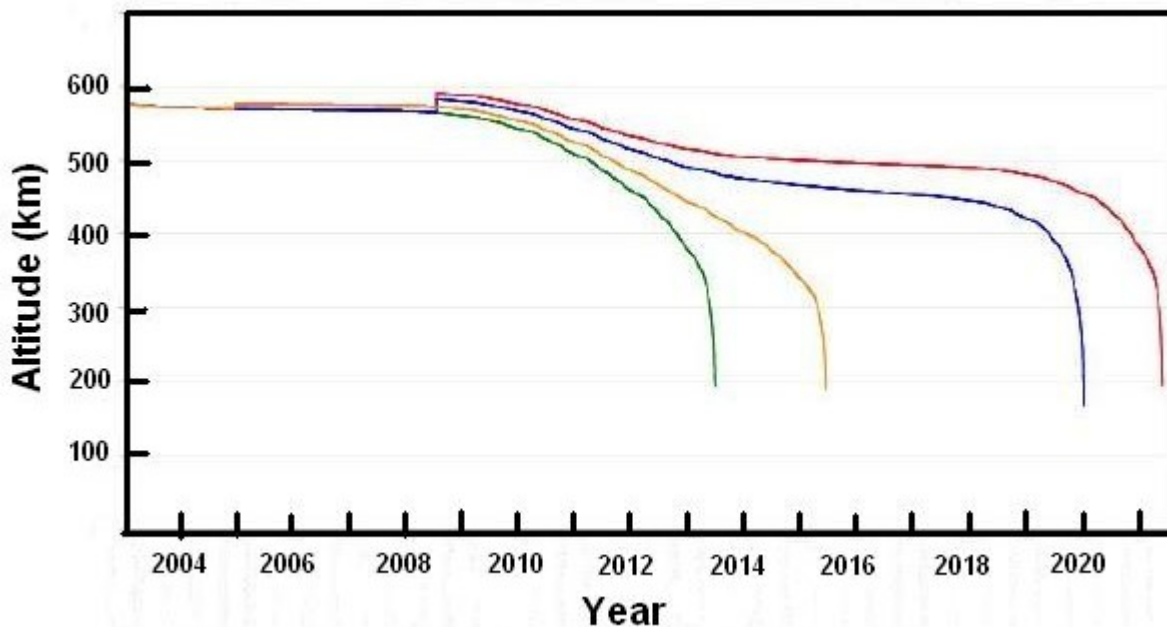


Problem 3 - The largest uncertainty is the strength of the next solar activity (sunspot) cycle. If it is stronger than the previous maximum between 2000-2001, the ISS altitude losses will be even larger in 2012-2014, and the ISS will be in extreme danger of re-entry before 2020.

The period after 2010, when the US loses access to space travel and has to rely on Russian low-capacity shuttles, will be a critical time for the ISS, and an intensely worrisome one for ISS managers.

Satellite Drag and the Hubble Space Telescope

The Hubble Space Telescope was never designed to operate forever. What to do with the observatory remains a challenge for NASA once its scientific mission is completed in ca 2012. Originally, a Space Shuttle was proposed to safely return it to Earth, where it would be given to the National Air and Space Museum in Washington DC. Unfortunately, after the last Servicing Mission, STS-125, scheduled for May, 2009, no further Shuttle visits are planned. In time, the HST orbit will decay, and the satellite will burn up in the atmosphere. The predictions for when this will happen depend on the just how intense the next sunspot cycle will be between 2009-2018. As solar activity increases, the upper atmosphere heats up and expands, causing greater friction for low-orbiting satellites like HST, and a more rapid re-entry. The curves show four re-entry scenarios. Green is with no re-boots at all. Brown is with one, 5-km re-boost in 2004. Blue is with one re-boost in 2009 by 18-km; Red is with one 5-km re boost in 2005 and one 18-km re-boost in 2009.



Problem 1 – The last Servicing Mission in 2009 will only extend the science operations by another 5 years. Which scenario keeps the HST operating just long enough to support the science goals?

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boots, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Problem 1 – The last Servicing Mission in 2009 will only extend the science operations by another 5 years. Which scenario keeps the HST operating just long enough to support the science goals?

Answer: The Servicing Mission will occur in 2009. The upgrades and gyro repairs will extend the satellite's operations by 5 more years, so if it re-enters after 2014 it will have maximized its usefulness. This occurs in the scenario where there is only one re-boost in 2004 and none in 2009, which is the brown curve.

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boasts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Answer: Use a millimeter ruler to determine the scale of the horizontal axis in weeks per millimeter. For the green curve, mark the point on the curve that corresponds to a vertical value of 400 km. Draw a line to the horizontal axis and measure its distance from 2013 in millimeters. Convert this to weeks using the scale factor you calculated.

"HST science lifetime could potentially be limited by HST spacecraft orbital decay. Long-term orbit decay predictions are developed based on atmospheric models and solar flux predictions. All contributing combinations of solar flux strength and timing are run in order to bound the orbit decay predictions from a best case atmosphere to a worst case ("unkind") atmosphere. The predictions also consider the effects of Space Shuttle re-boost during HST Servicing Missions. The figure shows the model results for a worst case, 2-sigma high solar cycle (Cycle 24), followed by an early Cycle 25 of average intensity. Figure 3 depicts four curves for various shuttle re-boost scenarios. For the case of no further HST re-boost in any future servicing mission, the prediction is that HST will reenter the Earth's atmosphere in late 2013 or early 2014. The HST science program will cease approximately one year prior to re-entry due to loss of the precise attitude control capability required for science observing, as the atmospheric drag increases. The earliest expected end of the HST science program due to orbital decay is thus late 2012. Further information about this topic is contained in the accompanying Hubble Fact Sheet, entitled "HST Orbit Decay and Shuttle Re-boost." [From "Expected HST Science Lifetime after SM4", HST Program Office; July 21, 2003]

Period	Age (years)	Days per year	Hours per day
Current	0	365	
Upper Cretaceous	70 million	370	
Upper Triassic	220 million	372	
Pennsylvanian	290 million	383	
Mississippian	340 million	398	
Upper Devonian	380 million	399	
Middle Devonian	395 million	405	
Lower Devonian	410 million	410	
Upper Silurian	420 million	400	
Middle Silurian	430 million	413	
Lower Silurian	440 million	421	
Upper Ordovician	450 million	414	
Middle Cambrian	510 million	424	
Ediacarin	600 million	417	
Cryogenian	900 million	486	

We learn that an 'Earth Day' is 24 hours long, and that more precisely it is 23 hours 56 minutes and 4 seconds long. But this hasn't always been the case. Detailed studies of fossil shells, and the banded deposits in certain sandstones, reveal a much different length of day in past eras! These bands in sedimentation and shell-growth follow the lunar month and have individual bands representing the number of days in a lunar month. By counting the number of bands, geologists can work out the number of days in a year, and from this the number of hours in a day when the shell was grown, or the deposits put down. The table above shows the results of one of these studies.

Problem 1 - Complete the table by calculating the number of hours in a day during the various geological eras. It is assumed that Earth orbits the sun at a fixed orbital period, based on astronomical models that support this assumption.

Problem 2 - Plot the number of hours lost compared to the modern '24 hours' value, versus the number of years before the current era.

Problem 3 - By finding the slope of a straight line through the points can you estimate by how much the length of the day has increased in seconds per century?

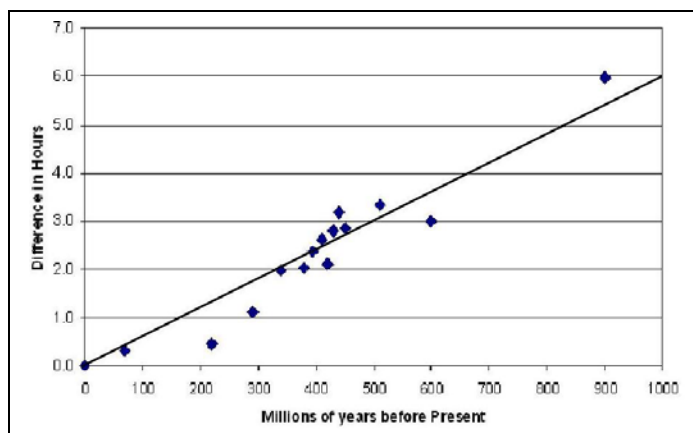
Answer Key

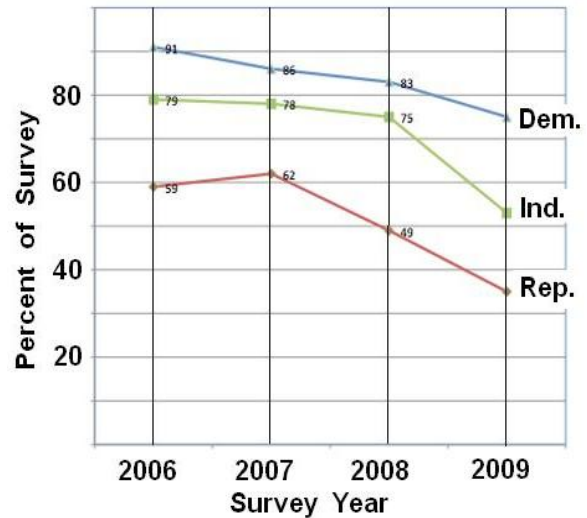
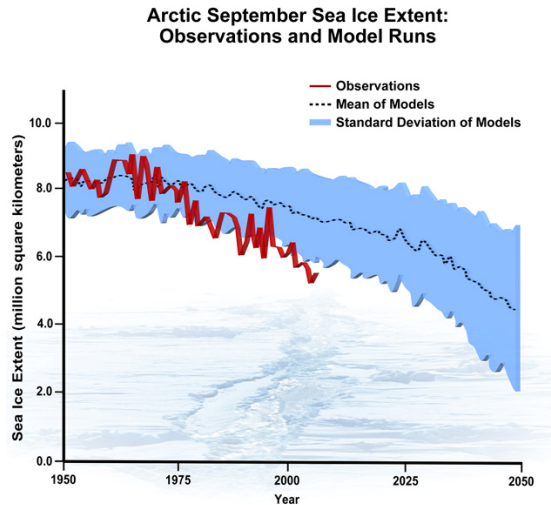
Period	Age (years)	Days per year	Hours per day
Current	0	365	24.0
Upper Cretaceous	70 million	370	23.7
Upper Triassic	220 million	372	23.5
Pennsylvanian	290 million	383	22.9
Mississippian	340 million	398	22.0
Upper Devonian	380 million	399	22.0
Middle Devonian	395 million	405	21.6
Lower Devonian	410 million	410	21.4
Upper Silurian	420 million	400	21.9
Middle Silurian	430 million	413	21.2
Lower Silurian	440 million	421	20.8
Upper Ordovician	450 million	414	21.2
Middle Cambrian	510 million	424	20.7
Ediacarin	600 million	417	21.0
Cryogenian	900 million	486	18.0

Problem 1 - Answer; See table above. Example for last entry: 486 days implies 24 hours \times (365/486) = 18.0 hours in a day.

Problem 2 - Answer; See figure below

Problem 3 - Answer: From the line indicated in the figure below, the slope of this line is $m = (y_2 - y_1) / (x_2 - x_1) = 6 \text{ hours} / 900 \text{ million years}$ or 0.0067 hours/million years. Since there are 3,600 seconds/ hour and 10,000 centuries in 1 million years (Myr), this unit conversion yields $0.0067 \text{ hr/Myr} \times (3600 \text{ sec/hr}) \times (1 \text{ Myr} / 10,000 \text{ centuries}) = 0.0024 \text{ seconds/century}$. This is normally cited as 2.4 milliseconds per century.





The graph above, based upon research by the National Sea Ice Data Center (Courtesy Steve Deyo, UCAR), shows the amount of Arctic sea ice in September (coldest Arctic month) for the years 1950-2006, based on satellite data (since 1979) and a variety of direct submarine measurements (1950 - 1978). The blue region indicates model forecasts based on climate models. Meanwhile, the figure on the right shows the results of polls conducted between 2006 and 2009 of 1,500 adults by the Pew Research Center for the People & the Press. The graph indicates the number of people, in all three major political parties, believing there is strong scientific evidence that the Earth has gotten warmer over the past few decades.

Problem 1 - Based on the red curve in the sea ice graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006?

Problem 2 - Based on the polling data, what are the three linear equations that model the percentage of Democrats (Dem.), Independents (Ind.) and Republicans (Rep.) who believed that strong evidence existed for global warming?

Problem 3 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?

Problem 4 - From your model for the polling data, by about what years will the average American in the Pew Survey, who identifies themselves as Democrats, Independents or Republicans, no longer believe that there is any scientific evidence at all for global warming?

Problem 1 - Based on the red curve in the graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006? Answer: The linear equation will be of the form $y = mx + b$. From the graph, the y-intercept for the actual data is 8.5 million km^2 for 1950. The value for 2006 is 5.5 million km^2 . The slope is $m = (5.5 - 8.5) / (2006 - 1950) = -0.053$, so the equation is given by **$Y = -0.053(x-1950) + 8.5$** in millions of km^2 .

Problem 2 - Based on the polling data, what are the three linear equations that model the percentage of Democrats (Dem.), Independents (Ind.) and Republicans (Rep.) who believed that strong evidence existed for global warming?

Answer:

Dems: $m = (75\% - 90\%)/(2009-2006) = -5.0$, so the model becomes

$$y = -5.0(x - 2006) + 90 \text{ percent ;}$$

Ind. $m = (52\% - 79\%)/(2009-2006) = -9.0$, so the model becomes

$$y = -9.0(x - 2006) + 79 \text{ percent.}$$

Rep. ; $(35\% - 60\%)/(2009-2006) = -8.3$, so the model becomes

$$y = -8.3(x - 2006) + 60 \text{ percent}$$

Problem 3 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?

Answer: In 1950-1975 there were about 8.5 million km^2 of sea ice in September. Half of this is 4.3 million km^2 . Set $y = 4.3$ and solve for x :

Solve $4.3 = -0.053(x-1950) + 8.5$ to get

$$-4.2 = -0.053(x-1950)$$

$$4.2 = 0.053(x-1950)$$

$$4.2/0.053 = x-1950$$

$$79 = x - 1950$$

And so $x = 2029$. So, during the year **2029 AD** there will only be half as much sea ice in the Arctic in September.

Note: If we use only the slope data since 1975 when the ice cover was 8.0 million km^2 , the slope would be $m = (5.5 - 8.0)/(2006-1975) = -0.083$, and linear equation is $y = -0.083(x-1975) + 8.0$. The year when half the ice is present would then be about 2023 AD, because the slope is steeper during the most recent 30 years. If the slope continues to steepen with time, the year when only half the ice is present will move closer to the current year.

Problem 4 - From your model for the polling data, by about what years will the Democrats, Independents and Republicans no longer believe that there is any scientific evidence at all for global warming?

Answer: Solve each linear model in Problem 2 for X , given that $y=0$:

Democrats: $0 = -5.0(x - 2006) + 90$ so $x =$ **2024 AD.**

Independents: $0 = -9.0(x-2006) + 79$ so $x =$ **2015 AD**

Republicans: $0 = -8.3(x-2006) + 60$ so $x =$ **2013 AD.**



NASA's Terra satellite flew over the Deepwater Horizon rig's oil spill in the Gulf of Mexico on Saturday, May 1 and captured the above natural-color image of the slick from space. The oil slick resulted from an accident at the Deepwater Horizon rig in the Gulf of Mexico. NOAA's estimated release rate of oil spilling into the Gulf is 200,000 gallons per day since April 20 when the accident occurred.

Problem 1 – Using a metric ruler, calculate the scale of this image in kilometers/mm.

Problem 2 – What is the approximate area of this oil leak in A) square kilometers? B) square centimeters?

Problem 3 - The estimated quantity of oil covering this area is about 2 million gallons. If one gallon of oil has a mass of 3.0 kg, what is the surface density, S , of oil in this patch in A) Gallons/meter²? B) kg/meter²?

Problem 4 – The density of crude oil is about $D=850 \text{ kg/m}^3$. From your estimate for S , what is the approximate thickness, h , of the oil layer covering the ocean water?

Problem 5 - Suppose that an average 'oil' molecule has a length of about 5 nanometers. About what is the average thickness of this oil layer in molecules if the molecules are lined up end to end?

Problem 1 – Using a metric ruler, calculate the scale of this image in kilometers/cm.

Answer: The '25km' legend mark on the Terra image is 1.7 cm long, so the scale is
 $25 \text{ km} / 1.7 \text{ cm} = \mathbf{15 \text{ km/cm}}$.

Problem 2 – What is the approximate area of this oil leak in A) square meters? B) square centimeters?

Answer: The oil spill is about 6 cm in diameter or $6 \text{ cm} \times 15 \text{ km/cm} = 90 \text{ km}$ in diameter. A) As a circle, the area is $A = \pi (45 \text{ km})^2 = 6,358 \text{ km}^2$ or to 1 significant figures $\mathbf{A = 6,000 \text{ km}^2}$.

B) $A = 6,000 \text{ km}^2 \times (1,000 \text{ m}/1 \text{ km})^2$ so $\mathbf{A = 6.0 \times 10^9 \text{ m}^2}$.

Problem 3 - The estimated quantity of oil covering this area is about 2 million gallons. If one gallon of oil has a mass of 3.0 kg, what is the surface density, S, of oil in this patch in A) Gallons/meter²? B) kg/meter²?

Answer: Mass = 2 million gallons x (3 kg/1 gallon) = 6 million kg. Then

A) $S = 2 \text{ million gallons} / 6.0 \times 10^9 \text{ m}^2$ so $\mathbf{S = 0.0003 \text{ gallons/m}^2}$.

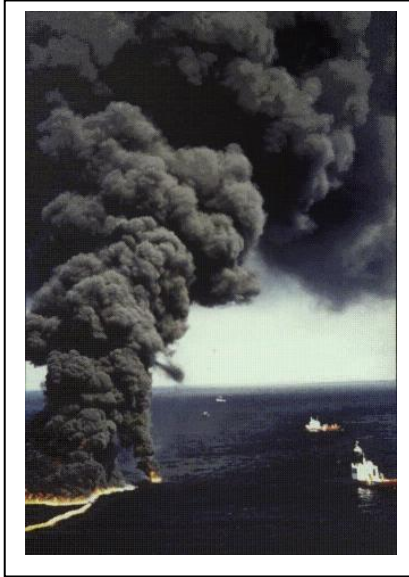
B) $S = 6 \text{ million kg} / 6.0 \times 10^9 \text{ m}^2$ so $\mathbf{S = 0.001 \text{ kg/m}^2}$.

Problem 4 – The density of crude oil is about $D=850 \text{ kg/m}^3$. From your estimate for S, what is the approximate thickness, h, of the oil layer covering the ocean water?

Answer: $h = S/D$ so $h = (0.001 \text{ kg/m}^2)/(850 \text{ kg/m}^3) = \mathbf{1.0 \times 10^{-6} \text{ meters}}$ (or 1 micron)

Problem 5 – Suppose that an average 'oil' molecule has a length of about 5 nanometers. About what is the average thickness of this oil layer in molecules if the molecules are lined up end to end?

Answer: Assuming that these cylindrical molecules are stacked up vertically along their maximum length, the layer is about $1.0 \times 10^{-6} \text{ meters} / 5.0 \times 10^{-9} \text{ meters} = \mathbf{200 \text{ molecules thick}}$.



On March 21, 2010 the Eyjafjalla Volcano in Iceland erupted, and the expanding ash cloud grounded over 3,000 flights in Europe. Then on April 20, a major oil spill began in the Gulf of Mexico. Although the preferred method for dealing with the oil spill is to collect it using skimmers, burning it is also a common option (see above left photo). A major concern in burning this oil is the addition of carbon dioxide to the atmosphere during the combustion process.

Problem 1 – The Gulf Oil Spill is predicted to generate 200,000 gallons of crude oil every day. If 50% of this is ultimately burned-off, how many tons/day of carbon dioxide are generated if the combustion of 1 gallon of oil generates 10 kg of carbon dioxide?

Problem 2 – Scientists have estimated that the Iceland volcano generated 15,000 tons of carbon dioxide per day, and this eruption continued for about 28 days. How many days will the Gulf Oil burn-off have to continue before its carbon dioxide contribution equals that of the total carbon dioxide generated by the Eyjafjalla Volcano?

Problem 3 – It has been estimated that the European aviation industry generates 344,000 tons of carbon dioxide each day. If 60% of this industry was shut down by the ash cloud from the Eyjafjalla Volcano, how many tons of carbon dioxide would have been produced by airline flights during the 5-day shut-down of the industry?

Problem 4 – What can you conclude by comparing your answers to Problem 1, 2 and 3?

Problem 1 – The Gulf Oil Spill is predicted to generate 200,000 gallons of crude oil every day. If 50% of this is ultimately burned-off, how many tons/day of carbon dioxide are generated if the combustion of 1 gallon of oil generates 10 kg of carbon dioxide?

Answer: $200,000 \text{ gallons/day} \times (0.50) \times (10 \text{ kg/ 1 gallon}) = 1,000,000 \text{ kg/day}$ or **1,000 tons/day**

Problem 2 – Scientists have estimated that the Iceland volcano generated 15,000 tons of carbon dioxide per day, and this eruption continued for about 28 days. How many days will the Gulf Oil burn-off have to continue before its carbon dioxide contribution equals that of the total carbon dioxide generated by the Eyjafjalla Volcano?

Answer: The volcano generated $15,000 \text{ tons/day} \times 28 \text{ days} = 420,000 \text{ tons of CO}_2$. The Gulf Oil burn-off generates 1,000 tons/day, so the Gulf Oil burn-off would have to continue for **420 days** before it equaled the emission of the volcano.

Problem 3 – It has been estimated that the European aviation industry generates 344,000 tons of carbon dioxide each day. If 60% of this industry was shut down by the ash cloud from the Eyjafjalla Volcano, how many tons of carbon dioxide would have been produced by airline flights during the 5-day shut-down of the industry?

Answer: $344,000 \text{ tons/day} \times (0.6) \times (5 \text{ days}) = 1 \text{ million tons}$.

Problem 4 – What can you conclude by comparing your answers to Problem 1, 2 and 3?

Answer: The total carbon dioxide generated by the volcano is only 40% of what was generated by the European airline industry during the time it was shut down, and the burn-off of the oil spill will only exceed what the volcano generated if the cleanup continues for over one year, which most experts say is very unlikely. The oil spill cleanup is a small part of the carbon dioxide generated by aviation or by the Icelandic volcano.

Note: Since the crude oil was destined to be burned to generate electricity, and combusted in cars, the fact that the clean-up is producing carbon dioxide is almost beside the point since this very same quantity of carbon dioxide would have been generated anyway once the crude oil is used under more controlled conditions.

Additional Resources

GLOBE Program - International project involving millions of students who take data on their local climate and geological conditions and publish their data in a growing, communal data base, that can then be analyzed by students in many different ways to explore the earth system.

<http://www.globe.gov>

NASA Earth Observing System - a huge data base of images and scientific data about earth together with news announcements and access to dozens of earth-orbiting satellites dedicated to measuring the atmosphere, oceans, crust and interior from space.

<http://eosps0.gsfc.nasa.gov>

NASA Earth Observatory -Provides an extensive collection of images, stories, and discoveries about climate and the environment that emerge from NASA research, including its satellite missions, in-the-field research, and climate models.

<http://earthobservatory.nasa.gov/>

Earth Science Picture of the Day - Every day a new image taken from satellites or ground-based studies of a different aspect of our home planet.

<http://epod.usra.edu>

Carbon Emission Calculators - The EPA and Nature Conservancy calculators are very detailed. The estimates for similar inputs to the forms can vary from 17 to 44 tons total each year, for a family of 4, living in Maryland with two cars, and taking occasional airline flights..

- US-EPA - http://www.epa.gov/climatechange/emissions/ind_calculator.html
- Nature Conservancy - <http://www.nature.org/initiatives/climatechange/calculator/>
- Carbon Fund - <http://www.carbonfund.org/Calculators/>
- Inconvenient Truth - <http://www.climatecrisis.net/takeaction/carboncalculator/>
- Fight Global Warming - <http://www.fightglobalwarming.com/carboncalculator.cfm>

A Note from the Author

Dear Teacher and Student,

It is hard to avoid the disturbing news that our environment and climate are both changing more rapidly now than at any other time in recent history. A few nay-sayers still maintain, against overwhelming scientific opinion, that there is no 'global warming' and that what we are experiencing is just a natural consequence of solar activity.

This book provides many of the quantitative skills your students will need to make sense out of what has now become the on-going front-page news story of our time. It is a story that will not go away, and the way that we approach this subject will say much about how our future will unfold. Ultimately, the way we will have to adjust to climate change will demand a great deal from us, and quite a few sacrifices. Habits will have to be changed, demands for certain types of consumer products will have to be altered, and the true long-term costs of our consumer-based 'buy it now' economy will have to be confronted.

To think quantitatively about climate change, we will need to be fluent in working with the two common temperature scales; Centigrade (Celsius) and Fahrenheit. A 2-degree C global temperature change by 2100 doesn't sound like much, but in Fahrenheit terms that's nearly 4-degrees F! We will also need to understand the difference between watts, kilowatts and kilowatt-hours; the difference between tons and gigatons, and between BTUs and tons of carbon dioxide. All of these units appear in modern news stories about global warming and human impacts upon the carbon dioxide and methane in our atmosphere.

It is true that our parents never had to worry about these terms, let alone their mathematical relationships, but those days of ignoring the balances in our environment that lead to stable climate, are now largely a thing of the past. Savvy consumers now worry about the energy efficiency of their cars and appliances, insulate their homes to reduce energy consumption, and have become increasingly familiar with their 'carbon footprint'.

To avoid the worst aspects of climate change in the future, we need to become more educated consumers today. This math guide uses NASA satellite resources to explore the mathematics behind living on a green, and healthy, planet.

Sincerely,

*Dr. Sten Odenwald
Space Math @ NASA*



National Aeronautics and Space Administration

Space Math @ NASA
Goddard Spaceflight Center
Greenbelt, Maryland 20771
spacemath.gsfc.nasa.gov

www.nasa.gov