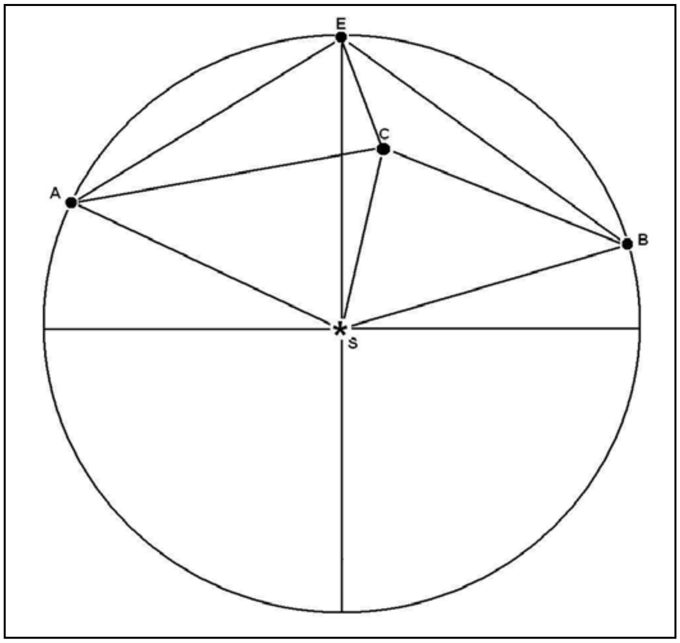
Seeing Solar Storms in STEREO - II



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

The two STEREO satellites are located at points A and B, with Earth located at Point E and the sun located at Point S, which is the center of a circle with a radius ES of 1.0 Astronomical unit (150 million kilometers). Suppose that the two satellites spot a Coronal Mass Ejection (CME) cloud at Point C. Satellite A measures its angle from the sun mSAC as 45 degrees while Satellite B measures the corresponding angle to be mSBC=50 degrees. In the previous math problem the astronomers knew the ejection angle of the CME, mESC, but in fact they didn't need to know this in order to solve the problem below!

Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

Givens from satellite orbits:

SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees

Find the measures of all of the angles and segment lengths in thee above diagram rounded to the nearest integer.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?

Answer Key

Givens from satellite orbits:

SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km

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use units of megakilometers i.e. 150 million km = 150 Mkm.

Method 1: Students construct a scaled model of the diagram based on the angles and measures, then use a protractor to measure the missing angles, and from the scale of the figure (in millions of kilometers per millimeter) they can measure the required segments. Segment EC is about 49 Mkm at an angle, mSEB of 28 degrees.

Method 2 use the Law of Cosines and the Law of Sines to solve for the angles and segment lengths exactly.

mASB = mASE+mBSE = 102 degrees
mASC = theta
mACS = 360 - mCAS - mASC = 315 - theta
mBSC = mASB - theta = 102 - theta
mBCS = 360 - mCBS - mBSC = 208 + theta

Use the Law of Sines to get

sin(mCAS)/L = sin(mACS)/150 Mkm and sin(mBCS)/L = sin(mBCS)/150 Mkm.

Eliminate L : 150sin(45)/sin(315-Theta) = 150 sin(50)/sin(208+theta)

Re-write using angle-addition and angle-subtraction:

sin50 [sin(315)cos(theta)-cos(315)sin(theta)] = sin(45) [sin(208)cos(theta) + cos(208)sin(theta)]

Compute numerical factors by taking indicated sines and cosines:

-0.541cos(theta) - 0.541sin(theta) = -0.332cos(theta) - 0.624sin(theta)

Simplify: cos(theta) = 0.397sin(theta)

Use definition of sine: $\cos(\theta)^2 = 0.158 (1 - \cos(\theta)^2)$

Solve for cosine: $\cos(\theta) = (0.158/1.158)^{1/2}$ so theta = 68 degrees. And so mASC=68

Now compute segment CS = 150 sin(45)/sin(315-68) = 115 Mkm.

BC = 115 sin(102-68)/sin(50) = 84 Mkm.

Then $EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(mEBS-mCBS)$

$EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(66-50)$ So EC = 47 Mkm.

mCEB from Law of Cosines: $84^2 = 122^2 + 47^2 - 2(122)(47)\cos(mCEB)$ so mCEB = 29 degrees

And since mAES = 180 - mASE - mEAS = 180 - 54 - 63 = 63 degrees

so mSEC = mAEB - mAES - mCEB = 129 - 63 - 29 = 37 degrees

So, the two satellites are able to determine that the CME is 49 million kilometers from Earth and approaching at an angle of 37 degrees from the sun.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?

Answer: 115 million kilometers / 2 million km/hr = 58 hours or 2.4 days.