Empirical reconstruction of fuzzy model of experiment in the Euclidean metric

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Abstract. In this paper we introduce a method for the fuzzy model reconstruction and a method for measurements reduction on the basis of test signals by maximization a posteriori possibility. It ensures the maximum accuracy of the measurements reduction. It is used the model of measurement errors with fuzzy constraints on its Euclidean norms.

Keywords: mathematical modeling, fuzzy sets, decision making, analysis and interpretation of data, measurement and computing systems

Introduction. In this paper we consider a fuzzy experiment conducted by the scheme

$$\xi = \Lambda\varphi + \nu. \tag{1}$$

The measurement result ξ of the Euclidean space $\mathcal{R}_n$ is accompanied by a additive noise ν. By a result of ξ of measure (1) it is required to estimate the value of the parameter vector $\eta = U\varphi$, where $U \in (\mathcal{R}_N \to \mathcal{R}_M)$ is defined linear operator [4].

In the paper [1] it is shown that if (1) vector $\varphi \in \mathcal{R}_N$ and a linear operator Λ priori arbitrary, and $\nu \in \mathcal{R}_n$ is the fuzzy vector in $\mathcal{R}_n$ with distribution possibilities $\pi^\nu(\cdot)$ [2,3]. And for the determination of the model of the measuring device Λ there are involved the measurement results $\xi_j = \Lambda f_j + \nu_j, \quad j = 1, \ldots, m$, of known test signals, $f_1, \ldots, f_m$, where the error $\nu_1, \ldots, \nu_m \in R_n$ of test measurements are fuzzy elements of $\mathcal{R}_n$ with the given distribution of possibilities. The estimates of the maximum possibility $\widehat{A}$ and $\widehat{f}$ are values of fuzzy elements Λ and φ respectively, as a solution of the maximin problem

$$(\widehat{A}, \widehat{f}) = \arg\max_{A,f} \min(\pi^\nu(x - Af), \pi^N(X - AF)), \tag{2}$$

and the estimate $\widehat{u}$ is value of the fuzzy element of η given by $\widehat{u} = U\widehat{f}$. Here, x, A, f, X are implementation of fuzzy elements of $\xi, \Lambda, \varphi, \Xi$, respectively. The same scheme of test measurements in matrix form is given by $\Xi = \Lambda F + N$.

Consider the solution of the problem (2), where the operator Λ and element φ are a priori arbitrary, so that $\pi^\Lambda(A) = 1$ for every $A \in (\mathcal{R}_N \to \mathcal{R}_n)$ and $\pi^\varphi(f) = 1$ for any $f \in \mathcal{R}_N$, and distribution possibilities of measurement error is given by

$$\pi^\nu(z) = \mu_0(\|z\|^2), \quad z \in \mathcal{R}_n; \quad \pi^N(Z) = \mu_0(\|Z\|_2^2), \quad Z \in (\mathcal{R}_m \to \mathcal{R}_n),$$

where $\mu_0(\cdot) : [0, \infty) \to [0, 1]$ is strictly decreasing function, $\mu_0(0) = 1$, $\lim\limits_{z\to\infty} \mu_0(z) = 0$.

Then the problem (2) leads to the following minimax problem:

$$\min_{A,f} \max(\|x - Af\|^2, \|X - AF\|_2^2). \tag{3}$$

Let us denote $J_1(A, f) = \|x - Af\|^2$, $J_2(A) = \|X - AF\|_2^2$, then $J(A, f) = \max(J_1(A, f), J_2(A))$, and the problem (3) can be rewritten as

$$J(A, f) = \max(J_1(A, f), J_2(A)) \sim \min_{A,f}. \tag{4}$$

Depending on which of the minimum values $J_1(\widehat{A}_0, \widehat{f}(\widehat{A}_0))$ or $J_2(\widehat{A}_0)$, is less it is selected different methods for solving the problem (3), they are considered in [1].

Example 1. Let the unknown operator A is defined by the matrix of size 2×2, $A = \begin{pmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{pmatrix}$, given m test signals, $f_1 = \begin{pmatrix} f_{11} \\ f_{21} \end{pmatrix}, \ldots, f_m = \begin{pmatrix} f_{1m} \\ f_{2m} \end{pmatrix}$, forming columns of the matrix $F \in (\mathcal{R}_m \to \mathcal{R}_2)$, and test results are given as vectors $x_1 = \begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}, \ldots, x_m = \begin{pmatrix} x_{1m} \\ x_{2m} \end{pmatrix}$, forming columns of the matrix $X \in (\mathcal{R}_m \to \mathcal{R}_2)$. Then the scheme of tests is given by

$$\Xi = AF + N,$$

where the j-th column of N defines the error of j-th test measure, $j = 1, \ldots, m$. For the error matrix N that defined by the distribution of possibilities of its values by the form $\pi^N(X) = \mu_0(\|Z\|_2^2)$, where $\mu_0(\cdot) : \mathcal{R}_+ \to [0, 1]$ is a monotonically decreasing function, $\mu_0(0) = 1$, $\mu_0(+\infty) = 0$.

Let the rank of F is equal to two. The signal f is measured according to the scheme $\xi = Af + \nu$. The result $x \in \mathcal{R}_2$ of this measurement is known. The distribution is $\pi^\nu(z) = \mu_0(\|z\|^2)$ of possibility of fuzzy vector $\nu \in \mathcal{R}_2$ for measurement error Af. It is required to determine the reduction of the vector ξ to the form which would be a measurement of the signal f by the instrument $I \in (\mathcal{R}_2 \to \mathcal{R}_2)$.

Let us write the problem of calculating the reduction as a minimax problem

$$\min_{A,f} (\max \|Af - x\|^2, \|AF - X\|_2^2).$$

Minimum of $J_2(A) = \|A_*F - X\|_2^2$ is achieved on a single matrix $\widehat{A}_0 = XF^-$, since the rank of F is equal to two and therefore holds $(I - FF^-) = 0$. If this matrix is nonsingular, then $J_1(\widehat{A}_0, \widehat{f}(\widehat{A}_0)) = 0 \leq J_2(\widehat{A}_0)$ and $(A_*, f_*) = (XF^-, (XF^-)^{-1}x)$, a result of the reduction is $f_* = (XF^-)^{-1}x$.

Example 2. Let the unknown operator A each the number of f associates a two-dimensional vector $Af = \begin{pmatrix} a_1 f \\ a_2 f \end{pmatrix}$, i.e. A is defined by the matrix of size

1×2, $A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$; is given m of test scalar signals $f_1, \ldots, f_m$, forming a matrix $F \in \mathcal{R}_m \to \mathcal{R}_1$, consisting of a single line, and test results are given as vectors $x_1 = \begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}, \ldots, x_m = \begin{pmatrix} x_{1m} \\ x_{2m} \end{pmatrix}$, forming columns of $X \in \mathcal{R}_m \to \mathcal{R}_2$. Scheme and model test and measurement reducible are the same as in previous Example 1.

Let us consider the minimax problem

$$\min_{A,f}(\max \|Af - x\|^2, \|AF - X\|_2^2). \tag{5}$$

In this case, the operator A such that its value space is the dimensional linear subspace of $\mathcal{R}_2$, containing the results of test and reduction measurement. The location of the points x and $x_1, \ldots, x_m$ such that inequality $\|(I - \widehat{A}_0\widehat{A}_0^-)x\|^2 \leq \|X(I - F^-F)\|_2^2$ is not satisfied and the point $(\widehat{A}_0, \widehat{f}_0)$ is not the point at which is achieved the minimax in (5). To specify the value space of the operator A from the geometric point view means to specify the line through the origin of coordinates. For any such line value of the vector a, which determines the action operator A with a given space of values, given length vector a along a given line.

To achieve the minimax we have to change the direction of the vector a, specifying the space of values of the operator A. Calculating $J_1(A, f)$ as the square of the distance from x to the line with direction vector a, and $J_2(A)$ are making such disposition of the space values of A, at which the equality $J_1(A, f) = J_2(A)$. The length of the projection of x on a one-dimensional subspace divided by the length of a vector a, yields a reduction of measurement x. This ensures a compromise between being able to test and reduction measurements.

Conclusions. In this paper we consider a method of empirical reconstruction and reduction of measurements for fuzzy model in restrictions on Euclidean norms of signals and the operator of the model. The information about the model is contained in a series of test experiments and reduction measurements. This work was supported by the Russian Foundation for Basic Research (project no. 11-07-00338-a, 09-01-96508 and 09-07-00505-a).

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