Scaling Laws for Linear Complexity Language Models

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`https://github.com/OpenNLPLab/ScalingLaws`

Abstract

The interest in linear complexity models for large language models is on the rise, although their scaling capacity remains uncertain. In this study, we present the scaling laws for linear complexity language models to establish a foundation for their scalability. Specifically, we examine the scaling behaviors of three efficient linear architectures. These include TNL (Qin et al., 2024c), a linear attention model with data-independent decay; HGRN2 (Qin et al., 2024e), a linear RNN with data-dependent decay; and cosFormer2 (Qin et al., 2022b, 2024a), a linear attention model without decay. We also include LLaMA as a baseline architecture for comparison with softmax attention. These models were trained with six variants, ranging from 70M to 7B parameters on a 300B-token corpus, and evaluated with a total of 1,376 intermediate checkpoints on various downstream tasks. These tasks include validation loss, commonsense reasoning, and information retrieval and generation. The study reveals that existing linear complexity language models exhibit similar scaling capabilities as conventional transformer-based models while also demonstrating superior linguistic proficiency and knowledge retention.

1 Introduction

The prosperity of large language models (LLMs) has necessitated the development of scaling laws (Kaplan et al., 2020) to optimize the tradeoff between increasing model size and expanding training data within finite computational resources. Scaling laws empirically study the correlation between model performance and factors including the number of parameters, training tokens, and FLOPs. Previous works (Kaplan et al., 2020; Henighan et al., 2020; Hoffmann et al., 2022; Clark et al., 2022) have established power laws to describe these scaling trends. Experiments are typically conducted on smaller models with relatively low training costs. From these observations, regression models are derived to guide the scaling of parameters, data, and computational resources. Establishing these scaling laws is crucial before expanding language models to the scale of LLMs, ensuring predictable results under controllable training costs. Scaling laws have guided the success of many recent LLMs, such as Chinchilla (Hoffmann et al., 2022) and GPT-4 (OpanAI, 2023). It is noteworthy that existing scaling laws are predominantly established for traditional softmax attention transformers (Vaswani et al., 2017).

Linear complexity language models (Katharopoulos et al., 2020; Qin et al., 2022b; Zheng et al., 2022; Wang et al., 2020; Choromanski et al., 2021; Zheng et al., 2022, 2023; Hua et al., 2022; Liu et al., 2022; Qin et al., 2023b, 2024c,d; Gu et al., 2021a, 2020, 2022; Fu et al., 2022; Qin et al., 2023a; Fu et al., 2023; Orvieto et al., 2023; Qin et al., 2023d, 2024e; Yang et al., 2023; Gu and Dao, 2023; Dao and Gu, 2024; Sun et al., 2023), have emerged as a promising alternative to traditional transformers in causal language modeling. However, the scalability of these models remains uncertain, which limits their applicability to large language models. To address this concern, in this paper we have developed pre-training scaling laws for efficient large language models. Following the approach outlined in (Hoffmann et al., 2022), we have used the training loss as a regression target to establish power law equations against FLOPs and infer the optimal model size and dataset size under constant computation budgets for linear complexity models. Our study focuses on investigating three efficient architectures, as detailed in Section 2: TNL (Qin et al., 2024c), HGRN2 (Qin et al., 2024e), and cosFormer2 (Qin et al., 2022b, 2023c, 2024a). Additionally, LLaMA (Touvron et al.,

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Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing, pages 16377–16426
November 12-16, 2024 ©2024 Association for Computational Linguistics

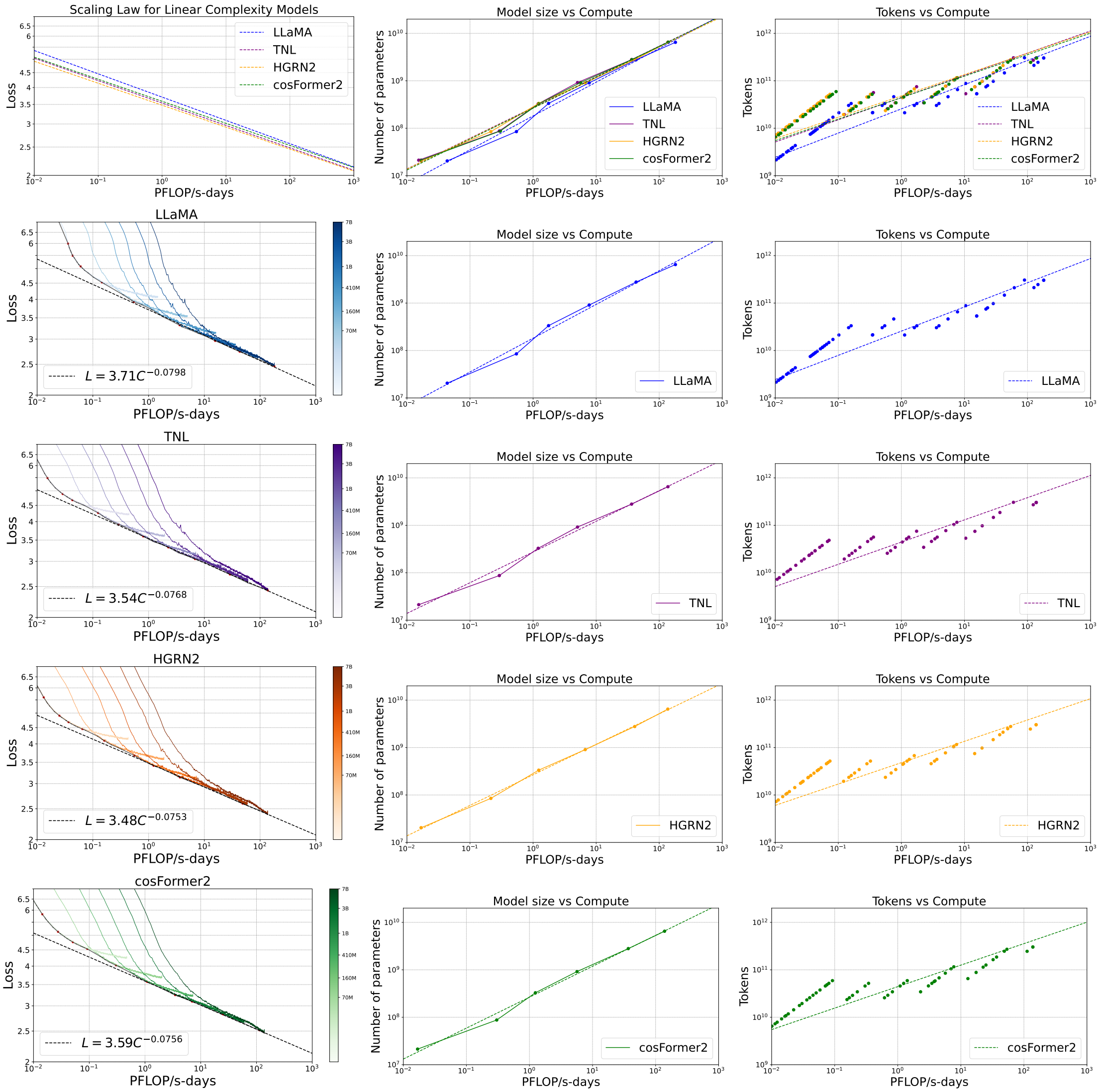

Figure 1: **Training Curve Fitting for Four Architectures.** In the master row, we present predicted training curves for various architectures, with each subsequent row representing a different architecture. On the left, the training curves for models ranging from 70M to 7B parameters are displayed. From these curves, we extract the envelope of minimum loss per FLOP, using these data points to estimate the optimal model size (center) for a specified compute budget, and the optimal number of training tokens (right).

2023b) is used as a baseline to represent softmax attention transformers. For a comprehensive analysis, we compare the scaling behavior of downstream task performance across different architectures. As outlined in Section 3, each model is evaluated in terms of linguistic proficiency, knowledge retention, and information retrieval and generation. Our findings reveal that linear complexity models exhibit similar scaling trends to conventional transformer-based models and consistently outperform LLaMA in cross-domain perplexity and average accuracy in commonsense reasoning under the same FLOPs budget but demonstrate weakness in retrieval tasks.

Our contributions are summarized as follows:

- We disclose scaling laws for linear complexity language models, focusing on three different architectures. Reveling the training loss L, model size N and dataset size D have power-law relationships with computation budget C.
- Our experiments showcase the advantage of linear complexity language models over traditional transformers on linguistic proficiency while inferior performance in retrieval tasks.
- We analyze the scaling trends for downstream task performance and observe the correlation of performance with computation budget.
- For linear models, aspect ratio (model dimension / number of layers) and context length affect model capacity. This is contradictory to previous scaling laws, where model shape makes a negligible impact.
- For linear models, data-dependent decay is beneficial in retrieval tasks and is not signifi-

cantly different from data-independent decay in other tasks.

2 Preliminary

2.1 Causal language modeling

Causal language modeling forecasts the next word in a sequence by analyzing prior words, commonly used in GPT models (Radford et al., 2018). It employs the cross-entropy loss function to assess model accuracy by comparing the predicted and actual word distributions—a lower score suggests better performance. Transformers that use softmax-based attention (Vaswani et al., 2017), are referred to as *vanilla* transformers. In our experiments, LLaMA (Touvron et al., 2023b,a) serves as the exemplar for this category of transformers.

2.2 Linear complexity sequence models

In order to tackle the high time complexity of traditional transformers, researchers are currently investigating new linear complexity sequence model architectures. These alternatives include linear attention (Qin et al., 2021), state space models (SSMs) (Gu et al., 2021b), long convolution (Qin et al., 2023a), and linear RNN (Qin et al., 2024f). According to Qin et al. (2024b), SSMs can be considered as linear attention variants, and long convolution can be accurately transformed into SSMs (Qin and Zhong, 2023). In this study, TNL (Qin et al., 2023b, 2024c,d) and cosFormer2 (Qin et al., 2024a) serve as representatives of linear attention, while HGRN2 (Qin et al., 2024e) is the chosen representative of linear RNN.

TNL uses *data-independent decay* to enhance Linear Attention in order to address the "dilution" problem. By employing these techniques, along with Lightning Attention (Qin et al., 2024c), TNL outperforms traditional softmax attention models in both efficiency and accuracy.

HGRN2 overcomes the limited expressiveness of traditional HGRNs (Qin et al., 2024f) by employing a state expansion mechanism inspired by linear attention, which enlarges the recurrent state size without extra parameters. It also incorporates *data-dependent decay* in its positional encoding components. This innovation enables HGRN2 to achieve superior performance in language modeling, image classification, and Long Range Arena benchmarks, demonstrating enhanced efficiency and accuracy compared to both its predecessor and other contemporary models.

cosFormer2 (cos2) represents an advanced iteration of the original cosFormer (Qin et al., 2022b) model without decay, incorporating several significant enhancements that optimize its performance and functionality: *1.* cos2 adopts complex-based LRPE positional encoding (Qin et al., 2023c) to facilitate a per-channel $\cos$ reweighting mechanism, an improvement over the uniform $\cos$ weighting applied across all features in the original cosFormer (Qin et al., 2022b). *2.* cos2 enhances its handling of relative positional information through the integration of TPE (Qin et al., 2024a). *3.* cos2 utilizes a low-rank output gate from TNL (Qin et al., 2024d), contributing to more efficient data processing. *4.* cos2 employs normalization (Qin et al., 2022a) for improved stability and performance instead of using a denominator. These enhancements enable cos2, with its *no-decay positional encoding*, to match the performance of top Transformer models like LLaMA.

2.3 Model size and FLOPs calculation

In calculating the model parameters N and compute budget C, previous studies have employed varying levels of simplification. For clarity, we denote model specifications as follows: l (number of layers), d (model dimension), and d_f (feed forward layer dimension). Kaplan et al. (2020) only accounts for the weights of linear layers (excluding input and output embedding) as model parameters. When $d_f = 4d$, the total number of model parameters is computed as: $N = 12ld^2$. The total forward and backward FLOPs can be approximated as: $C \approx 6N$. On the other hand, Hoffmann et al. (2022) takes a more detailed approach by incorporating embedding parameters into the model parameter count and factoring in the computational load of softmax operation, input and output embedding in the FLOPS count. To underscore the distinction between *vanilla* transformers and linear complexity sequence models, as well as the variations among linear complexity sequence models, we employ a detailed method for computing N and C as outlined in Table 1.

3 Experimental setup

3.1 Corpus

The training dataset for this work comprises 300 billion tokens sampled from a self-collected and curated corpus of approximately 2 trillion tokens. The data is bilingual, consisting of English and Chinese

Table 1: **Checklist of Model Parameters and FLOPs.** Detailed calculations are deduced in Appendix A.1. Here h is the number of attention heads. Compared to the full equations, we exclude embedding parameters and other subleading terms in our calculation for better fitting results of the scaling law equations.

Architecture	Parameter count	FLOPs count
LLaMA	$12ld^2$	$72bnld^2(1 + \frac{n}{6d} + \frac{5}{18d})$
TNL	$12ld^2 + 2ld^2/h$	$72bnld^2(1 + \frac{1}{2h} + \frac{5}{18d})$
HGRN2	$12ld^2 + ld$	$72bnld^2(1 + \frac{1}{3h} + \frac{29}{72d})$
cosFormer2	$12ld^2 + 2ld^2/h + d^2/h$	$72bnld^2(1 + \frac{3}{4h} + \frac{23}{72d})$

texts in a 2:1 ratio. Sources of data span various categories, including academic publications, books, and selected web pages. The corpus was refined using several cleaning strategies (Qin et al., 2023b, 2024d), such as rule-based filtering, deduplication, and a proprietary self-cleaning scheme.

3.2 Training procedures

Our experiments were implemented using the Metaseq training framework (Zhang et al., 2022) built atop PyTorch (Paszke et al., 2019). The LLaMA model was equipped with FlashAttention-2 (Dao, 2023), whereas the TNL, cosFormer2 model incorporated Lightning Attention (Qin et al., 2024c). HGRN2 employs Flash Linear Attention (FLA) (Yang and Zhang, 2024). We conducted all experiments on H100/H800 80G GPUs.

For all model architectures and training sequence lengths, we maintained a consistent global batch size of 4 million tokens. We utilized the Adam optimizer, with a learning rate of 3e-4 and a weight decay of 0.1. A fixed learning rate scheduler was used for all experiments within constrained computation resources. We use tiktoken (Openai) as the tokenizer, featuring a vocabulary size of 100,280.

3.3 Model configurations

We investigate four distinct model architectures: LLaMA, TNL, cosFormer2, and HGRN2, across a spectrum of scales—70M, 160M, 410M, 1B, 3B, and 7B. Each model is trained on a corpus of up to 300 billion tokens with a context length of 8192, aligning with the methodology proposed by Hoffmann et al. (Hoffmann et al., 2022), where training loss serves as a direct proxy for test loss.

In our continued exploration of linear complexity models, we have extended the pre-training context lengths for the 1B models to encompass 2048, 4096, and 16384. Additionally, we have introduced variations in the hidden dimensions of the 1B models, testing sizes 1536, 1792, 2048, and 3072, to assess the impact of these adjustments on pre-training loss and subsequent performance.

Table 2: **Specifications of Model Variants.** We outlines the specifications for various model variants, detailing their hidden dimensions (Hidden) and the dimensions of attention heads (H. Dim).

Size	Layers	Hidden	Head	H. Dim
70M	6	512	8	128
160M	12	768	8	128
410M	26	1024	8	128
1B	32	1536	16	128
3B	35	2560	20	128
7B	32	4096	32	128

3.4 Evaluation metrics

Perplexity is a key metric used to evaluate the word prediction capabilities of causal language models. We use training loss and validation perplexity as evaluation metrics, with WIKITEXT-2 (Merity, 2016) and LAMBADA (Paperno et al., 2016) serving as benchmarks for comprehending complex, informative text and assessing their ability in narrative comprehension and contextual prediction, respectively. Lower perplexity indicates better predictive performance, suggesting that the model can more accurately capture language structure.

Knowledge retention Common sense reasoning (CSR) measures a model's ability to reason and understand everyday scenarios, indicating its practical real-world applicability. We report BoolQ (Clark et al., 2019), PIQA (Bisk et al., 2019), SIQA (Sap et al., 2019), HellaSwag (Zellers et al., 2019), WinoGrande (Sakaguchi et al., 2019), ARC easy and challenge (Clark et al., 2018) and OpenBookQA (Mihaylov et al., 2018). We report 0-shot results for all benchmarks using LM-Eval-Harness (Gao et al., 2021). We do not use the MMLU (Hendrycks et al., 2021) benchmark as it is more suitable for instruction-tuned models.

Information retrieval and generation The Needle in A Haystack (NIAH) benchmark is designed to evaluate the in-context retrieval capabilities of LLMs. We extend NIAH to present two difficulty levels. In Easy mode, both the question and its corresponding answer (QA pair) are embedded within a lengthy text, challenging the model to identify and respond to the query by locating the QA pair. This mode is particularly accessible for base models that have not undergone instruction tuning. In contrast, the standard mode places only the answer within the long context. Here, the model must comprehend the question, locate the relevant answer in the text, and provide a response.

We quantify NIAH using three metrics: accuracy at a specific context length, weighted average accuracy, and NIAH score, as detailed in Appendix B.1. We use weighted average accuracy as our main evaluation metric for NIAH. Previous work (Hsieh et al., 2024) assigns weights to context lengths in a linear scale. We assign weights for both depths and context lengths using a geometric progression for clearer distinction. We adopt $w_{d_i} = w_{d_0}\alpha_d^{i-1}$ for depth and $w_{c_i} = w_{c_0}\alpha_c^{i-1}$ for context length, where w_{d_i} and w_{c_i} are the weights for the i-th step, α_d and α_c are constants greater than 1. These weights form a map that modifies the average accuracy calculation.

In addition to NIAH, our evaluation also includes the SCROLLS benchmark (Shaham et al., 2022). SCROLLS assesses the model's abilities in information retrieval and generation across three distinct tasks: summarization, question answering, and natural language inference. We utilize the LM-Eval-Harness (Gao et al., 2021) by configuring 0-shot, greedy-search evaluations and truncation of pretraining context length.

4 Scaling laws

The concept of scaling laws involves four key factors: loss L, specifically the cross-entropy loss in a causal language modeling setting; model size N, which is determined by the number of model parameters; dataset size D, calculated as the number of training tokens; and computation budget C, represented by the total FLOPs used for training. N_{opt} and D_{opt} are the optimal model size and dataset size given a fixed computation budget.

Initially, we establish power law equations between L and C. In this analysis, we adopt the approach introduced by (Hoffmann et al., 2022), treating the training loss as an unbiased estimate of the test loss. Subsequently, based on the fitted curve, we ascertain the optimal loss for specific FLOPs, enabling us to obtain coefficients for $N_{opt} \propto C^a$ and $D_{opt} \propto C^b$. When modeling the scaling trend of loss against factors such as N, D, and C, the original scaling laws (Kaplan et al., 2020) utilize the power function $L(X) = (X_0/X)^{\alpha_X}$, where X represents the factor of interest. Subsequent studies (Henighan et al., 2020; Clark et al., 2022; Hoffmann et al., 2022; Gao et al., 2024) employ a more general power-law plus constant form, $L(X) = \epsilon + (X_0/X)^{\alpha_X}$, to achieve improved fitting. Here, the constant ϵ is interpreted as irreducible loss or the entropy of natural text (Hoffmann et al., 2022). In our case, we have simplified all forms of the power law and unified them into $L(X) = \beta_X X^{\alpha_X}$, which allows for a more intuitive comparison of the scaling capabilities of different models based on coefficients α_X and β_X.

4.1 Training loss

We aim to obtain the scaling laws of different models under the condition that only the model architectures are different. Therefore, we record the training losses of all models at the same interval, classify and fit them according to the power law mentioned above, and finally obtain the relation between L and C, shown in the left column in Fig.1 and Table 3.

4.2 Optimal model size and dataset size

Given a fixed computation budget, we study how to allocate it to model parameter size and dataset size. Following (Hoffmann et al., 2022), we extract the minimal loss for each FLOP and consider this as the optimal loss for a given computation budget. To find model scaling exponent a and data scaling exponent b that satisfy $N_{opt} \propto C^a$ and $D_{opt} \propto C^b$, we use non-embedding parameters (Kaplan et al., 2020) as our vocabulary size is large and accounts for a large proportion of the parameters of the small model. Based on the above content, we fit all models to obtain the relationship between C and N_{opt} and D_{opt}, which can be seen in the last two columns of Fig.1 and Table 3.

4.3 Downstream tasks

Traditional scaling laws primarily focus on the relationship between computation power and training loss, typically measured by cross-entropy. However, this measure alone does not fully capture the capabilities of large language models (Kaplan et al., 2020; Hoffmann et al., 2022). To address this, we expand our investigation to include scaling

Table 3: **Summary of Scaling Laws:** it illustrates the relationship between loss (L, left), the number of parameters (N_{opt}, middle), and corpus size (D_{opt}, right) with computation budget (C). It can be seen intuitively that under the same computation budget, linear complexity models consume more parameters and tokens while obtain lower loss.

Arch	$L(C)$	$N_{opt}(C)$	$D_{opt}(C)$
LLaMA	$3.7087C^{-0.0798}$	$(1.82 \times 10^8)C^{0.7118}$	$(2.56 \times 10^{10})C^{0.5102}$
TNL	$3.5391C^{-0.0768}$	$(2.74 \times 10^8)C^{0.6470}$	$(4.43 \times 10^{10})C^{0.4684}$
HGRN2	$3.4788C^{-0.0753}$	$(2.66 \times 10^8)C^{0.6427}$	$(4.80 \times 10^{10})C^{0.4500}$
cosFormer2	$3.5877C^{-0.0756}$	$(2.65 \times 10^8)C^{0.6516}$	$(4.23 \times 10^{10})C^{0.4529}$

laws that correlate computation power with validation perplexity and common sense reasoning (CSR) scores in Fig. 2. Additionally, we evaluate the performance of retrieval and generation capabilities using benchmarks such as Needle in A Haystack (NIAH) in Table 4 and SCROLLS in Table 4. Each metric in these tasks provides a unique perspective on the strengths and limitations of LLMs.

Validation perplexity Validation perplexity consistently decreases across all architectures as the number of model parameters increases, observed in both the WIKITEXT-2 and LAMBADA datasets. This trend underscores the scalability of linear complexity sequence models compared to vanila transformer models. When analyzing specific parameter sizes, HGRN2 architecture shows the best performance, closely followed by TNL. In the WIKITEXT-2 dataset, cosFormer2 surpasses LLaMA, while LLaMA performs better than cosFormer2 on the LAMBADA dataset.

CSR score The CSR scores for all linear complexity sequence models demonstrate scaling capabilities comparable to the transformer models. Specifically, HGRN2 is the only model that surpasses LLaMA at the 70M parameter level. In terms of 7B parameters, however, all linear complexity sequence models outscore LLaMA, suggesting that linear complexity sequence models exhibit enhanced scaling capabilities as the number of parameters increases. Notably, HGRN2 and TNL consistently outshine the other models. In contrast, cosFormer2 shows fluctuating performance compared to LLaMA.

Both CSR scores and validation perplexity highlight the strong scaling potential of HGRN2, TNL, and cosFormer2 in addressing linguistic and knowledge-based tasks in downstream tasks.

NIAH In evaluating the easy mode of the Needle in a Haystack (NIAH) task in 16K contexts, different architectures perform differently. Models with parameter sizes below 160M struggle to perform the tasks effectively. The vanilla transformer LLaMA maintains a success rate of about 50%. TNL begins to show results in NIAH only after reaching 1B parameters, achieving a maximum success rate of about 10%. Both HGRN2 and cosFormer2 start to display scaling capabilities in NIAH with over 410M parameters. Specifically, cosFormer2 achieves a maximum retrieval success rate of 25% in 4K context, while HGRN2 performs slightly better with 30% in 4.8K context. Linear complexity sequence models like cosFormer2 and HGRN2 tend to retrieve information from contexts shorter than their pre-training length of 8K. In terms of performance, the order is LLaMA > cosFormer2 = HGRN2 >> TNL. Additionally, these linear complexity sequence models require large parameter sizes to effectively handle NIAH tasks.

SCROLLS Similar to NIAH, all architectures begin to effectively address the SCROLLS task starting at minimal 410 million parameters. All models with linear complexity sequences display a consistent scaling power comparable to LLaMA. TNL also requires a large parameter size (1B) for SCROLLS. The overall performance ranking is LLaMA and cosFormer2 at the top, followed by HGRN2, and then TNL.

5 Discussion

5.1 Aspect ratio and model capacity

Under the same model parameters, we can tweak the model architecture by adjusting the aspect ratio (hidden dimension and layers) and the dimension of attention heads. We analyze the aspect ratio for a 1B parameter model in Table 5. Since we need to fix the parameters, a higher hidden dimension indicates fewer layers. For both LLaMA and cosFormer2, a hidden dimension below 2048 proves beneficial for CSR and validation perplexity.

In tasks involving retrieval and generation, LLaMA and cosFormer2 consistently show similar results for CSR and validation perplexity. However,

Table 4: **Benchmark of Downstream Task: Common Sense Reasoning (CSR), Validation Perplexity, Needle in A Haystack (easy mode) and SCROLLS.** For CSR, Needle in A Haystack, and SCROLLS, higher scores indicate better performance. For Validation Perplexity, lower scores are preferable. PS: parameter size (billion). HS: HellaSwag. WG: WinoGrande. OBQA: OpenBookQA. WIKI: WIKITEXT-2. λ: LAMBADA. acc_n.: acc_norm. We provide the average score for CSR, the weighted average accuracy for NIAH, and the average score for SCROLLS. Detailed score breakdowns can be found in the "Experiments" section of the Appendix C.

Arch	P.S.	BoolQ	PIQA	HS	WG	ARC-E	ARC-C	OBQA	CSR	WIKI	λ	NIAH	SCORLLS
	B	acc	acc	acc_n.	acc	acc	acc_n.	acc_n.	avg ↑	ppl.↓	ppl. ↓	w.a. ↑	avg ↑
LLaMA	0.07	46.48	58.87	27.82	48.46	39.98	21.42	25.60	38.38	82.7	291.3	0.4	**7.43**
TNL	0.07	43.18	58.87	27.77	50.12	39.77	21.76	23.80	37.90	77.0	369.1	**0.6**	6.13
HGRN2	0.07	56.57	59.19	28.05	52.01	38.64	22.61	26.00	**40.44**	**73.0**	**270.1**	0.2	7.32
cos2	0.07	47.61	60.94	28.12	49.72	37.33	22.18	23.60	38.50	88.8	369.9	0.1	6.67
LLaMA	0.16	52.94	63.66	30.67	51.78	44.32	23.29	26.60	41.89	51.1	69.9	6.0	8.37
TNL	0.16	53.82	63.82	31.22	50.20	45.92	23.21	28.80	**42.43**	44.9	71.1	**7.5**	7.77
HGRN2	0.16	54.01	63.06	31.04	52.41	45.08	23.38	27.00	42.28	**43.8**	**52.8**	0.2	8.29
cos2	0.16	45.47	63.28	29.72	52.41	44.49	22.53	27.20	40.73	49.0	83.8	1.3	**8.71**
LLaMA	0.41	54.04	67.19	38.75	52.17	49.24	23.72	30.00	45.02	29.8	25.1	**52.3**	10.51
TNL	0.41	60.31	66.65	38.98	51.70	52.61	25.17	30.00	46.49	28.0	23.3	14.2	7.55
HGRN2	0.41	60.86	67.74	40.32	51.78	54.21	24.83	31.20	**47.28**	**27.0**	**19.3**	4.8	**10.93**
cos2	0.41	57.40	66.27	36.65	50.59	51.81	23.72	29.00	45.06	30.3	30.3	9.6	9.06
LLaMA	1	56.42	69.97	47.04	52.72	57.07	28.16	32.60	49.14	26.5	12.8	**44.1**	11.01
TNL	1	59.85	71.49	48.70	52.57	57.07	27.73	33.20	50.09	21.7	12.2	28.0	9.65
HGRN2	1	59.17	71.65	49.52	54.38	60.27	28.07	33.40	**50.92**	**21.0**	**10.9**	10.0	**11.08**
cos2	1	44.28	70.73	45.55	50.51	55.22	27.30	31.00	46.37	21.2	15.5	10.9	10.88
LLaMA	3	61.31	73.18	57.88	59.59	63.93	31.40	34.00	54.47	23.0	7.4	**45.1**	13.88
TNL	3	56.76	75.03	60.87	61.33	65.49	33.02	36.40	**55.56**	16.4	6.6	11.1	12.26
HGRN2	3	55.47	74.10	61.48	58.64	65.61	34.47	35.60	55.06	**15.6**	**6.5**	17.9	**15.43**
cos2	3	50.92	74.27	57.38	57.30	63.22	31.40	35.20	52.81	16.0	8.4	25.8	12.75
LLaMA	7	57.46	75.19	64.39	61.88	67.55	35.41	35.00	56.70	15.2	5.9	**59.7**	14.57
TNL	7	62.63	76.22	66.29	61.48	67.76	38.23	37.80	58.63	14.1	5.5	20.5	10.74
HGRN2	7	62.69	76.50	66.96	61.40	69.02	36.86	38.00	**58.78**	13.8	**5.2**	30.8	13.46
cos2	7	65.02	76.33	63.93	59.19	66.96	36.43	37.60	57.92	**13.5**	6.4	23.6	**15.15**

Table 5: **Benchmark of Aspect Ratio and Model Capacity:** it covers models with *1B* parameters each, featuring an *8K* pre-training context. Key metrics include CSR, Validation PPL, NIAH (easy mode), and SCROLLS. HS: HellaSwag. WG: WinoGrande. OBQA: OpenBookQA. WIKI: WIKITEXT-2. λ: LAMBADA. acc_n.: acc_norm.

Arch	Dim	L.	BoolQ	PIQA	HS	WG	ARC-E	ARC-C	OBQA	CSR	WIKI	λ	NIAH	SCORLLS
			acc	acc	acc_n.	acc	acc	acc_n.	acc_n.	avg ↑	ppl.↓	ppl. ↓	w.a. ↑	avg ↑
LLaMA	1536	32	60.98	69.91	46.74	54.85	56.94	28.50	30.00	**49.70**	26.2	13.4	44.1	11.01
LLaMA	1792	24	49.85	70.51	47.29	53.20	57.37	28.16	32.00	48.34	**19.8**	13.4	**54.2**	**12.45**
LLaMA	2048	18	56.42	69.97	47.04	52.72	57.07	28.16	32.60	49.14	26.5	**12.8**	45.0	12.20
LLaMA	3072	8	55.44	69.53	44.25	51.62	52.78	26.54	30.20	47.19	24.8	16.7	46.0	10.88
cos2	1536	32	56.42	70.57	45.99	52.01	57.49	26.11	32.00	48.66	21.3	15.3	10.9	**10.88**
cos2	1792	24	61.83	70.67	46.04	51.70	56.69	27.39	32.40	**49.53**	**21.0**	**14.0**	8.8	10.31
cos2	2048	18	44.28	70.73	45.55	50.51	55.22	27.30	31.00	46.37	21.2	15.5	**12.3**	10.75
cos2	3072	8	43.52	69.86	43.38	50.83	53.79	26.62	32.60	45.80	23.4	18.3	6.5	9.85

Table 6: **Benchmark of Pre-training Context Length:** it involves CSR, Validation PPL, NIAH (easy mode), and SCROLLS. All models tested have a parameter size of *1B* and a hidden dimension of *1536*. HS: HellaSwag. WG: WinoGrande. OBQA: OpenBookQA. WIKI: WIKITEXT-2. λ: LAMBADA. acc_n.: acc_norm.

Arch	Len	BoolQ	PIQA	HS	WG	ARC-E	ARC-C	OBQA	CSR	WIKI	λ	NIAH	SCORLLS
		acc	acc	acc_n.	acc	acc	acc_n.	acc_n.	avg ↑	ppl.↓	ppl. ↓	w.a. ↑	avg ↑
TNL	2K	61.96	72.03	49.94	54.38	57.58	28.33	31.20	50.77	26.4	**10.5**	8.0	9.02
TNL	4K	61.80	72.31	49.88	55.33	57.91	29.10	32.20	**51.22**	23.9	10.7	12.1	**11.79**
TNL	8K	54.53	71.60	49.94	55.41	58.50	29.01	34.20	50.45	**20.6**	11.7	**28.0**	9.65
TNL	16K	49.05	71.06	49.51	51.46	57.53	28.16	31.60	48.34	**20.6**	11.3	14.2	8.74
HGRN2	2K	62.23	72.25	50.68	54.70	60.02	30.03	33.40	**51.90**	22.2	**10.0**	2.3	11.60
HGRN2	4K	61.77	70.95	51.21	53.59	60.19	30.89	31.20	51.40	20.9	10.6	2.1	11.46
HGRN2	8K	59.54	71.82	50.65	54.85	60.40	29.61	34.20	51.58	20.3	10.7	**10.0**	11.08
HGRN2	16K	54.92	72.03	50.37	55.25	59.01	28.92	32.00	50.36	**20.1**	11.2	8.8	**12.23**
cos2	2K	60.95	70.35	47.37	53.43	56.44	27.30	31.00	**49.55**	22.7	**12.0**	6.8	10.93
cos2	4K	55.66	70.08	47.05	50.99	55.35	27.13	33.00	48.46	**21.3**	12.5	6.5	11.79
cos2	8K	56.42	70.57	45.99	52.01	57.49	26.11	32.00	48.66	**21.3**	15.3	**10.9**	10.88
cos2	16K	62.51	69.86	44.61	52.72	54.25	26.02	32.20	48.88	22.1	16.9	9.6	**13.04**

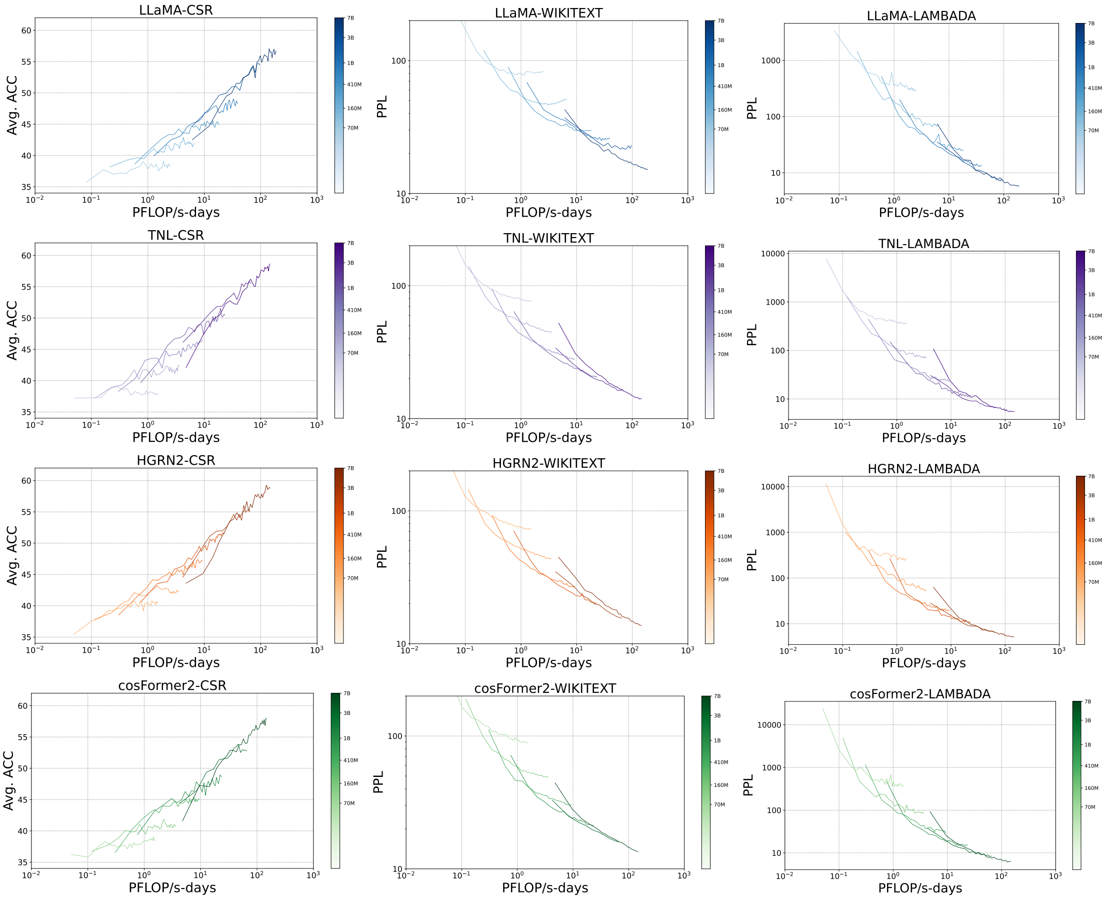

Figure 2: **Comparative performance across distinct benchmarks** illustrating the scaling trends observed in evaluation metrics. The figure highlights the progressive improvement in model performance as the complexity and size of the models increase, underscoring the significance of scaling in enhancing benchmark outcomes.

a larger aspect ratio can lead to failures in these tasks. Specifically, cosFormer2 with a 3072 dimension results in a collapse in NIAH and SCROLLS evaluations. Models with linear complexity in their sequences are more sensitive to aspect ratio changes than the vanilla transformer models.

5.2 Pre-training context length

We further examined the impact of pre-training on the performance of downstream tasks. Table 6 indicates that CSR and validation perplexity for all linear complexity sequence models remains unaffected by pre-training context lengths of 2K, 4K, and 8K. However, extending the context length to 16K slightly degrades performance.

When increasing the pre-training context length from 8K to 16K, all linear complexity sequence models fail to retrieve longer contexts in both NIAH and SCROLLS tasks. In contrast, LLaMA's retrieval capabilities double when the pre-training context length is increased to 16K from 8K. Moreover, shorter pre-training context lengths have a detrimental effect on retrieval tasks for linear complexity sequence models.

5.3 Decay types for linear complexity models

As outlined in our preliminary study, TNL, HGRN2, and cosFormer2 utilize three distinct decay strategies for linear complexity sequence models: data-independent decay, data-dependent decay, and no decay. Analyzing Tables 4 and 6, we find that cosFormer2, which employs a no-decay linear attention mechanism, performs worse than TNL (which uses data-independent decay) in terms of CSR and validation perplexity. However, cosFormer2 shows superior retrieval capabilities in NIAH and SCROLLS tasks. Meanwhile, HGRN2, which uses data-dependent decay RNN, displays performance on par with no-decay linear attention in retrieval and generation tasks and matches the performance of data-independent decay in CSR and validation perplexity.

5.4 Retrieval capacity of linear models

Our experimental results suggest that linear complexity models have limited capacity in retrieval tasks. This is because linear models maintain a fixed size of hidden space, which makes it difficult to retain input information precisely. Then a natural question arises: "Why does softmax attention have this capability? Can we address this limitation for linear models?"

To answer this question, let us consider a softmax attention:

$$\mathbf{O} = \mathrm{Softmax}(\mathbf{Q}\mathbf{K}\top/\sqrt{d})\mathbf{V}. \tag{1}$$

It can be rewritten into a linear recurrent form as:

$$\begin{gathered} s_t^0 = 0, s_t^j = s_t^{j-1} + \exp(\mathbf{q}_t \mathbf{k}_j^T/\sqrt{d}), \\ \mathbf{o}_t^j = (s_t^{j-1}/s_t^j)\mathbf{o}_t^{j-1} + (1 - s_t^{j-1}/s_t^j)\mathbf{v}_j, \\ \mathbf{o}_t = \mathbf{o}_t^t, j = 1, \ldots, t. \end{gathered} \tag{2}$$

Note that the linear recurrence form of Linear Attention is as follows:

$$\begin{gathered} \mathbf{kv}_t = 0, \mathbf{kv}_t = \mathrm{diag}\{\lambda_t\}\mathbf{kv}_{t-1} + \mathbf{k}_t\mathbf{v}_t^\top \\ \mathbf{o}_t = \mathbf{kv}_t^\top \mathbf{q}_t, j = 1, \ldots, t. \end{gathered} \tag{3}$$

The softmax attention can be interpreted as an additive linear RNN (Qin et al., 2024a). At each time step t, the hidden space is recomputed starting from the initial time $t_0 = 1$, a process referred to as "Going Through a Book (GTB)". This approach allows the model to accurately retain input information by revisiting previous information. For linear models, there is no recomputation. Therefore, linear models struggle to accurately retain input data without a GTB process.

6 Related work

Scaling laws in large language models aim for an ideal balance between increasing the number of parameters and enlarging the training corpus, given limited computation resources (Kaplan et al., 2020; Henighan et al., 2020; Hernandez et al., 2021; Hoffmann et al., 2022; Clark et al., 2022). The initial scaling laws (Kaplan et al., 2020) use the test-time cross-entropy loss as a regression target to investigate its power-law correlations with model size, dataset size and training computation budget. Hoffmann et al. (2022) use three approaches to find the optimal model size and dataset size given a fixed computation budget. By 1) freezing model size and varying number of training tokens, 2) fixing FLOPs and changing model sizes and dataset sizes and 3) directly solving a constrained optimization equation, they conclude that models and the training corpus should be scaled equally when enlarging computing resources. They use the revised scaling law to train a compute-optimal model, *Chinchilla*, that stands out across various benchmarks. Other works extend scaling laws to multiple modalities (Henighan et al., 2020), mixture of expert models (Clark et al., 2022) and reinforcement learning (Hilton et al., 2023). Recently, Su et al. (2024); Bi et al. (2024) studied the influence of additional factors such as learning rate, context length, and batch size on the scaling-law coefficients. (Isik et al., 2024) studies scaling laws of downstream task performance in a transfer learning setting for the machine translation task.

7 Conclusion

Our comprehensive study has demonstrated that linear complexity language models, including TNL, HGRN2, and cosFormer2, exhibit competitive scaling capabilities akin to transformer-based models while also showcasing enhanced linguistic proficiency and knowledge retention. With rigorous training across a vast parameter range and extensive evaluation of diverse tasks, our findings validate these models as promising contenders for future large-scale language model development.

Limitations

- We train all models on a fixed dataset, thus overlooking the influence of data distribution on scaling laws.
- For each model architecture, we only experiment with six different model sizes, resulting in fewer data points than previous works in terms of fitting the loss-computation curve.
- We use fixed learning rate scheduler and batch size across experiments.

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A Appendix

A.1 Model parameters and FLOPs

Here we provided detailed FLOPs and the number of model parameters calculation for each model architecture. Some operations are omitted for simplicity, e.g. the FLOPs and parameters related to positional encoding, normalization, activation functions, and softmax of the final head, if applicable. We parameterize models with the following notations:

- d: attention hidden dimension.
- h: number of heads in attention.
- g: GLU hidden dimension. (In all scenarios, we use $g = 8/3d$.)
- l: number of layers.
- n: input sequence length.
- v: vocabulary size.
- b: batch size.
- t: output gate bottleneck dimension.
- B lightning attention/flash linear attention block size. (In all scenarios, we use $B = d/h$.)
- e: Tpe hidden dimension.

Similar to (Kaplan et al., 2020; Hoffmann et al., 2022), we use a factor of 2 to represent the multiplication-accumulation in matrice products, and a factor of 3 to include both the forward and the backward pass. In the following discussion, we assume f is the swish function.

A.1.1 Transformer-LLaMA

Equation Input embedding:

$$\begin{aligned}&\mathbf{X}^{(1)} = \mathrm{Lookup}(\mathbf{X}^{(0)}, \mathbf{W}_{in}),\\&\mathbf{X}^{(0)} \in \mathbb{R}^{n}, \mathbf{W}_{in} \in \mathbb{R}^{d\times v}.\end{aligned} \tag{4}$$

Token mixer(s-th layer):

$$\begin{aligned}&\bar{\mathbf{X}}^{(s)} = \mathrm{Norm}(\mathbf{X}^{(s)}),\\&\mathbf{Q}_i^{(s)}, \mathbf{K}_i^{(s)}, \mathbf{V}_i^{(s)} = \bar{\mathbf{X}}^{(s)}\mathbf{W}_{q_i}^{(s)}, \bar{\mathbf{X}}^{(s)}\mathbf{W}_{k_i}^{(s)}, \bar{\mathbf{X}}^{(s)}\mathbf{W}_{v_i}^{(s)},\\&\mathbf{O}_i^{(s)} = \mathrm{Softmax}\left(\mathbf{Q}_i^{(s)}\mathbf{K}_i^{(s)\top}/\sqrt{d/h}\right)\mathbf{V}_i^{(s)},\\&\mathbf{O}^{(s)} = \mathrm{Concat}[\mathbf{O}_1^{(s)}, \ldots, \mathbf{O}_h^{(s)}]\mathbf{W}_o^{(s)} + \mathbf{X}^{(s)},\\&\mathbf{X}^{(s)} \in \mathbb{R}^{n\times d}, \mathbf{W}_o^{(s)} \in \mathbb{R}^{d\times d},\\&\mathbf{W}_{q_i}^{(s)}, \mathbf{W}_{k_i}^{(s)}, \mathbf{W}_{v_i}^{(s)} \in \mathbb{R}^{d\times d/h}, i = 1, \ldots, h.\end{aligned} \tag{5}$$

Channel mixer(s-th layer):

$$\begin{aligned}&\bar{\mathbf{O}}^{(s)} = \mathrm{Norm}(\mathbf{O}^{(s)}),\\&\mathbf{U}^{(s)}, \mathbf{V}^{(s)} = \bar{\mathbf{O}}^{(s)}\mathbf{W}_u^{(s)}, \bar{\mathbf{O}}^{(s)}\mathbf{W}_v^{(s)},\\&\mathbf{X}^{(s+1)} = [\mathbf{U}^{(s)} \odot f(\mathbf{V}^{(s)})]\mathbf{W}_{down}^{(s)} + \mathbf{O}^{(s)},\\&\mathbf{O}^{(s)} \in \mathbb{R}^{n\times d},\\&\mathbf{W}_u^{(s)}, \mathbf{W}_v^{(s)} \in \mathbb{R}^{d\times g}, \mathbf{W}_{down}^{(s)} \in \mathbb{R}^{g\times d}.\end{aligned} \tag{6}$$

Output embedding:

$$\begin{aligned}&\mathbf{O} = \mathbf{X}^{(l+1)}\mathbf{W}_{out},\\&\mathbf{X}^{(l+1)} \in \mathbb{R}^{n\times d}, \mathbf{W}_{out} \in \mathbb{R}^{d\times v}.\end{aligned} \tag{7}$$

FLOPs

- Input embedding: $[n] \times [d, v] \implies 2ndv$.
- Token mixer: $8nd^2 + 4n^2d + 3n^2h + 4nd$.
  - qkv projection:
    $$[n, d] \times [d, 3d] \implies 3 \times 2nd^2.$$
  - qk multiplication:
    $$[h, n, d/h] \times [h, d/h, n] \implies 2n^2d.$$
  - RoPE: $4nd$.
  - Softmax: exp, sum, divide $\implies 3n^2h$.
  - (qk)v multiplication:
    $$[h, n, n] \times [h, d/h, n] \implies 2n^2d.$$
  - output projection:
    $$[n, d] \times [d, d] \implies 2nd^2.$$
- Channel mixer: $6ndg + ng$.
  - u,v projection:
    $$[n, d] \times [d, g] \implies 4 \times ndg.$$
  - gating:
    $$[n, g] \odot [n, g] \implies ng.$$
  - down projection:
    $$[n, g] \times [g, d] \implies 2 \times ndg.$$
- Output embedding:
  $$[n, d] \times [d, v] \implies 2ndv.$$
- Forward FLOPs: $bl \times (8nd^2 + 4n^2d + 3n^2h + 4nd + 6ndg + ng) + 4ndv$

Table 7: **Model Parameters and FLOPs.** The listed numbers of parameters do not include the embedding parameters.

Configuration	b	n	l	d	h	g	v	Parameters (Million)	PFLOPs / Step
LLaMA-70M	480	8192	6	512	4	1536	100280	20.5	1.6
LLaMA-160M	480	8192	12	768	6	2048	100280	85.0	5.6
LLaMA-410M	480	8192	26	1024	8	2816	100280	334.1	18.0
LLaMA-1B	480	8192	32	1536	16	5632	100280	906.2	40.4
LLaMA-3B	480	8192	35	2560	20	6912	100280	2775.8	99.6
LLaMA-7B	480	8192	32	4096	32	11008	100280	6476.5	202.7
TNL-70M	480	8192	6	512	4	1536	100280	21.2	0.5
TNL-160M	480	8192	12	768	6	2048	100280	87.3	2.2
TNL-410M	480	8192	25	1024	8	2816	100280	327.7	7.9
TNL-1B	480	8192	32	1536	16	5632	100280	918.6	22.3
TNL-3B	480	8192	35	2560	20	6912	100280	2798.4	66.6
TNL-7B	480	8192	32	4096	32	11008	100280	6509.6	154.4
HGRN2-70M	480	8192	6	512	4	1536	100280	20.5	0.5
HGRN2-160M	480	8192	12	768	6	2048	100280	84.9	2.1
HGRN2-410M	480	8192	26	1024	8	2816	100280	334.0	8.0
HGRN2-1B	480	8192	32	1536	16	5632	100280	906.0	22.0
HGRN2-3B	480	8192	35	2560	20	6912	100280	2775.5	66.0
HGRN2-7B	480	8192	32	4096	32	11008	100280	6476.1	153.6
cos2-70M	480	8192	6	512	4	1536	100280	21.3	0.5
cos2-160M	480	8192	12	768	6	2048	100280	87.5	2.3
cos2-410M	480	8192	25	1024	8	2816	100280	328.0	8.1
cos2-1B	480	8192	32	1536	16	5632	100280	919.1	22.7
cos2-3B	480	8192	35	2560	20	6912	100280	2799.0	67.4
cos2-7B	480	8192	32	4096	32	11008	100280	6511.0	155.6

- Total training FLOPS: $bl \times (24nd^2 + 12n^2d + 9n^2h + 18ndg + 3ng) + 12ndv$
- Substituting $g = 8/3d$ yields:

$$bl(72nd^2 + 12n^2d + 9n^2h + 20nd) + 12ndv$$
$$= \underbrace{72bnld^2\left(1 + \frac{n}{6d} + \frac{5}{18d} + \frac{h}{8d^2}\right)}_{\text{Non-embedding term}} + \underbrace{12ndv}_{\text{Embedding term}}.$$

Parameters

- Input & output embedding (shared weights): dv.
- Token mixer: $4d^2$.
- Channel mixer: $3dg$.
- Total parameters: $4ld^2 + 3ldg + dv$.
- Substituting $g = 8/3d$ yields: $12ld^2 + dv$.

A.1.2 Linear Attention - TNL (data independent decay)

For TNL, we use $0 < \lambda_i < 1$ as the decay of head i and use LA as the abbreviation for Lightning Attention.

Equation Input embedding:

$$\begin{aligned}&\mathbf{X}^{(1)} = \mathrm{Lookup}(\mathbf{X}^{(0)}, \mathbf{W}_{in}),\\&\mathbf{X}^{(0)} \in \mathbb{R}^{n}, \mathbf{W}_{in} \in \mathbb{R}^{d\times v}.\end{aligned} \tag{8}$$

Token mixer(s-th layer):

$$\begin{aligned}&\bar{\mathbf{X}}^{(s)} = \mathrm{Norm}(\mathbf{X}^{(s)}),\\&\mathbf{Q}_i^{(s)}, \mathbf{K}_i^{(s)}, \mathbf{V}_i^{(s)}\\&\quad = f(\bar{\mathbf{X}}^{(s)}\mathbf{W}_{q_i}^{(s)}), f(\bar{\mathbf{X}}^{(s)}\mathbf{W}_{k_i}^{(s)}), \bar{\mathbf{X}}^{(s)}\mathbf{W}_{v_i}^{(s)},\\&\mathbf{G}^{(s)} = \mathrm{Sigmoid}\left(\mathbf{X}\mathbf{W}_{gdown}^{(s)}\mathbf{W}_{gup}^{(s)}\right),\\&\mathbf{O}_i^{(s)} = \mathrm{LA}\left(\mathbf{Q}_i^{(s)}, \mathbf{K}_i^{(s)}, \mathbf{V}_i^{(s)}, \lambda_i\right),\\&\mathbf{O}^{(s)} = \mathrm{Norm}\left(\mathrm{Concat}[\mathbf{O}_1^{(s)}, \ldots, \mathbf{O}_h^{(s)}]\right) \odot \mathbf{G}^{(s)} + \mathbf{X}^{(s)},\\&\mathbf{X}^{(s)} \in \mathbb{R}^{n\times d},\\&\mathbf{W}_{gdown}^{(s)} \in \mathbb{R}^{d\times t}, \mathbf{W}_{gup}^{(s)} \in \mathbb{R}^{t\times d}, \mathbf{W}_o^{(s)} \in \mathbb{R}^{d\times d},\\&\mathbf{W}_{q_i}^{(s)}, \mathbf{W}_{k_i}^{(s)}, \mathbf{W}_{v_i}^{(s)} \in \mathbb{R}^{d\times d/h}, i = 1, \ldots, h.\end{aligned} \tag{9}$$

Channel mixer(s-th layer):

$$\begin{aligned}&\bar{\mathbf{O}}^{(s)} = \mathrm{Norm}(\mathbf{O}^{(s)}),\\&\mathbf{U}^{(s)}, \mathbf{V}^{(s)} = \bar{\mathbf{O}}^{(s)}\mathbf{W}_u^{(s)}, \bar{\mathbf{O}}^{(s)}\mathbf{W}_v^{(s)},\\&\mathbf{X}^{(s+1)} = [\mathbf{U}^{(s)} \odot \mathbf{V}^{(s)}]\mathbf{W}_{down}^{(s)} + \mathbf{O}^{(s)},\\&\mathbf{O}^{(s)} \in \mathbb{R}^{n\times d},\\&\mathbf{W}_u^{(s)}, \mathbf{W}_v^{(s)} \in \mathbb{R}^{d\times g}, \mathbf{W}_{down}^{(s)} \in \mathbb{R}^{g\times d}.\end{aligned} \tag{10}$$

Output embedding:

$$\begin{aligned}&\mathbf{O} = \mathbf{X}^{(l+1)}\mathbf{W}_{out},\\ &\mathbf{X}^{(l+1)} \in \mathbb{R}^{n\times d}, \mathbf{W}_{out} \in \mathbb{R}^{d\times v}.\end{aligned} \tag{11}$$

FLOPs

- Input embedding: $[n] \times [d, v] \implies 2ndv.$
- Token Mixer: $C = 8nd^2 + \frac{4nd^2}{h} + \frac{nd^2}{Bh} + 4nBd + nBh + 4ntd + 2nd.$
  - qkv projection:
    $$[n, d] \times [d, 3d] \implies 3 \times 2nd^2.$$
  - lightning attention intra block:
    $$\text{repeat } \frac{n}{B} \text{ times} \implies 4B^2d + B^2h.$$
  - lightning attention inter block:
    $$\text{repeat } \frac{n}{B} \text{ times} \implies \frac{2Bd^2}{h}.$$
  - kv update: repeat
    $$\frac{n}{B} \text{ times} \implies \frac{2Bd^2}{h} + \frac{d^2}{h}.$$
  - attention output update:
    $$\text{repeat } \frac{n}{B} \text{ times} \implies Bd.$$
  - output gate:
    $$[n, d] \times [d, t],\ [n, t] \times [t, d] \implies 4ntd.$$
  - gating:
    $$[n, d] \odot [n, d] \implies nd.$$
  - output projection:
    $$[n, d] \times [d, d] \implies 2nd^2.$$
- Channel Mixer: $6ndg + ng.$
  - u,v projection:
    $$[n, d] \times [d, g] \implies 4 \times ndg.$$
  - gating:
    $$[n, g] \odot [n, g] \implies ng.$$
  - down projection:
    $$[n, g] \times [g, d] \implies 2 \times ndg.$$
- Output embedding: $[n, d] \times [d, v] \implies 2ndv.$
- Forward FLOPs: $l \times (C + 6ndg + ng) + 4ndv.$
- Total training FLOPs: $bl(3C + 18ndg + 3ng) + 12bndv.$
- Substituting $g = 8/3d, B = t = d/h$ yields:

$$\begin{aligned}&C = 8nd^2 + 12\frac{nd^2}{h} + 4nd,\\ &\text{FLOPs}\\ &= bl\left(72nd^2 + \frac{36nd^2}{h} + 20nd\right) + 12ndv\\ &= \underbrace{72bnld^2\left(1 + \frac{1}{2h} + \frac{5}{18d}\right)}_{\text{Non-embedding term}}\\ &+ \underbrace{12ndv}_{\text{Embedding term}}.\end{aligned}$$

Parameters

- Input & output embedding (shared weights): $dv.$
- Token Mixer: $4d^2 + 2dt.$
- Channel Mixer: $3dg.$
- Total parameters: $4ld^2 + 2ldt + 3ldg + dv.$
- Substituting $g = 8/3d, t = d/h$ yields: $12ld^2 + 2ld^2/h + dv.$

A.1.3 Linear RNN - HGRN2 (data dependent decay)

We use FLA as the abbreviation for Flash Linear Attention.

Equation Input embedding:

$$\begin{aligned}&\mathbf{X}^{(1)} = \text{Lookup}(\mathbf{X}^{(0)}, \mathbf{W}_{in}),\\ &\mathbf{X}^{(0)} \in \mathbb{R}^{n}, \mathbf{W}_{in} \in \mathbb{R}^{d\times v}.\end{aligned} \tag{12}$$

Lower bound:

$$\begin{aligned}&\overline{\mathbf{LR}}_i = \text{Softmax}(\mathbf{LR}_i, \dim = 0),\\ &\mathbf{Lr}_i^{(s)} = \text{Cumsum}(\overline{\mathbf{LR}}_i, \dim = 0)[s],\\ &\mathbf{LR}_i \in \mathbb{R}^{L\times d/h}, i = 1, \ldots, h.\end{aligned}$$

Token mixer(s-th layer):

$$
\begin{aligned}
&\bar{\mathbf{X}}^{(s)} = \mathrm{Norm}(\mathbf{X}^{(s)}),\\
&\mathbf{Og}_i^{(s)}, \mathbf{Fg}_i^{(s)}, \mathbf{H}_i^{(s)} = \bar{\mathbf{X}}^{(s)}\mathbf{W}_{og_i}^{(s)}, \bar{\mathbf{X}}^{(s)}\mathbf{W}_{fg_i}^{(s)}, \bar{\mathbf{X}}^{(s)}\mathbf{W}_{h_i}^{(s)},\\
&\mathbf{Fg}_i^{(s)} = \mathbf{Lr}_i^{(s)} + (1 - \mathbf{Lr}_i^{(s)})(\mathrm{Sigmoid}(\mathbf{Fg}_i^{(s)})),\\
&\mathbf{O}_i^{(s)} = \mathrm{FLA}\left(\mathbf{Og}_i^{(s)}, \mathbf{Fg}_i^{(s)}, \mathbf{H}_i^{(s)}, 1 - \mathbf{Fg}_i^{(s)}\right),\\
&\mathbf{O}^{(s)} = \mathrm{Norm}\left(\mathrm{Concat}[\mathbf{O}_1^{(s)}, \ldots, \mathbf{O}_h^{(s)}]\right) + \mathbf{X}^{(s)},\\
&\mathbf{X}^{(s)} \in \mathbb{R}^{n\times d},\\
&\mathbf{W}_{gdown}^{(s)} \in \mathbb{R}^{d\times t}, \mathbf{W}_{gup}^{(s)} \in \mathbb{R}^{t\times d}, \mathbf{W}_o^{(s)} \in \mathbb{R}^{d\times d},\\
&\mathbf{W}_{og_i}^{(s)}, \mathbf{W}_{fg_i}^{(s)}, \mathbf{W}_{h_i}^{(s)} \in \mathbb{R}^{d\times d/h}, i = 1, \ldots, h.
\end{aligned}
\tag{13}
$$

Channel mixer(s-th layer):

$$
\begin{aligned}
&\bar{\mathbf{O}}^{(s)} = \mathrm{Norm}(\mathbf{O}^{(s)}),\\
&\mathbf{U}^{(s)}, \mathbf{V}^{(s)} = \bar{\mathbf{O}}^{(s)}\mathbf{W}_u^{(s)}, \bar{\mathbf{O}}^{(s)}\mathbf{W}_v^{(s)},\\
&\mathbf{X}^{(s+1)} = [\mathbf{U}^{(s)} \odot \mathbf{V}^{(s)}]\mathbf{W}_{down}^{(s)} + \mathbf{O}^{(s)},\\
&\mathbf{O}^{(s)} \in \mathbb{R}^{n\times d},\\
&\mathbf{W}_u^{(s)}, \mathbf{W}_v^{(s)} \in \mathbb{R}^{d\times g}, \mathbf{W}_{down}^{(s)} \in \mathbb{R}^{g\times d},
\end{aligned}
\tag{14}
$$

Output embedding:

$$
\begin{aligned}
&\mathbf{O} = \mathbf{X}^{(l+1)}\mathbf{W}_{out},\\
&\mathbf{X}^{(l+1)} \in \mathbb{R}^{n\times d}, \mathbf{W}_{out} \in \mathbb{R}^{d\times v}.
\end{aligned}
\tag{15}
$$

FLOPs

- Input embedding: $[n] \times [d, v] \implies 2ndv.$
- Lower bound: $4ld.$
- Token Mixer: $C = 8nd^2 + \frac{4nd^2}{h} + \frac{nd^2}{Bh} + 4nBd + nBh + 5nd.$
  - hidden state projection:
    $$[n, d] \times [d, 3d] \implies 3 \times 2nd^2.$$
  - forget gate compute: $4nd.$
  - fla intra block:
    $$\text{repeat } \frac{n}{B} \text{ times} \implies 4B^2d + B^2h.$$
  - fla inter block:
    $$\text{repeat } \frac{n}{B} \text{ times} \implies \frac{2Bd^2}{h}.$$
  - state update: repeat
    $$\frac{n}{B} \text{ times} \implies \frac{2Bd^2}{h} + \frac{d^2}{h}.$$
  - attention output update:
    $$\text{repeat } \frac{n}{B} \text{ times} \implies Bd.$$
  - output projection:
    $$[n, d] \times [d, d] \implies 2nd^2.$$
- Channel Mixer: $6ndg + ng.$
  - u,v projection:
    $$[n, d] \times [d, g] \implies 4 \times ndg.$$
  - gating:
    $$[n, g] \odot [n, g] \implies ng.$$
  - down projection:
    $$[n, g] \times [g, d] \implies 2 \times ndg.$$
- Output embedding: $[n, d] \times [d, v] \implies 2ndv.$
- Forward FLOPs: $l \times (C + 6ndg + ng) + 4ndv + 4dl.$
- Total training FLOPs: $bl(3C + 18ndg + 3ng) + 12bndv + 12dl.$
- Substituting $g = 8/3d, B = d/h.$ yields:

$$
\begin{aligned}
&C = 8nd^2 + \frac{8nd^2}{h} + 7nd,\\
&\text{FLOPs}\\
=&bl\left(72nd^2 + \frac{24nd^2}{h} + 29nd\right)\\
&+ 12ndv + 12dl\\
=&\underbrace{72bnld^2\left(1 + \frac{1}{3h} + \frac{29}{72d}\right) + 12dl}_{\text{Non-embedding term}}\\
&+ \underbrace{12ndv}_{\text{Embedding term}}.
\end{aligned}
$$

Parameters

- Lower bound: $ld.$
- Input & output embedding (shared weights): $dv.$
- Token Mixer: $4d^2.$
- Channel Mixer: $3dg.$
- Total parameters: $4ld^2 + 3ldg + dv + ld.$
- Substituting $g = 8/3d$ yields: $12ld^2 + dv + ld.$

A.1.4 Linear Attention - cosFormer2

For cosFormer2, we use $\theta_i \in \mathbb{R}^{d/h}$ as the Lrpe parameter of head i and use LA as the abbreviation for Lightning Attention.

Equation Input embedding:

$$\begin{aligned} &\mathbf{X}^{(1)} = \mathrm{Lookup}(\mathbf{X}^{(0)}, \mathbf{W}_{in}), \\ &\mathbf{X}^{(0)} \in \mathbb{R}^{n}, \mathbf{W}_{in} \in \mathbb{R}^{d\times v}. \end{aligned} \tag{16}$$

Tpe:

$$\mathbf{X}^{(1)} = \mathrm{Tpe}(\mathbf{X}^{(1)}).$$

Token mixer(s-th layer):

$$\begin{aligned} &\bar{\mathbf{X}}^{(s)} = \mathrm{Norm}(\mathbf{X}^{(s)}), \\ &\mathbf{Q}_i^{(s)}, \mathbf{K}_i^{(s)}, \mathbf{V}_i^{(s)} = f(\bar{\mathbf{X}}^{(s)}\mathbf{W}_{q_i}^{(s)}), f(\bar{\mathbf{X}}^{(s)}\mathbf{W}_{k_i}^{(s)}), \bar{\mathbf{X}}^{(s)}\mathbf{W}_{v_i}^{(s)}, \\ &\mathbf{Q}_i^{(s)} = \mathrm{Concat}[\cos(\theta_i^{(s)})\mathbf{Q}_i^{(s)}, \sin(\theta_i^{(s)})\mathbf{Q}_i^{(s)}], \\ &\mathbf{K}_i^{(s)} = \mathrm{Concat}[\cos(\theta_i^{(s)})\mathbf{K}_i^{(s)}, \sin(\theta_i^{(s)})\mathbf{K}_i^{(s)}], \\ &\mathbf{G}^{(s)} = \mathrm{Sigmoid}\left(\mathbf{X}\mathbf{W}_{gdown}^{(s)}\mathbf{W}_{gup}^{(s)}\right), \\ &\mathbf{O}_i^{(s)} = \mathrm{LA}\left(\mathbf{Q}_i^{(s)}, \mathbf{K}_i^{(s)}, \mathbf{V}_i^{(s)}\right), \\ &\mathbf{O}^{(s)} = \mathrm{Norm}\left(\mathrm{Concat}[\mathbf{O}_1^{(s)}, \ldots, \mathbf{O}_h^{(s)}]\right) \odot \mathbf{G}^{(s)} + \mathbf{X}^{(s)}, \\ &\mathbf{X}^{(s)} \in \mathbb{R}^{n\times d}, \\ &\mathbf{W}_{gdown}^{(s)} \in \mathbb{R}^{d\times t}, \mathbf{W}_{gup}^{(s)} \in \mathbb{R}^{t\times d}, \mathbf{W}_{o}^{(s)} \in \mathbb{R}^{d\times d}, \\ &\mathbf{W}_{q_i}^{(s)}, \mathbf{W}_{k_i}^{(s)}, \mathbf{W}_{v_i}^{(s)} \in \mathbb{R}^{d\times d/h}, i = 1, \ldots, h. \end{aligned} \tag{17}$$

Channel mixer(s-th layer):

$$\begin{aligned} &\bar{\mathbf{O}}^{(s)} = \mathrm{Norm}(\mathbf{O}^{(s)}), \\ &\mathbf{U}^{(s)}, \mathbf{V}^{(s)} = \bar{\mathbf{O}}^{(s)}\mathbf{W}_u^{(s)}, \bar{\mathbf{O}}^{(s)}\mathbf{W}_v^{(s)}, \\ &\mathbf{X}^{(s+1)} = [\mathbf{U}^{(s)} \odot \mathbf{V}^{(s)}]\mathbf{W}_{down}^{(s)} + \mathbf{O}^{(s)}, \\ &\mathbf{O}^{(s)} \in \mathbb{R}^{n\times d}, \\ &\mathbf{W}_u^{(s)}, \mathbf{W}_v^{(s)} \in \mathbb{R}^{d\times g}, \mathbf{W}_{down}^{(s)} \in \mathbb{R}^{g\times d}, \end{aligned} \tag{18}$$

Output embedding:

$$\begin{aligned} &\mathbf{O} = \mathbf{X}^{(l+1)}\mathbf{W}_{out}, \\ &\mathbf{X}^{(l+1)} \in \mathbb{R}^{n\times d}, \mathbf{W}_{out} \in \mathbb{R}^{d\times v}. \end{aligned} \tag{19}$$

FLOPs

- Input embedding: $[n] \times [d, v] \implies 2ndv$.
- Tpe: $4nde$.
  - Up projection: $[n, d] \times [d, e] \implies 2nde$.
  - Recurrence: nde.
  - Down projection: $[n, d, e] \implies nde$.
- Token Mixer: $C = 8nd^2 + \frac{8nd^2}{h} + \frac{2nd^2}{Bh} + 6nBd + nBh + 4ntd + 2nd$.
  - qkv projection:

    $$[n, d] \times [d, 3d] \implies 3 \times 2nd^2.$$

  - Lrpe: $4nd$.
  - lightning attention intra block:

    $$\text{repeat } \frac{n}{B} \text{ times} \implies 6B^2d + B^2h.$$

  - lightning attention inter block:

    $$\text{repeat } \frac{n}{B} \text{ times} \implies \frac{4Bd^2}{h}.$$

  - kv update: repeat

    $$\frac{n}{B} \text{ times} \implies \frac{4Bd^2}{h} + \frac{2d^2}{h}.$$

  - attention output update:

    $$\text{repeat } \frac{n}{B} \text{ times} \implies Bd.$$

  - output gate:

    $$[n, d] \times [d, t],\ [n, t] \times [t, d] \implies 4ntd.$$

  - gating:

    $$[n, d] \odot [n, d] \implies nd.$$

  - output projection:

    $$[n, d] \times [d, d] \implies 2nd^2.$$

- Channel Mixer: $6ndg + ng$.
  - u,v projection:

    $$[n, d] \times [d, g] \implies 4 \times ndg.$$

  - gating:

    $$[n, g] \odot [n, g] \implies ng.$$

  - down projection:

    $$[n, g] \times [g, d] \implies 2 \times ndg.$$

- Output embedding: $[n, d] \times [d, v] \implies 2ndv$.
- Forward FLOPs: $l \times (C + 6ndg + ng) + 4ndv + 4nde$.
- Total training FLOPs: $bl(3C + 18ndg + 3ng) + 12bndv + 12bnde$.

Table 8: **Supplemental Benchmark of Downstream Task of Mamba1 and Mamba2: Common Sense Reasoning (CSR) and Validation Perplexity.** For CSR, higher scores indicate better performance. For Validation Perplexity, lower scores are preferable. PS: parameter size (billion). HS: HellaSwag. WG: WinoGrande. OBQA: OpenBookQA. WIKI: WIKITEXT-2. acc_n.: acc_norm. We provide the average score for CSR.

Arch	P.S.	BoolQ	PIQA	HS	WG	ARC-E	ARC-C	OBQA	CSR	WIKI
	B	acc	acc	acc_n.	acc	acc	acc_n.	acc_n.	avg ↑	ppl.↓
Mamba1	0.41	60.21	67.85	40.75	51.93	52.99	24.83	31.40	47.14	27.0
Mamba2	0.41	52.75	68.50	41.27	53.04	53.28	25.77	31.80	46.63	24.0
Mamba1	1	60.73	71.49	50.08	54.46	58.67	29.35	30.80	50.8	20.1
Mamba2	1	62.14	71.98	50.98	55.17	58.21	27.39	34.80	51.52	19.4
Mamba1	3	60.46	74.81	61.72	60.22	65.19	35.49	35.00	56.13	15.1
Mamba2	3	59.48	75.19	62.12	59.75	65.57	34.30	36.40	56.12	14.3

- Substituting $g = 8/3d, B = e = d/h$ yields:

$$
\begin{aligned}
&C = 8nd^2 + 18\frac{nd^2}{h} + 5nd,\\
&\text{FLOPs}\\
&=bl\left(72nd^2 + \frac{54nd^2}{h} + 23nd\right)\\
&\quad + 12ndv + \frac{12bnd^2}{h}\\
&=\underbrace{72bnld^2\left(1 + \frac{3}{4h} + \frac{23}{72d}\right) + \frac{12bnd^2}{h}}_{\text{Non-embedding term}}\\
&\quad + \underbrace{12ndv}_{\text{Embedding term}}.
\end{aligned}
$$

Parameters

- Input & output embedding (shared weights): dv.
- Tpe: de.
- Token Mixer: $4d^2 + 2dt$.
- Channel Mixer: $3dg$.
- Total parameters: $4ld^2+2ldt+3ldg+dv+de$.
- Substituting $g = 8/3d, t = e = d/h$ yields: $12ld^2 + 2ld^2/h + dv + d^2/h$.

B Evaluations

B.1 Metrics of Needle in A Haystack

We use four types metrics in NIAH evaluation:

Accuracy at a context length. This averages the retrieval accuracy at a chosen context length across all depth steps (acc@seq_len in Table 12).

Accuracy less or equal to a context length. This metric calculates the mean accuracy over a range of context lengths, across all depth steps (acc≤seq_len in Table 12). The total averaged accuracy is the accuracy less or equal to the maximum context length, which is 16k in all our experiments.

Weighted average accuracy. To further represent the levels of complexity for retrieving a needle in different depths and context lengths, we assign weights to each depth and context length. We assume larger weights for deeper and longer texts. We use geometric progression as a weight function for both aspects. Specifically, the weights are calculated as: $w_{d_i} = w_{d_0}{\alpha_d}^{i-1}$, $w_{c_i} = w_{c_0}{\alpha_c}^{i-1}$, where w_{d_i} (w_{c_i}) is the weight for i-th depth step (context length), α_d (α_c) is a constant greater than 1. Using the outer product, we obtain a weight map for all depth-length combinations. The weight map is applied when calcualting the average accuracy (weighted avg acc in Table 12).

NIAH score. Through our experiments, we observe cases when two models achieve the same average accuracy but display different patterns in the NIAH heatmap. Using weighted average can assist in this situation. To better evaluate the model ability in such cases, we develop a penalty mechanism. We first binarize the NIAH score array for success and failure, which is originally ranged from 1 to 10 using a threshold. Then for each context length, we penalize the situations when models do not consistently succeed or fail in retrieving the needle across different depths. For each column of the score array, we find the longest continuous sequence of 1s (success). If the sequence does not exist, the largest penalty is assigned ($p = 0$); If the sequence length equals the number of depth steps, no penalty is assigned ($p = 1$); Otherwise, we count the number n of continuous segments of either 1s (success) or 0s (failure), and assign penalty as $p = 2^{(1-n)/2}$. Combining weighted average and penalty, we have the NIAH score (niah score in Table 12).

Table 9: **SCROLLS Benchmark Overview.** P.S.: Parameter Size. R.1/R.2/R.L: rouge-1/rouge-2/rouge-l. The term '-' indicates a failure in the specified task.

Arch	P.S. (Billion)	GovReport (R.1/R.2/R.L)	SummScreenfd (R.1/R.2/R.L)	QmSum (R.1/R.2/R.L)	Qasper (F1)	NarrativeQA (F1)	Quality (EM)	ContractNLI (EM)	SCROLLS (Avg)
LLaMA	0.07	6.49/1.46/5.07	8.52/0.96/7.05	5.13/0.79/4.58	10.45	4.70	26.46	14.95	7.43
TNL	0.07	2.64/0.8/2.27	6.08/0.49/4.95	2.51/0.59/2.22	8.00	4.06	27.18	17.94	6.13
HGRN2	0.07	10.88/2.09/8.19	7.13/0.58/5.98	7.14/1.02/6.38	7.08	3.34	26.70	9.45	7.32
cos2	0.07	6.21/1.36/5.08	6.43/0.66/5.59	4.98/0.58/4.36	8.27	3.04	27.47	12.63	6.67
LLaMA	0.16	5.5/2.15/4.44	10.91/1.24/8.45	7.98/1.46/7.37	9.10	8.93	26.75	14.56	8.37
TNL	0.16	-	9.88/1.21/8.09	3.36/0.82/2.76	8.19	6.39	27.90	9.06	7.77
HGRN2	0.16	13.69/2.71/10.1	6.61/0.5/6.02	7.33/0.99/6.61	8.24	7.18	25.55	10.90	8.29
cos2	0.16	7.01/1.89/5.44	8.07/0.9/6.85	9.28/1.52/8.14	8.33	5.66	26.41	10.70	7.71
LLaMA	0.41	8.21/3.54/6.21	11.31/1.56/8.65	10.66/2.07/9.42	17.82	15.39	27.95	13.89	10.51
TNL	0.41	2.96/1.12/2.54	10.54/1.15/7.95	6.34/1.33/5.08	11.41	9.87	27.61	10.32	7.55
HGRN2	0.41	15.33/3.54/10.91	7.35/0.76/6.17	8.32/1.22/7.4	12.36	10.87	26.37	31.53	10.93
cos2	0.41	6.11/2.51/4.87	12.02/1.83/9.41	10.25/2.19/8.62	14.04	9.60	27.23	9.06	9.06
LLaMA	1	12.91/3.06/9.38	9.47/0.84/7.72	10.93/2.24/9.43	22.77	16.03	28.43	9.93	11.01
TNL	1	5.86/2.02/4.74	9.39/1.34/7.32	5.81/1.43/4.8	14.23	13.83	28.19	26.52	9.65
HGRN2	1	14.86/4.21/10.45	11.4/1.44/9.16	10.9/2.28/9.68	16.21	15.09	27.76	10.61	11.08
cos2	1	7.97/3.51/6.15	12.25/1.95/9.38	11.91/2.7/9.96	16.94	13.93	27.76	17.07	10.88
LLaMA	3	11.16/4.88/8.14	11.89/1.9/9.3	16.08/4.25/12.87	28.57	20.77	30.44	20.15	13.88
TNL	3	-	9.65/1.56/7.17	11.37/2.97/9.14	21.20	17.70	28.95	12.92	12.26
HGRN2	3	21.7/6.62/14.09	14.55/2.13/10.79	12.48/2.69/10.58	25.41	18.75	28.86	31.92	15.43
cos2	3	14.69/5.37/9.98	11.33/1.74/8.77	15.38/3.53/12.68	25.10	18.05	29.72	9.35	12.75
LLaMA	7	17.4/7.33/11.43	12.92/1.75/9.95	14.59/3.7/11.8	32.35	22.31	33.84	10.03	14.57
TNL	7	5.36/2.29/4.41	11.17/1.72/8.46	12.02/3.1/9.46	24.12	19.24	29.15	9.16	10.74
HGRN2	7	14.93/5.21/10.16	15.43/2.4/11.1	14.3/2.97/11.78	27.07	19.60	30.06	10.03	13.46
cos2	7	19.97/7.36/12.92	14.31/2.4/10.57	13.72/3.27/11.34	23.94	18.70	30.73	27.68	15.15

C Experiments

C.1 Downstream tasks

We conduct a thorough evaluation of downstream tasks for Mamba1 and Mamba2 in Table 8. Both models were trained using the same dataset, and our assessment includes metrics such as validation perplexity and common sense reasoning (CSR) scores.

C.2 SCROLLS

We assess models such as LLaMA, TNL, HGRN2, and cosFormer2 using the SCROLLS benchmark, focusing on different parameter sizes (refer to Table 9), aspect ratios (see Table 10), and context lengths (consult Table 11).

The table 9 provides a detailed comparison of various models such as LLaMA, TNL, HGRN2, and cosFormer2 across multiple metrics on the SCROLLS benchmark. It outlines the performance of these models based on parameter size (ranging from 0.07 to 7 billion), as detailed in each row. Table 9 highlights that across all linear complexity sequence models and LLaMA, there is a general improvement in performance with increasing parameter sizes. Models exhibit varying sensitivity to parameter size across tasks; for example, LLaMA's NarrativeQA F1 score jumps from 4.70 to 22.31 as parameters increase. At higher sizes, HGRN2 tends to outperform TNL consistently, highlighting its superior scaling capability. Additionally, models show task-specific strengths, with cos2 excelling in ContractNLI at 7 billion parameters, showcasing its effectiveness with legal texts.

C.3 NIAH analysis

The Needle in A Haystack (NIAH) evaluates language models in two modes: Easy Mode, where both the question and answer are embedded in a text for straightforward retrieval, and Standard Mode, where only the answer is embedded, requiring the model to comprehend the question and locate the answer, thereby adding complexity. In table 12, 13 and 14, the upper sub-table displays the NIAH benchmark results in easy mode, while the lower sub-table shows the results in standard mode.

Overall, LLaMA consistently outperforms other linear complexity sequence models in a variety of conditions, excelling in both retrieval-only and comprehension-inclusive tasks. Additionally, HGRN2 and cosFormer2 also demonstrate strong scaling capabilities, particularly in easy mode.

Table 10: **SCROLLS Benchmark by Aspect Ratio.** The term '-' indicates a failure in the specified task.

Arch	Dim	GovReport (R.1/R.2/R.L)	SummScreenfd (R.1/R.2/R.L)	QmSum (R.1/R.2/R.L)	Qasper (F1)	NarrativeQA (F1)	Quality (EM)	ContractNLI (EM)	SCROLLS (Avg)
LLaMA	1536	12.91/3.06/9.38	9.47/0.84/7.72	10.93/2.24/9.43	22.77	16.03	28.43	9.93	11.01
LLaMA	1792	14.68/6.0/10.03	11.5/1.63/9.03	9.72/2.18/8.45	23.02	16.68	27.85	21.02	12.45
LLaMA	2048	7.62/3.59/5.89	11.07/1.61/8.5	14.49/3.51/11.85	19.06	16.62	27.85	26.90	12.20
LLaMA	3072	17.04/5.78/11.41	9.16/1.29/7.35	5.79/0.98/4.96	17.89	12.12	28.00	19.67	10.88
cos2	1536	7.97/3.51/6.15	12.25/1.95/9.38	11.91/2.7/9.96	16.94	13.93	27.76	17.07	10.88
cos2	1792	6.1/2.52/4.88	11.06/1.49/8.41	12.46/2.87/10.24	17.80	13.90	27.76	14.56	10.31
cos2	2048	7.63/3.22/5.89	12.94/1.71/9.84	11.35/2.33/9.69	17.68	13.39	27.76	16.30	10.75
cos2	3072	9.22/2.85/7.01	9.03/1.05/7.45	12.96/2.99/10.96	13.89	12.16	27.47	11.09	9.85

Table 11: **SCROLLS Benchmark by Pre-training Context Length.** The term '-' indicates a failure in the specified task.

Arch	Len	GovReport (R.1/R.2/R.L)	SummScreenfd (R.1/R.2/R.L)	QmSum (R.1/R.2/R.L)	Qasper (F1)	NarrativeQA (F1)	Quality (EM)	ContractNLI (EM)	SCROLLS (Avg)
TNL	2k	5.72/2.23/4.6	7.37/1.0/5.42	5.88/1.24/4.7	13.34	14.42	28.24	23.14	9.02
TNL	4k	7.61/2.8/5.96	13.01/1.98/9.76	10.87/2.97/9.07	19.45	14.89	27.18	27.77	11.79
TNL	8k	5.86/2.02/4.74	9.39/1.34/7.32	5.81/1.43/4.8	14.23	13.83	28.19	26.52	9.65
TNL	16k	4.54/1.66/3.67	9.41/0.96/7.4	7.12/1.61/5.79	16.61	13.68	28.09	13.11	8.74
HGRN2	2k	15.25/4.18/10.5	10.58/1.23/8.77	11.19/2.0/9.54	18.46	13.60	27.71	17.74	11.60
HGRN2	4k	14.97/4.69/10.27	10.08/1.26/8.38	11.39/2.26/9.65	17.43	15.05	26.27	17.26	11.46
HGRN2	8k	14.86/4.21/10.45	11.4/1.44/9.16	10.9/2.28/9.68	16.21	15.09	27.76	10.61	11.08
HGRN2	16k	21.7/5.67/14.25	11.25/1.25/9.07	11.69/2.43/10.07	20.70	15.01	26.80	9.06	12.23
cos2	2k	9.08/2.95/6.79	10.43/1.15/8.3	13.06/2.88/10.67	15.18	13.70	27.90	19.96	10.93
cos2	4k	10.56/3.83/7.64	12.38/1.96/9.51	12.6/2.75/10.86	17.32	13.59	28.38	21.89	11.79
cos2	8k	7.97/3.51/6.15	12.25/1.95/9.38	11.91/2.7/9.96	16.94	13.93	27.76	17.07	10.88
cos2	16k	17.92/5.71/12.17	-	11.43/2.31/9.91	21.00	12.66	26.51	10.80	13.04

TNL shows a more varied performance, performing decently in some contexts but not as uniformly strong as other models.

Across all architectures, performances are generally higher in the easy mode compared to the standard mode, which includes both retrieval and comprehension components. This suggests that the addition of comprehension tasks adds significant complexity and challenge.

The NIAH score, indicating efficiency in managing sparse or relevant information, is consistently highest for LLaMA, especially at larger context scales in easy mode. Both weighted average accuracy and average accuracy tend to follow similar trends, suggesting these metrics might be adjusted based on task difficulty or importance across various context scales.

C.4 NIAH heatmaps by easy mode

The figures below provide a heatmap visualization of NIAH in easy mode.

C.5 NIAH heatmap by standard mode

The figures below provide a heatmap visualization of NIAH in standard mode.

Table 12: **Benchmark of Needle In A Haystack:** it presents accuracy metrics at four context scales: 2K, 4K, 8K, and 16K. Accuracies below the 4K and 8K thresholds are presented in the middle columns. Both average accuracy and weighted average accuracy are detailed, along with the NIAH score, in the rightmost columns.

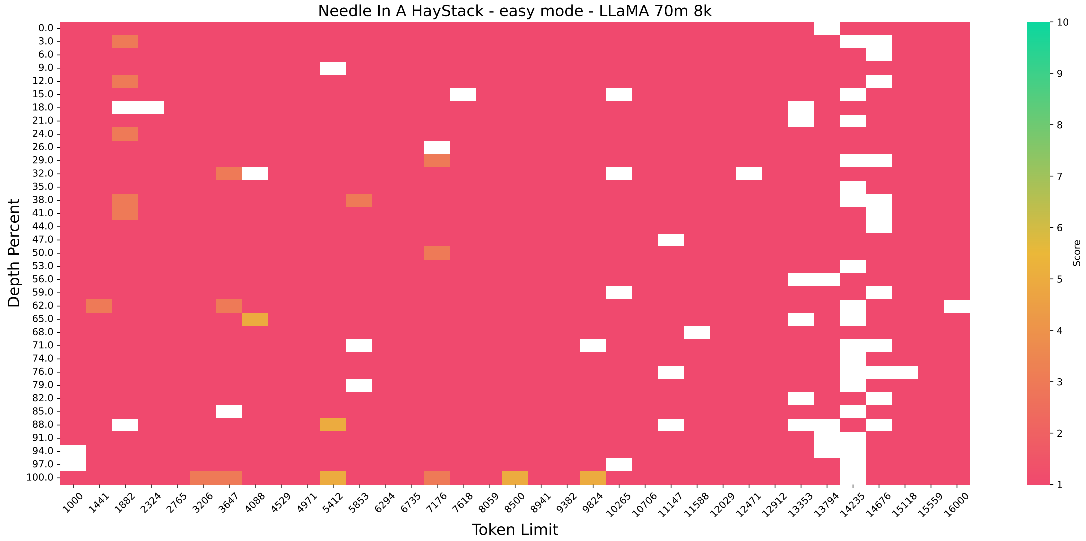
Arch	P.S. B	Acc @2k	Acc @4k	Acc @8k	Acc @16k	Acc ≤4k	Acc ≤8k	Acc avg	Weighted avg acc	NIAH score
Easy Mode (Retrieval Only)										
LLaMA	0.07	3.2	1.3	0.0	0.0	1.0	0.7	0.4	0.4	0.2
TNL	0.07	0.0	1.3	0.6	0.6	0.4	0.7	0.5	0.6	0.0
HGRN2	0.07	0.0	0.0	0.0	0.0	0.0	0.3	0.2	0.2	0.0
cos2	0.07	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1
LLaMA	0.16	7.3	0.6	0.0	0.0	13.7	11.7	6.0	6.0	1.8
TNL	0.16	2.5	3.5	24.4	4.4	5.6	10.1	7.3	7.5	1.7
HGRN2	0.16	0.6	0.0	0.0	0.0	1.1	0.7	0.3	0.3	0.0
cos2	0.16	0.0	5.4	0.0	0.0	2.5	2.1	1.2	1.3	0.9
LLaMA	0.41	100.0	97.1	97.8	0.0	99.3	99.5	56.4	52.3	45.5
TNL	0.41	27.9	5.7	3.8	10.2	18.1	15.4	13.8	14.2	2.8
HGRN2	0.41	8.6	6.3	1.3	0.0	17.0	9.3	4.9	4.8	1.4
cos2	0.41	37.1	11.4	2.9	0.0	25.5	18.5	9.7	9.6	2.1
LLaMA	1	100.0	71.4	73.3	0.0	92.5	90.9	47.8	44.1	28.1
TNL	1	43.5	8.9	21.3	8.6	30.8	28.7	27.6	28.0	2.7
HGRN2	1	17.1	5.7	2.9	3.5	18.3	13.4	9.7	10.0	3.6
cos2	1	54.9	5.7	2.9	0.0	37.8	22.7	11.5	10.9	2.0
LLaMA	3	97.1	100.0	82.9	0.6	95.4	93.9	48.8	45.1	29.9
TNL	3	0.0	26.0	3.2	9.5	9.7	12.9	10.4	11.1	2.3
HGRN2	3	58.4	11.4	2.9	7.3	46.4	28.9	18.0	17.9	4.2
cos2	3	97.1	34.3	8.6	0.0	86.8	54.7	27.8	25.8	14.3
LLaMA	7	100.0	100.0	87.0	0.0	100.0	98.4	62.9	59.7	44.7
TNL	7	43.5	14.3	12.4	18.1	38.8	26.8	20.2	20.5	7.8
HGRN2	7	100.0	28.6	14.6	11.4	83.8	50.1	31.3	30.8	17.1
cos2	7	97.1	37.1	8.6	0.0	78.6	48.0	25.1	23.6	11.6
Standard Mode (Retrieval and Comprehension)										
LLaMA	0.07	1.9	0.0	0.6	0.0	0.6	0.3	0.2	0.2	0.1
TNL	0.07	0.0	0.6	0.0	0.0	0.3	0.4	0.5	0.5	0.1
HGRN2	0.07	0.0	0.6	0.0	0.0	0.4	0.3	0.2	0.2	0.1
cos2	0.07	0.0	0.0	0.0	0.0	0.2	0.1	0.1	0.1	0.0
LLaMA	0.16	1.9	0.6	11.4	0.0	11.8	7.9	4.0	3.6	0.7
TNL	0.16	1.3	7.0	23.5	0.0	5.2	7.0	6.3	6.4	1.3
HGRN2	0.16	0.0	0.0	0.0	0.0	0.8	0.4	0.2	0.2	0.0
cos2	0.16	0.0	0.0	0.0	0.0	0.7	0.4	0.2	0.2	0.1
LLaMA	0.41	100.0	100.0	92.7	0.0	100.0	99.6	55.4	51.4	43.5
TNL	0.41	12.4	5.1	5.7	8.9	12.8	12.0	10.3	10.4	1.3
HGRN2	0.41	0.0	0.0	0.0	0.0	8.3	4.2	2.3	2.1	0.4
cos2	0.41	0.0	8.6	3.5	0.0	3.7	3.3	1.8	1.9	1.1
LLaMA	1	100.0	82.9	47.6	0.0	97.9	83.7	43.2	40.1	24.4
TNL	1	8.3	7.0	34.6	22.2	20.9	22.9	23.6	24.0	1.2
HGRN2	1	12.7	5.7	14.6	0.0	15.9	11.6	7.5	7.4	1.8
cos2	1	76.8	5.7	2.9	0.0	24.3	13.6	7.0	6.6	2.0
LLaMA	3	99.4	100.0	42.9	0.0	98.3	85.5	43.4	40.1	20.0
TNL	3	2.9	17.1	2.9	44.8	8.7	11.6	10.3	10.8	3.1
HGRN2	3	54.0	5.7	2.9	8.9	32.2	18.0	12.7	12.7	3.1
cos2	3	54.9	9.2	8.6	0.0	38.7	24.1	12.8	12.5	4.7
LLaMA	7	100.0	100.0	85.4	0.0	100.0	96.2	58.6	55.4	45.5
TNL	7	28.9	13.7	19.7	44.1	29.4	21.8	18.8	19.4	6.3
HGRN2	7	65.4	5.7	5.1	4.1	48.5	28.6	18.4	18.3	7.5
cos2	7	74.3	5.7	11.4	0.0	51.6	31.0	16.8	16.0	7.6

Table 13: **Benchmark of Needle In A Haystack by Aspect Ratio**

Arch	Dim	Acc	Acc	Acc	Acc	Acc	Acc	Acc	Weighted	NIAH
		@2k	@4k	@8k	@16k	≤4k	≤8k	avg	avg acc	score
Easy Mode (Retrieval Only)										
LLaMA	1536	100.0	71.4	73.3	0.0	92.5	90.9	47.8	44.1	28.1
LLaMA	1792	100.0	100.0	82.9	0.0	100.0	96.8	57.4	54.2	45.0
LLaMA	2048	100.0	100.0	62.9	0.0	100.0	92.9	48.7	45.0	39.2
LLaMA	3072	100.0	100.0	94.3	0.3	92.5	95.6	49.8	46.0	38.1
cos2	1536	54.9	5.7	2.9	0.0	37.8	22.7	11.5	10.9	2.0
cos2	1792	20.0	5.7	5.7	0.0	31.8	18.2	9.2	8.8	3.6
cos2	2048	82.9	5.7	2.9	0.0	49.6	26.4	13.4	12.3	4.7
cos2	3072	1.0	5.7	2.9	0.0	21.7	13.6	6.9	6.5	1.6
Standard Mode (Retrieval and Comprehension)										
LLaMA	1536	100.0	82.9	47.6	0.0	97.9	83.7	43.2	40.1	24.4
LLaMA	1792	100.0	100.0	57.1	0.0	100.0	92.3	51.5	48.3	38.8
LLaMA	2048	100.0	100.0	57.1	0.6	100.0	90.8	48.3	44.9	38.3
LLaMA	3072	97.1	100.0	74.3	0.0	97.2	92.3	47.1	43.3	26.5
cos2	1536	76.8	5.7	2.9	0.0	24.3	13.6	7.0	6.6	2.0
cos2	1792	5.7	3.5	1.9	0.0	11.5	6.9	3.8	3.9	2.0
cos2	2048	24.8	5.7	2.9	0.0	25.0	13.3	6.8	6.4	4.1
cos2	3072	0.3	2.9	0.0	0.0	5.9	3.9	2.0	2.0	1.0

Table 14: **Benchmark of Needle In A Haystack by Pre-training Context Length**

Arch	Len	Acc	Acc	Acc	Acc	Acc	Acc	Acc	Weighted	NIAH
		@2k	@4k	@8k	@16k	≤4k	≤8k	avg	avg acc	score
Easy Mode (Retrieval Only)										
TNL	2k	58.1	7.3	2.9	4.8	31.0	15.5	8.7	8.0	1.6
TNL	4k	25.7	17.1	2.9	11.1	23.9	15.4	11.6	12.1	4.2
TNL	8k	43.5	8.9	21.3	8.6	30.8	28.7	27.6	28.0	2.7
TNL	16k	43.2	13.3	16.8	4.4	21.4	16.5	13.7	14.2	2.7
HGRN2	2k	0.0	2.9	3.5	0.0	1.1	2.9	2.1	2.3	1.0
HGRN2	4k	5.7	2.9	1.3	0.6	4.0	3.3	2.1	2.1	0.8
HGRN2	8k	17.1	5.7	2.9	3.5	18.3	13.4	9.7	10.0	3.6
HGRN2	16k	20.0	8.3	11.1	3.8	23.0	13.0	8.7	8.8	3.2
cos2	2k	62.9	0.0	0.0	0.0	34.2	16.1	7.8	6.8	2.1
cos2	4k	65.4	10.5	0.0	0.0	30.3	14.8	7.2	6.5	1.7
cos2	8k	54.9	5.7	2.9	0.0	37.8	22.7	11.5	10.9	2.0
cos2	16k	17.1	2.9	2.9	0.0	22.9	14.2	9.4	9.6	3.9
Standard Mode (Retrieval and Comprehension)										
TNL	2k	14.0	3.5	3.8	17.5	16.3	9.0	6.4	6.5	2.0
TNL	4k	5.7	10.2	3.5	12.1	11.4	8.5	7.9	8.4	2.7
TNL	8k	8.3	7.0	34.6	22.2	20.9	22.9	23.6	24.0	1.2
TNL	16k	10.2	12.1	16.5	9.2	17.7	14.5	11.3	11.3	1.4
HGRN2	2k	0.6	0.0	3.5	0.0	0.7	1.6	1.3	1.4	0.6
HGRN2	4k	3.2	1.3	1.3	0.0	4.6	3.6	2.0	1.8	0.2
HGRN2	8k	12.7	5.7	14.6	0.0	15.9	11.6	7.5	7.4	1.8
HGRN2	16k	5.4	9.2	0.6	17.5	11.3	6.5	4.7	4.7	1.1
cos2	2k	37.5	0.0	0.0	0.0	21.9	10.3	5.0	4.4	0.4
cos2	4k	34.3	5.7	0.0	0.0	21.4	10.2	5.0	4.5	1.5
cos2	8k	76.8	5.7	2.9	0.0	24.3	13.6	7.0	6.6	2.0
cos2	16k	5.7	2.9	2.9	0.0	4.6	3.5	2.8	3.0	1.6


Needle In A HayStack - easy mode - LLaMA 70m 8k
Depth Percent
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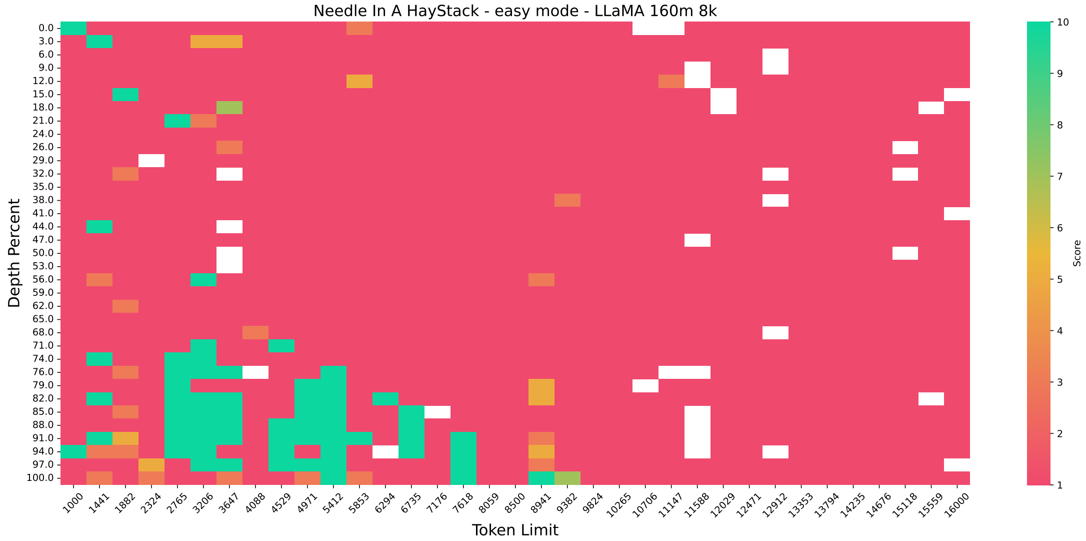
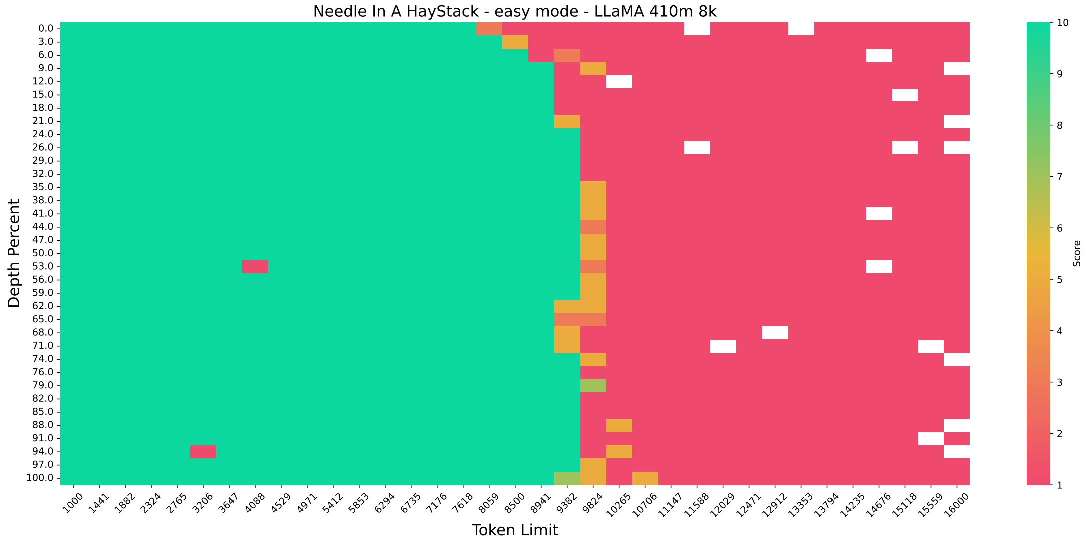
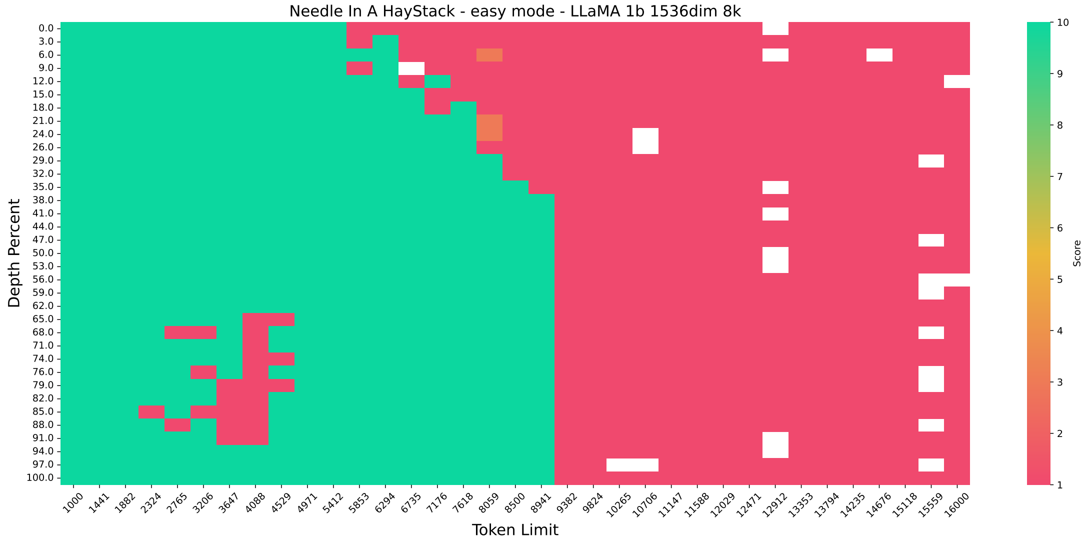
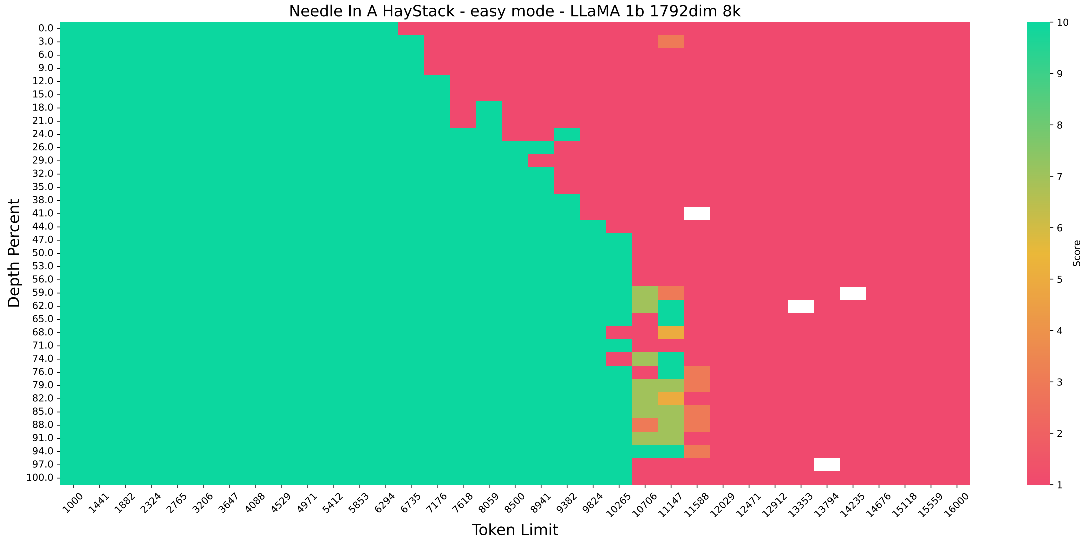
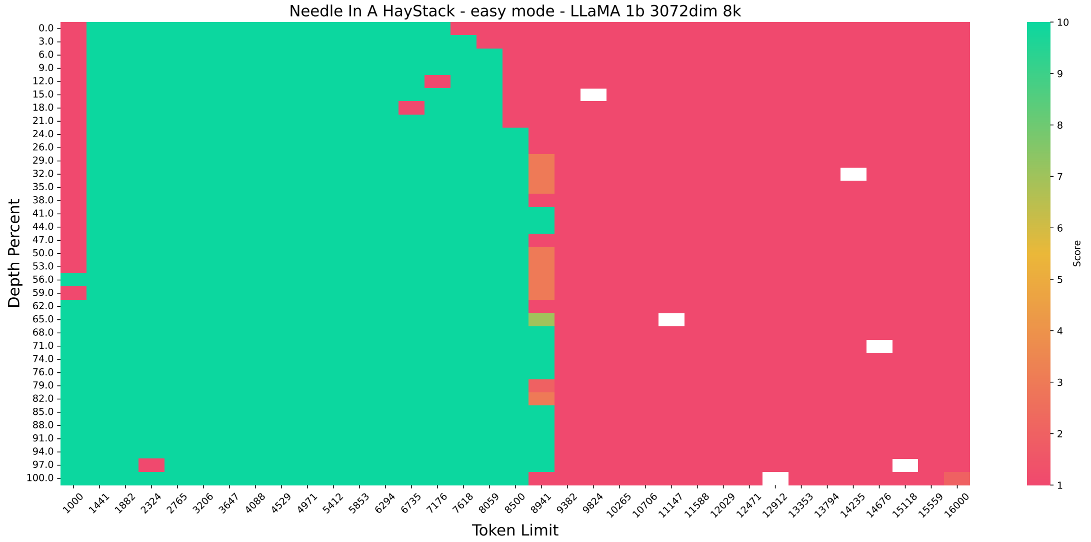
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1

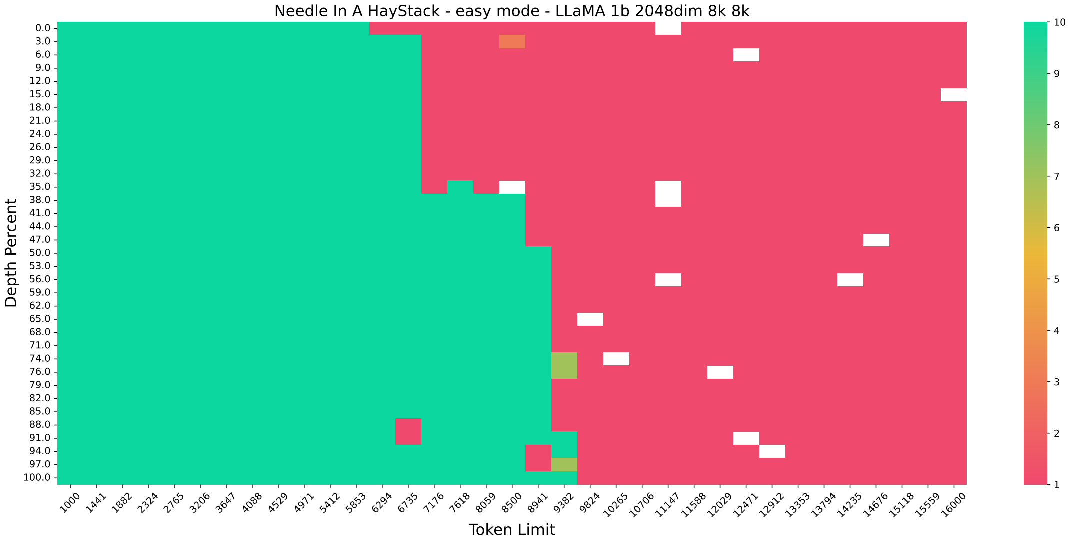

Needle In A HayStack - easy mode - LLaMA 160m 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1


Needle In A HayStack - easy mode - LLaMA 410m 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1


Needle In A HayStack - easy mode - LLaMA 1b 1536dim 8k


Needle In A HayStack - easy mode - LLaMA 1b 1792dim 8k


Needle In A HayStack - easy mode - LLaMA 1b 3072dim 8k


Needle In A HayStack - easy mode - LLaMA 1b 2048dim 8k 8k
Depth Percent
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
8059
8500
8941
9382
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11588
12029
12471
12912
13353
13794
14235
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15118
15559
16000
Token Limit
Score
10
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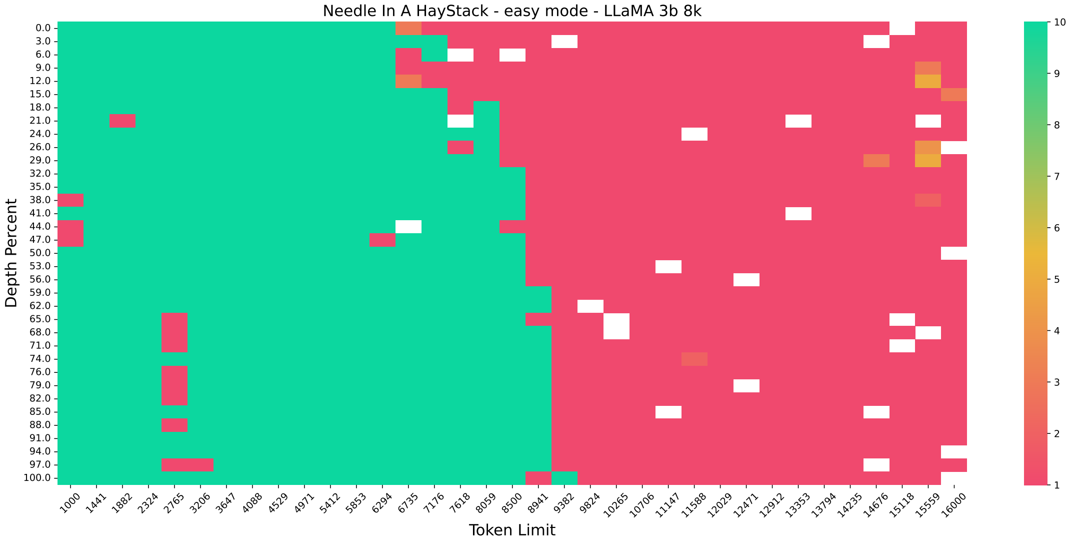
1


Needle In A HayStack - easy mode - LLaMA 3b 8k
Depth Percent
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
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53.0
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Token Limit
Score
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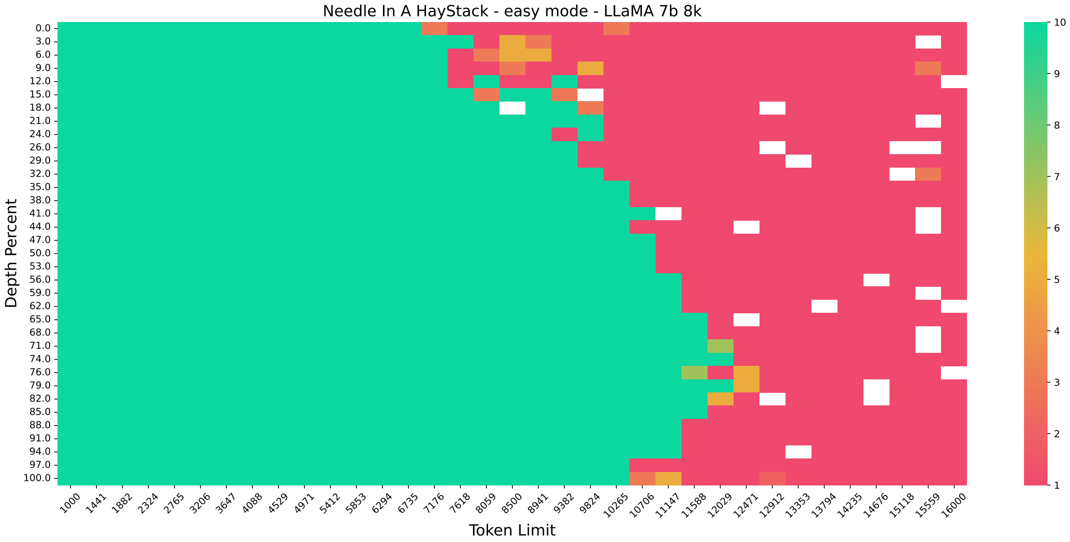
Needle In A HayStack - easy mode - LLaMA 7b 8k
Depth Percent
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3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
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65.0
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71.0
74.0
76.0
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1000
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2324
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3206
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6294
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Token Limit
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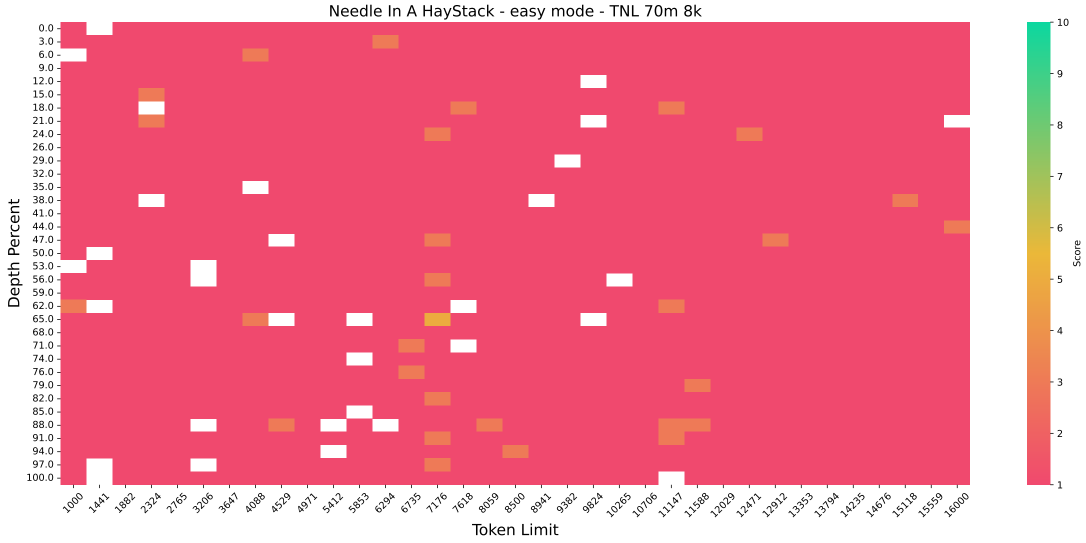
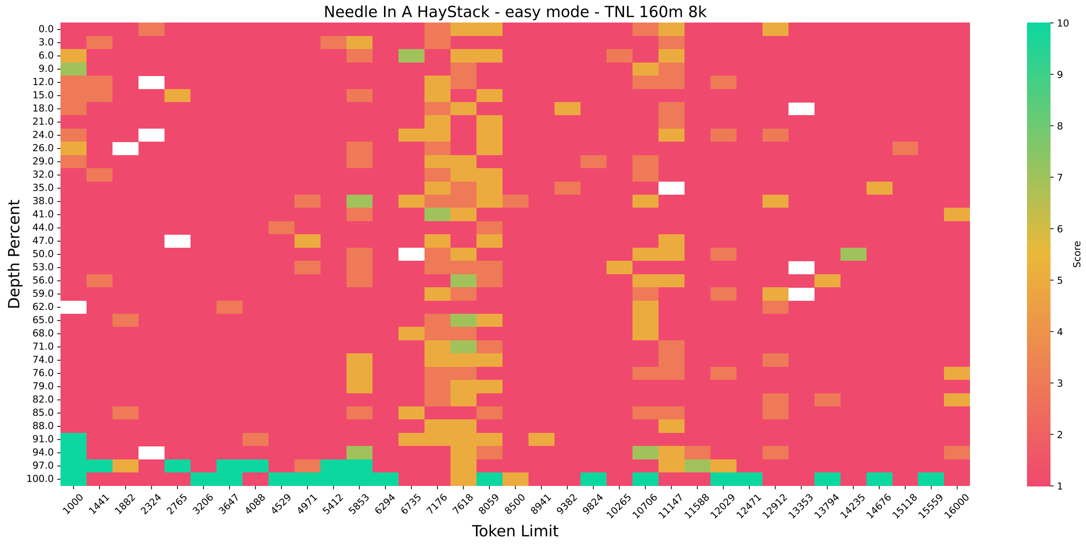
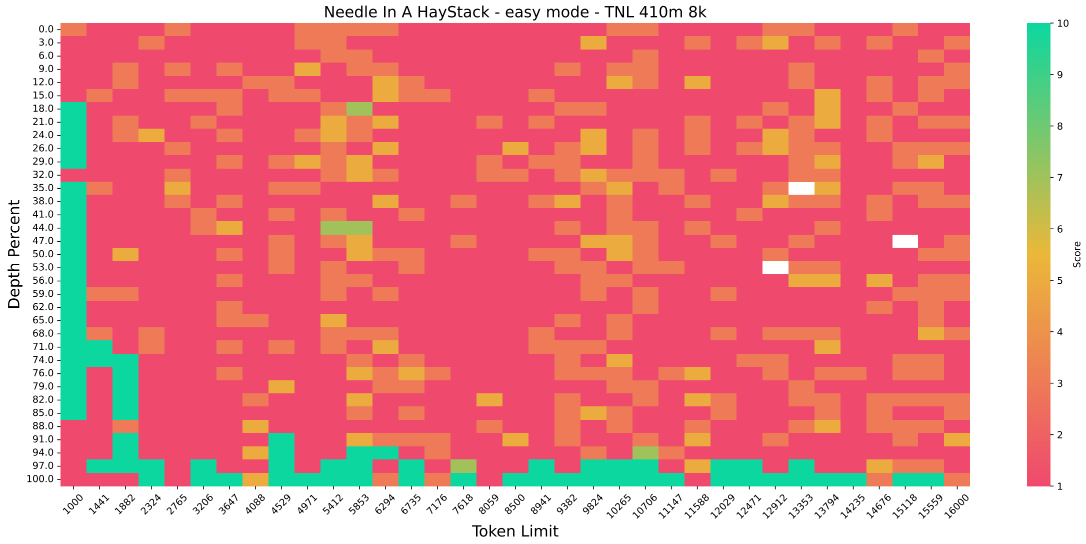


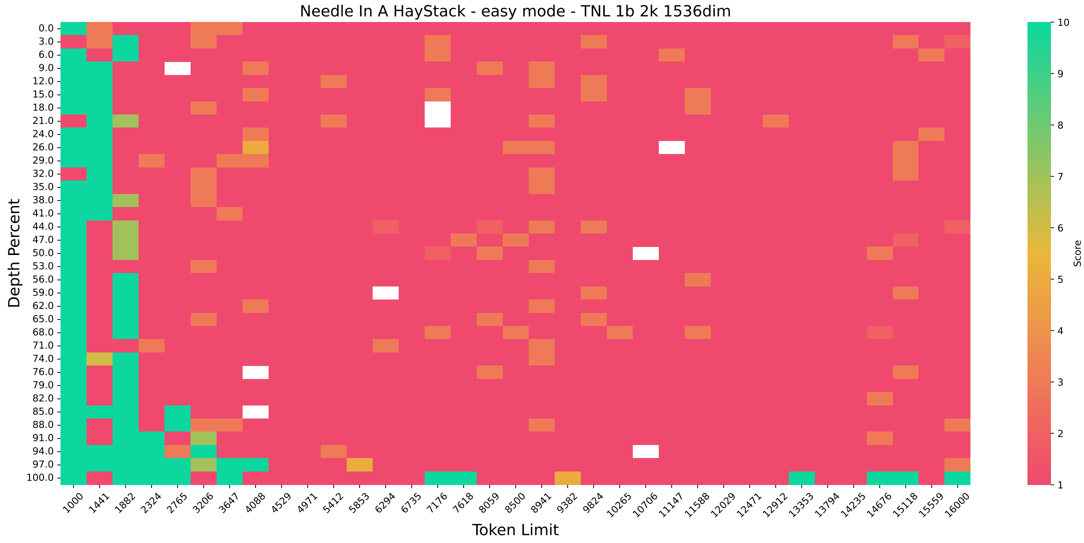
Needle In A HayStack - easy mode - TNL 70m 8k



Needle In A HayStack - easy mode - TNL 160m 8k



Needle In A HayStack - easy mode - TNL 410m 8k


Needle In A HayStack - easy mode - TNL 1b 2k 1536dim
Depth Percent
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
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71.0
74.0
76.0
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16000
Token Limit
Score
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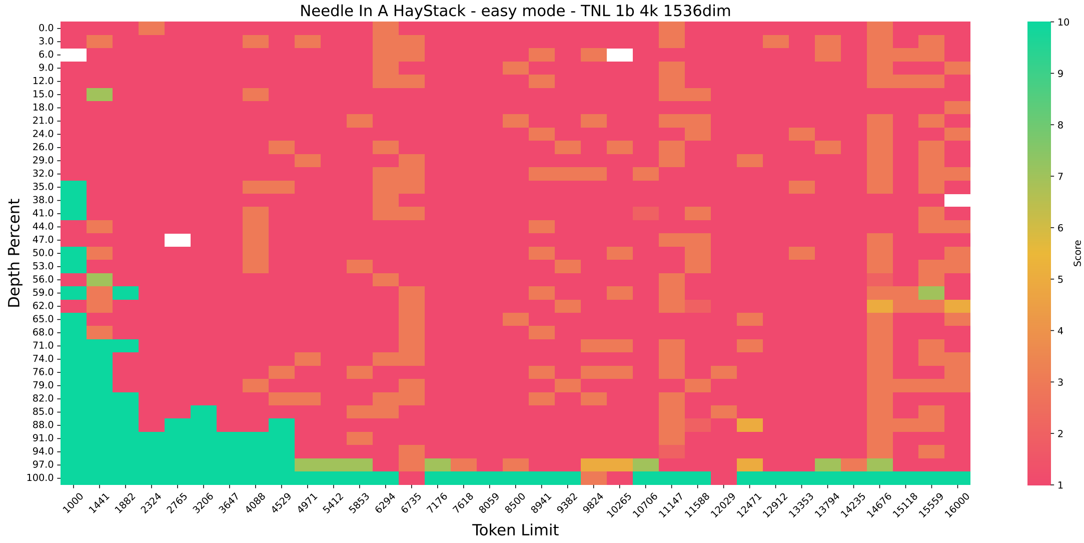
Needle In A HayStack - easy mode - TNL 1b 4k 1536dim
Depth Percent
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3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
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59.0
62.0
65.0
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100.0
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Token Limit
Score
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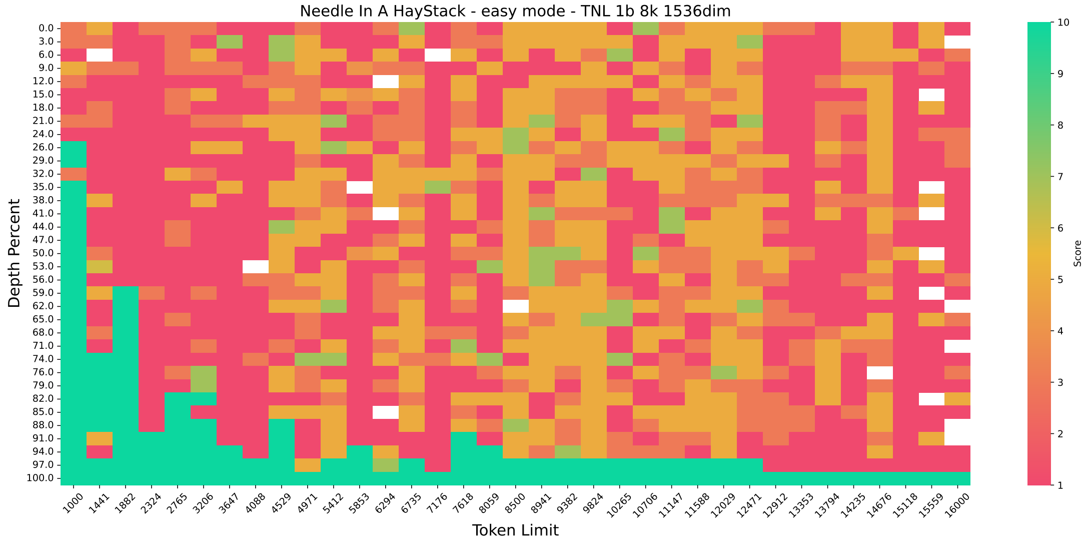
Needle In A HayStack - easy mode - TNL 1b 8k 1536dim
Depth Percent
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9.0
12.0
15.0
18.0
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24.0
26.0
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71.0
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82.0
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97.0
100.0
1000
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10706
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12029
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16000
Token Limit
Score
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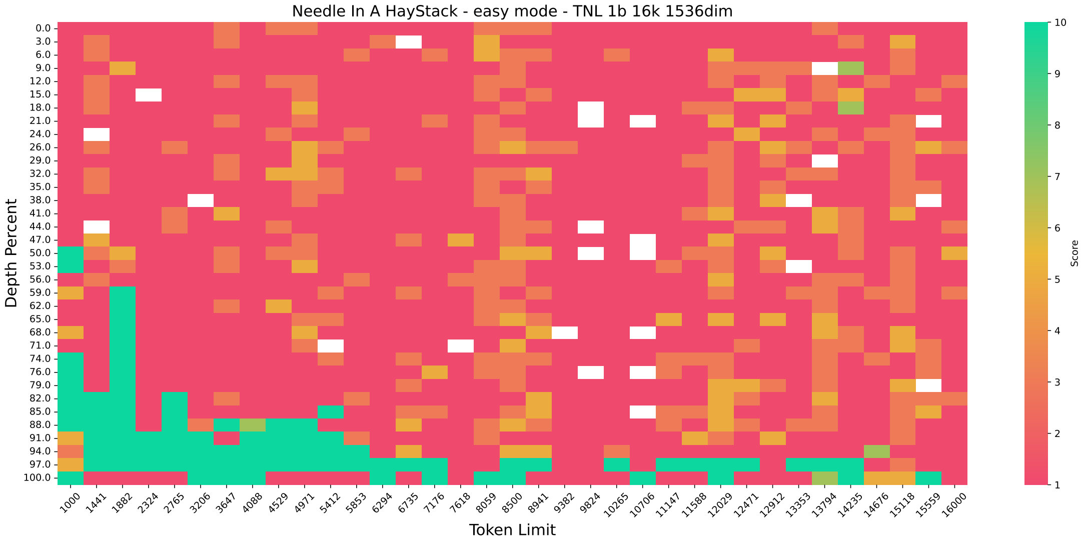
Needle In A HayStack - easy mode - TNL 1b 16k 1536dim
Depth Percent
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6.0
9.0
12.0
15.0
18.0
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24.0
26.0
29.0
32.0
35.0
38.0
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65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
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2765
3206
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4971
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6294
6735
7176
7618
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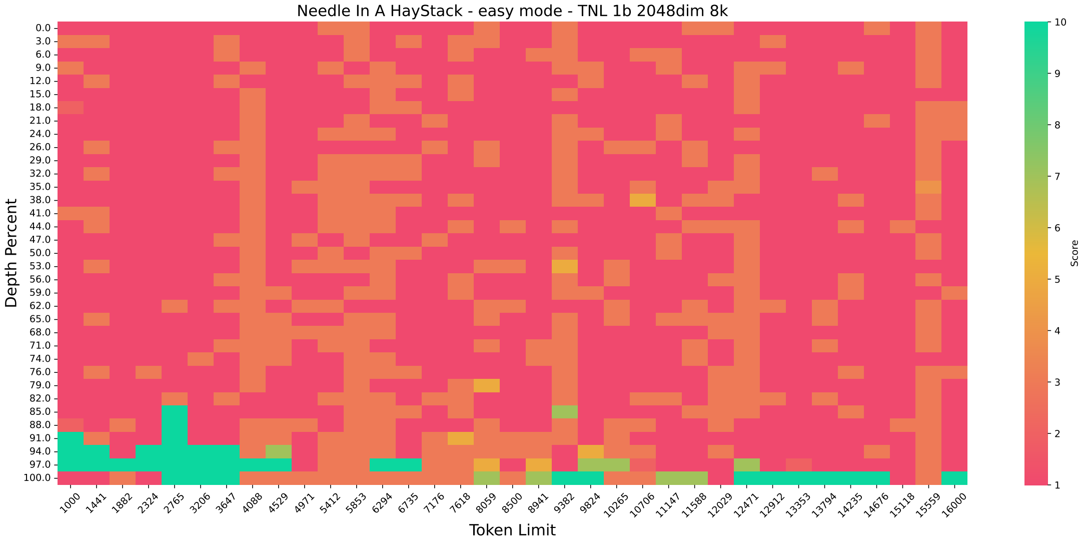
Needle In A HayStack - easy mode - TNL 1b 2048dim 8k
Depth Percent
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6.0
9.0
12.0
15.0
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21.0
24.0
26.0
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38.0
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65.0
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82.0
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97.0
100.0
Token Limit
1000
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2324
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7618
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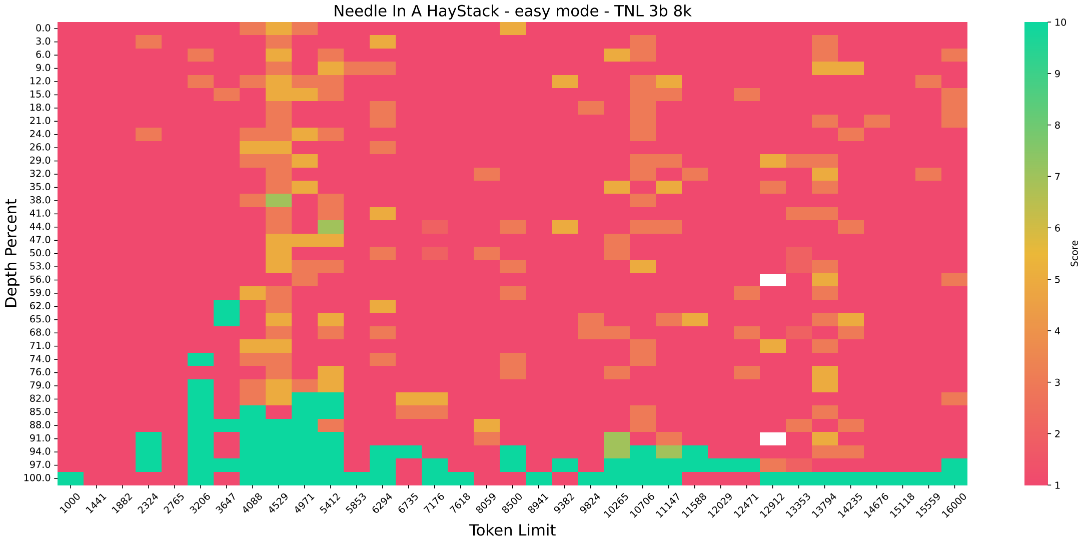
Needle In A HayStack - easy mode - TNL 3b 8k
Depth Percent
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9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
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68.0
71.0
74.0
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85.0
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100.0
Token Limit
1000
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Score
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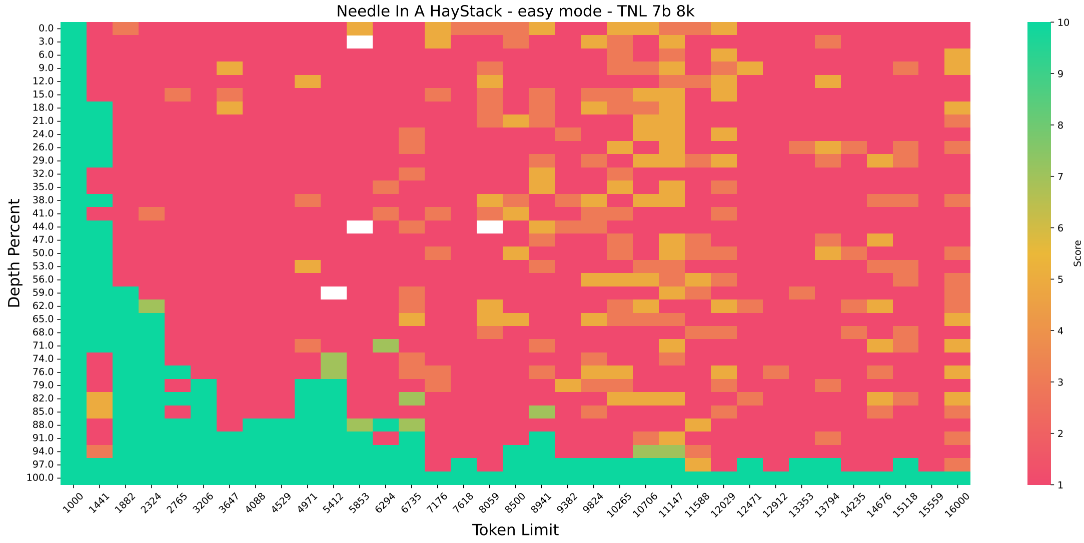
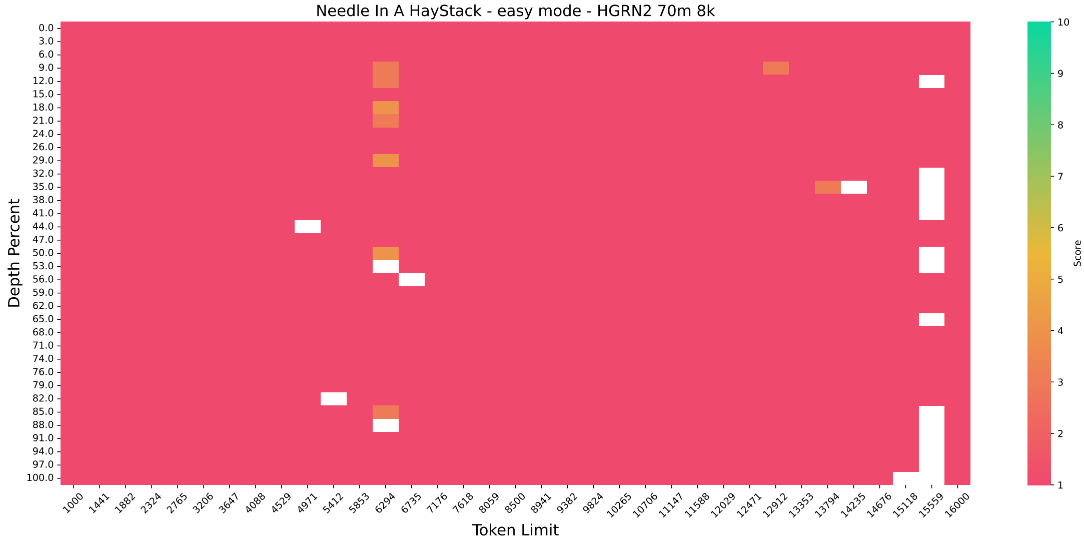
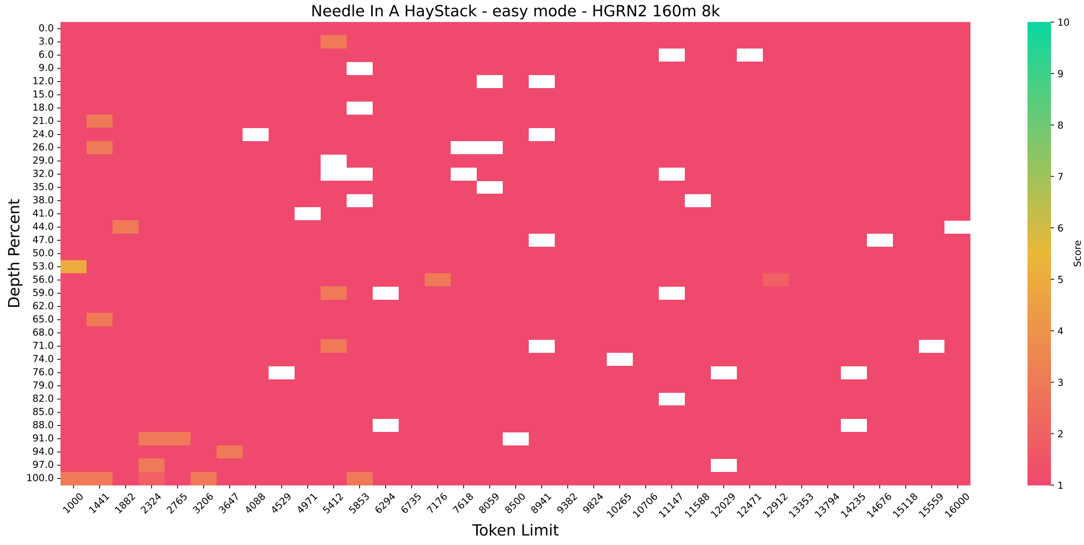
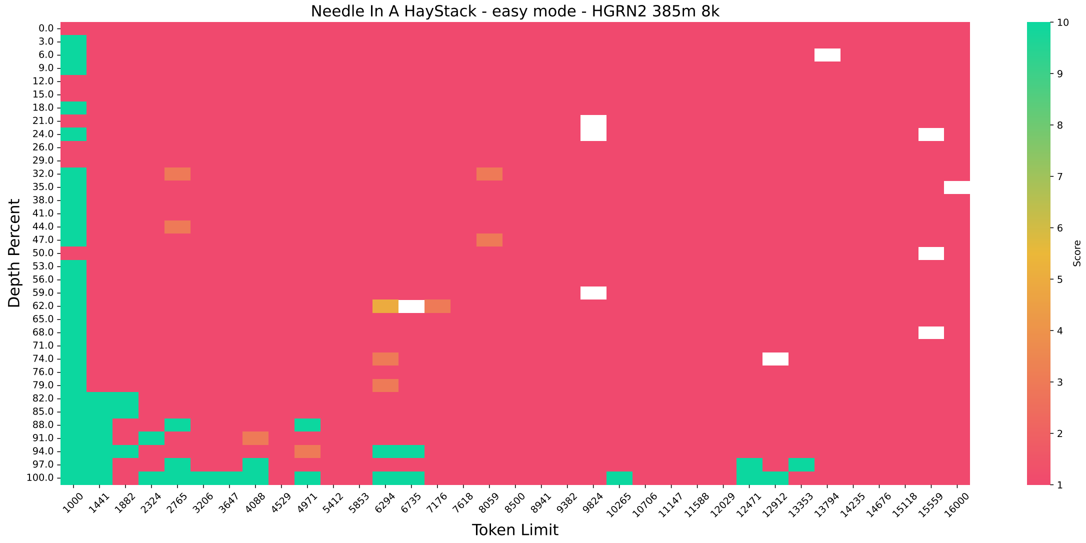
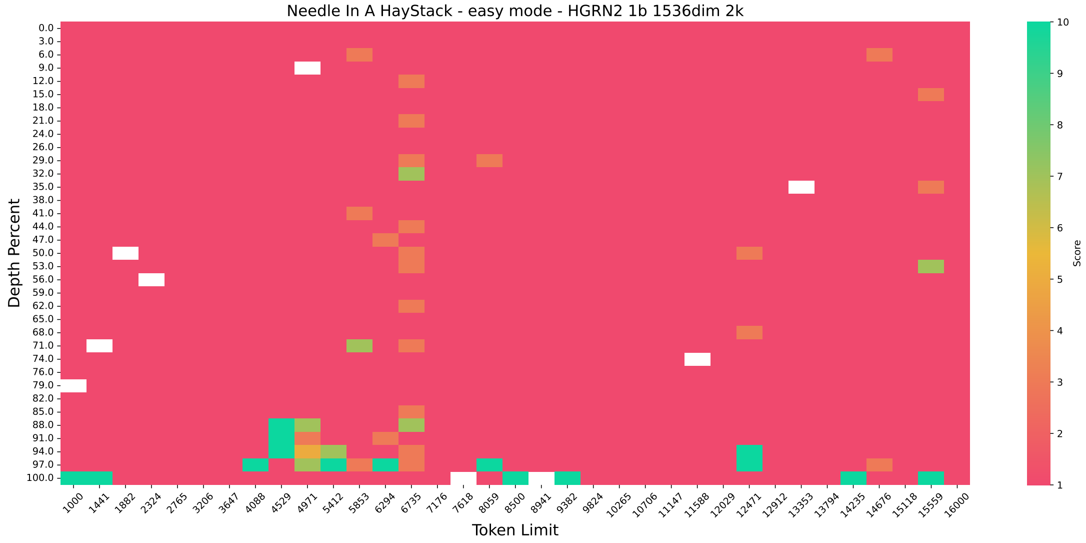

Needle In A HayStack - easy mode - TNL 7b 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - easy mode - HGRN2 70m 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10

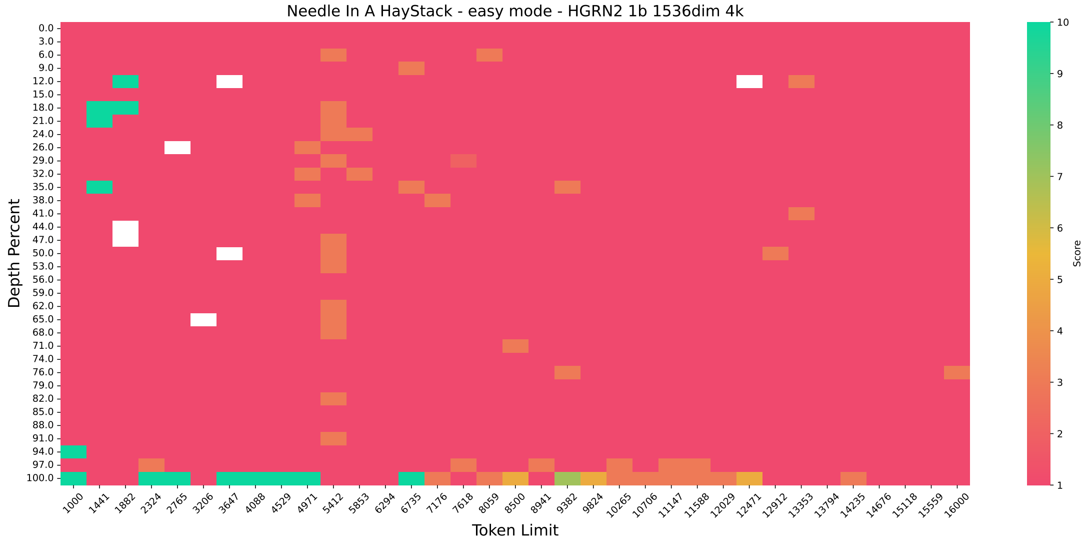

Needle In A HayStack - easy mode - HGRN2 160m 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10



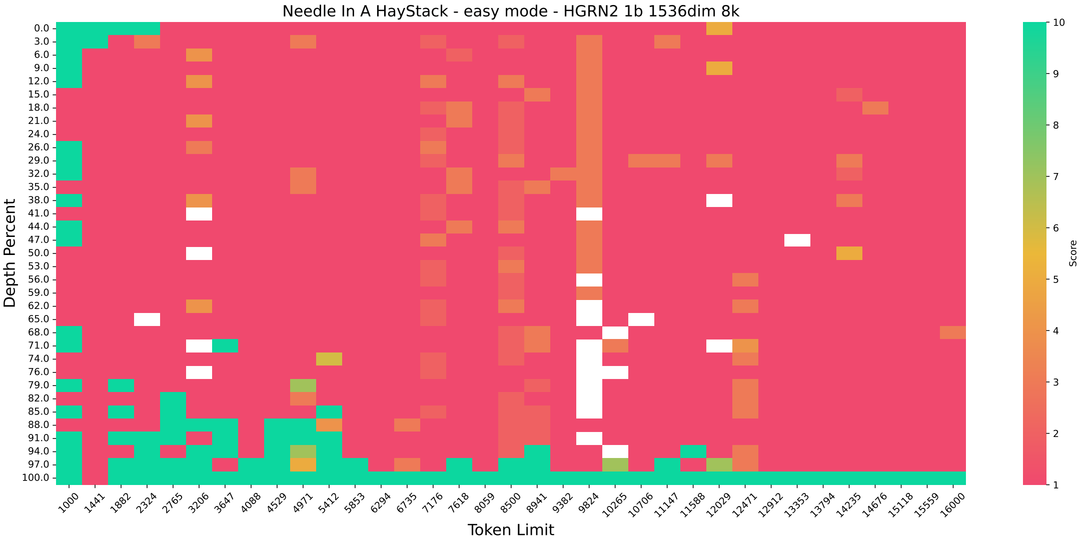
Needle In A HayStack - easy mode - HGRN2 385m 8k



Needle In A HayStack - easy mode - HGRN2 1b 1536dim 2k



Needle In A HayStack - easy mode - HGRN2 1b 1536dim 4k


Needle In A HayStack - easy mode - HGRN2 1b 1536dim 8k
Depth Percent
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
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56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
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9824
10265
10706
11147
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12029
12471
12912
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16000
Score
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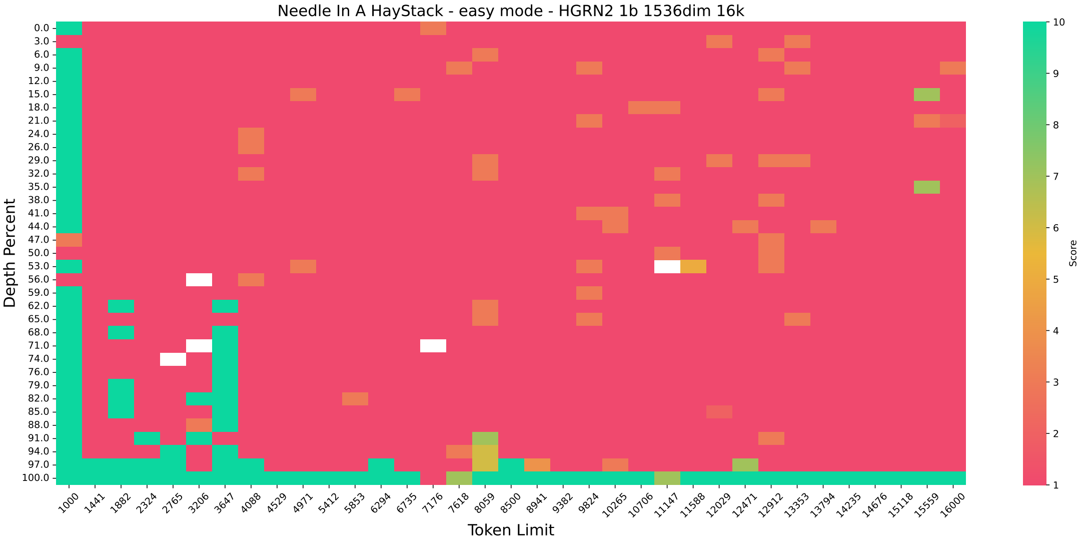
Needle In A HayStack - easy mode - HGRN2 1b 1536dim 16k
Depth Percent
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3.0
6.0
9.0
12.0
15.0
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21.0
24.0
26.0
29.0
32.0
35.0
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65.0
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74.0
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Token Limit
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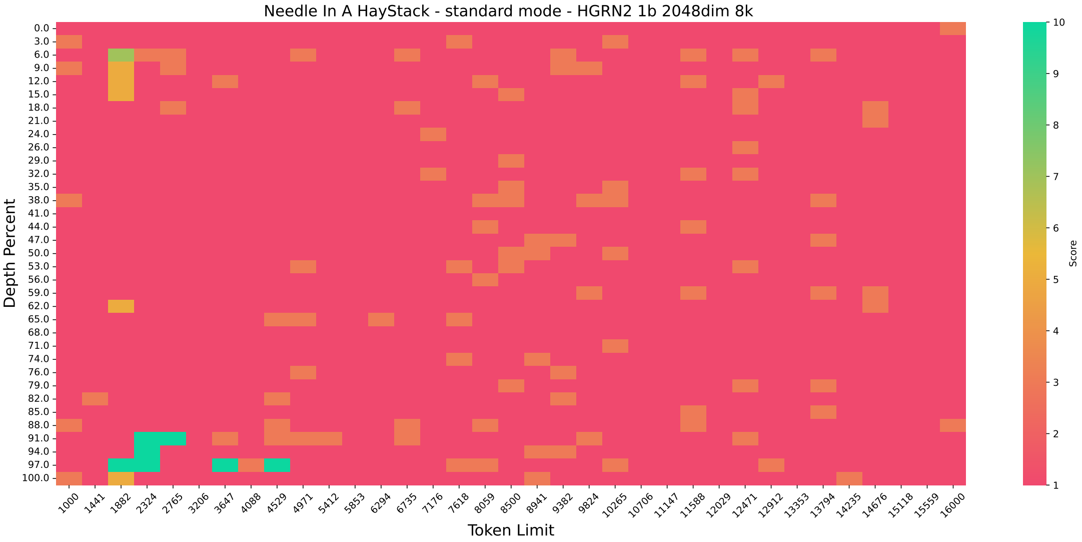
Needle In A HayStack - standard mode - HGRN2 1b 2048dim 8k
Depth Percent
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3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
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53.0
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79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
8059
8500
8941
9382
9824
10265
10706
11147
11588
12029
12471
12912
13353
13794
14235
14676
15118
15559
16000
Score
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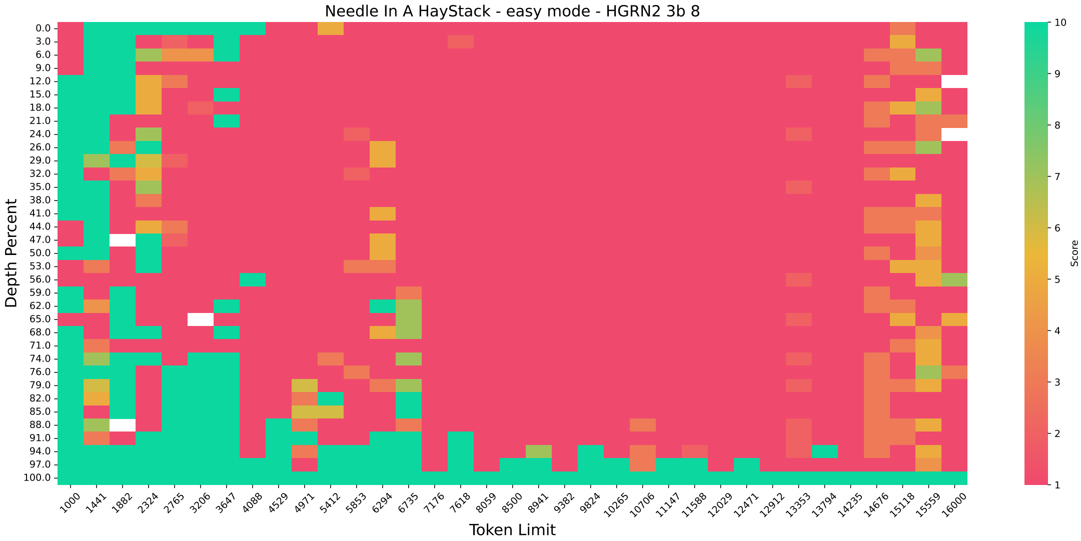
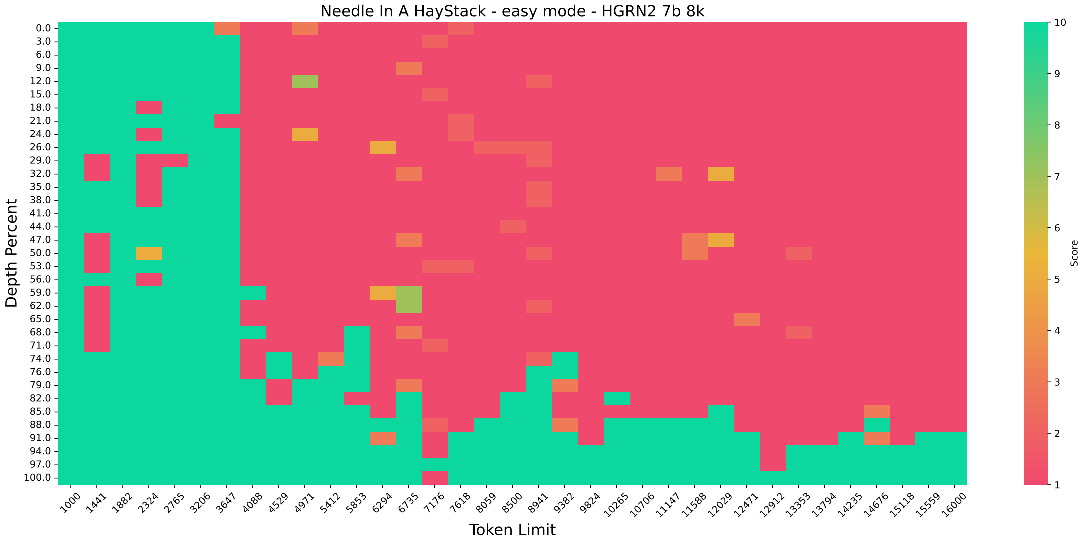
6
5
4
3
2
1

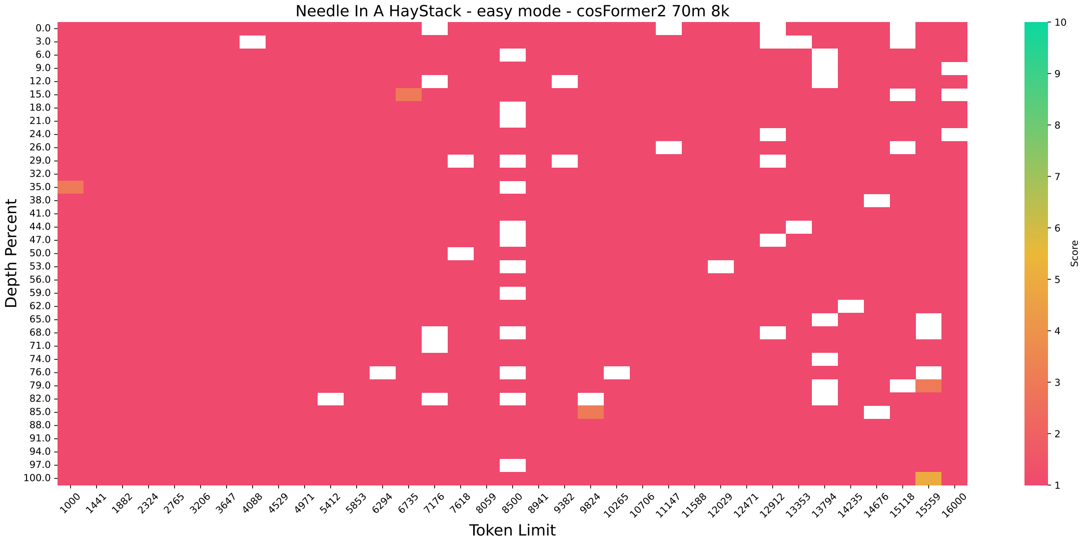
Needle In A HayStack - easy mode - HGRN2 3b 8

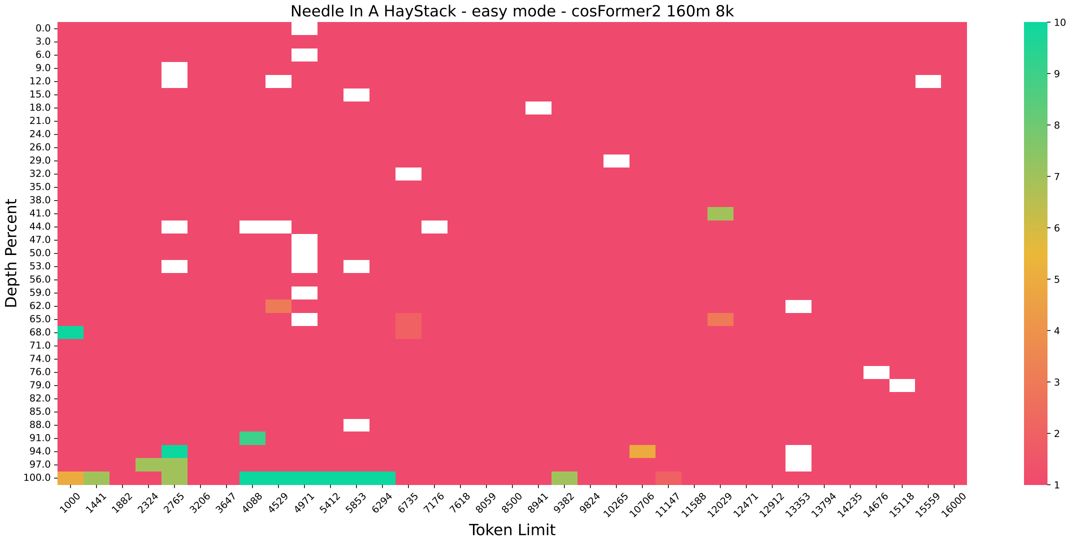


Needle In A HayStack - easy mode - HGRN2 7b 8k



Needle In A HayStack - easy mode - cosFormer2 70m 8k





Needle In A HayStack - easy mode - cosFormer2 160m 8k
Depth Percent
0.0
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6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
8059
8500
8941
9382
9824
10265
10706
11147
11588
12029
12471
12912
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15118
15559
16000
Score
10
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7
6
5
4
3
2
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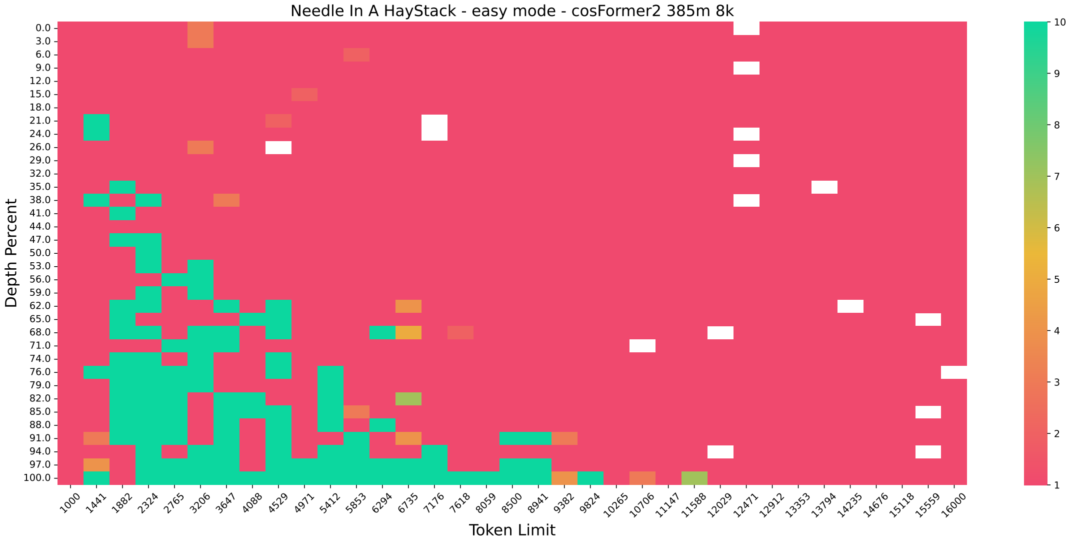

Needle In A HayStack - easy mode - cosFormer2 385m 8k
Depth Percent
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6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
8059
8500
8941
9382
9824
10265
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11147
11588
12029
12471
12912
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Score
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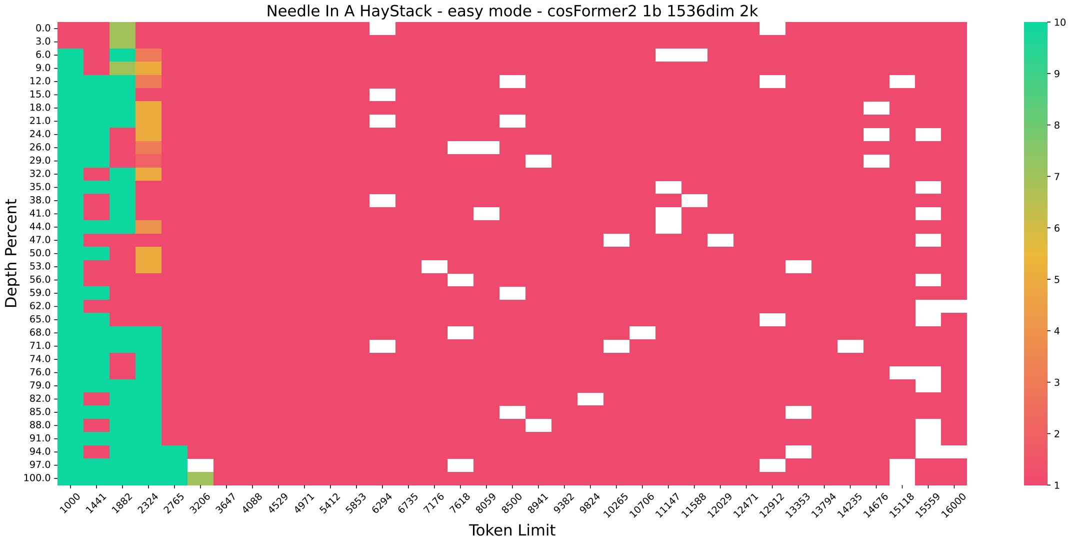

Needle In A HayStack - easy mode - cosFormer2 1b 1536dim 2k
Depth Percent
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6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
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8500
8941
9382
9824
10265
10706
11147
11588
12029
12471
12912
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14676
15118
15559
16000
Score
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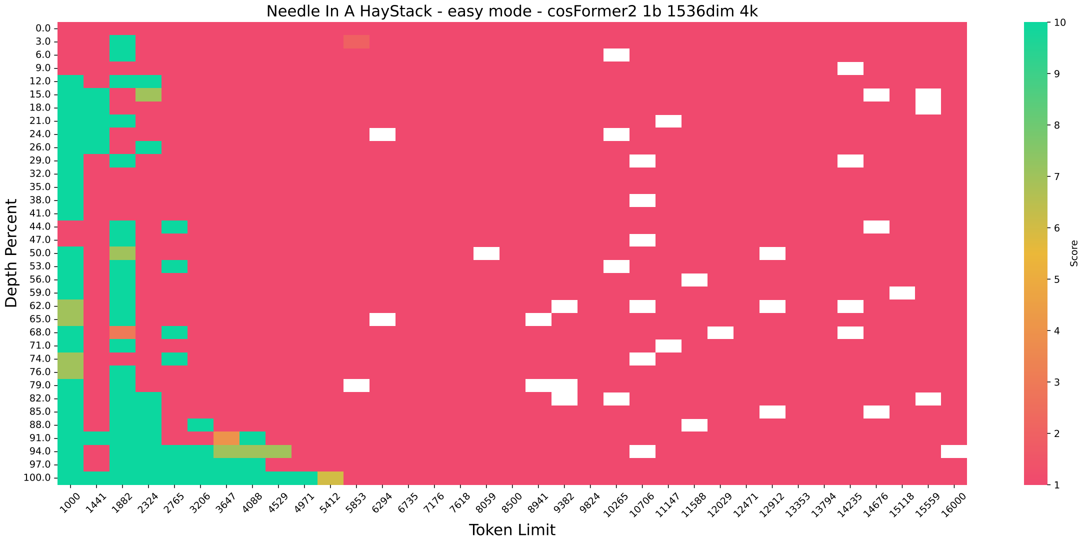
Needle In A HayStack - easy mode - cosFormer2 1b 1536dim 4k
Depth Percent
Token Limit
Score
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3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
1000
1441
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2324
2765
3206
3647
4088
4529
4971
5412
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6735
7176
7618
8059
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8941
9382
9824
10265
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12912
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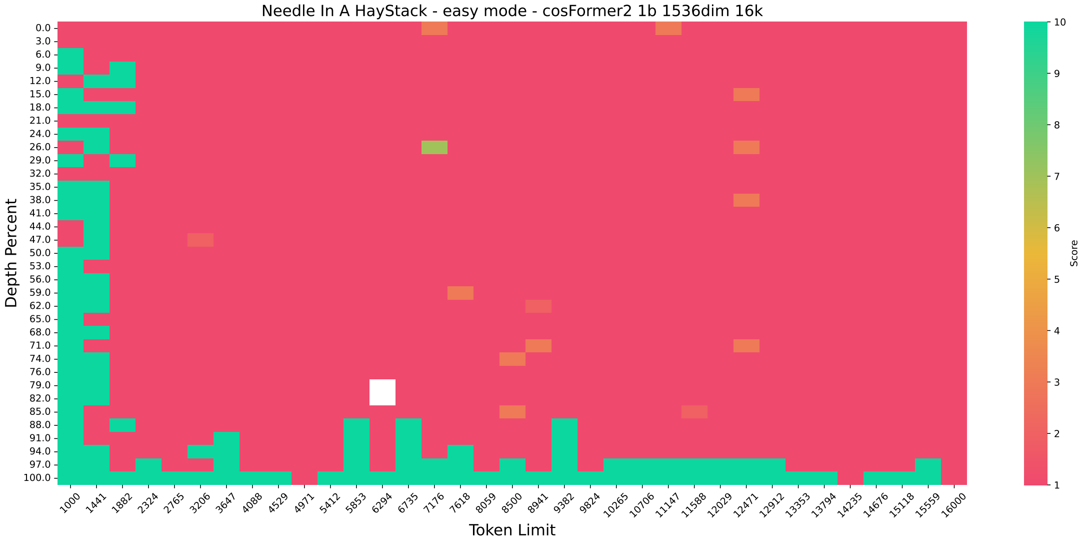
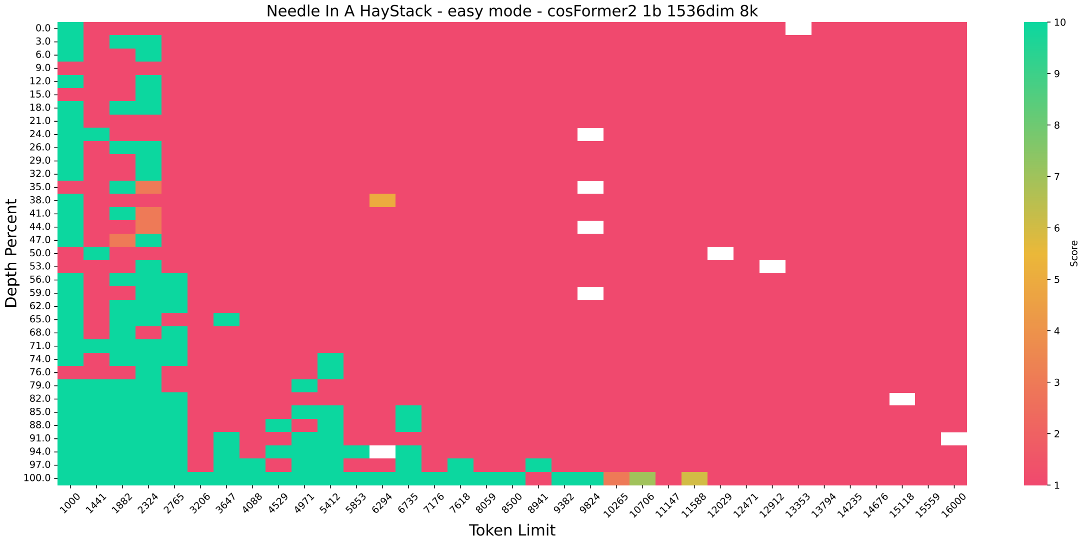
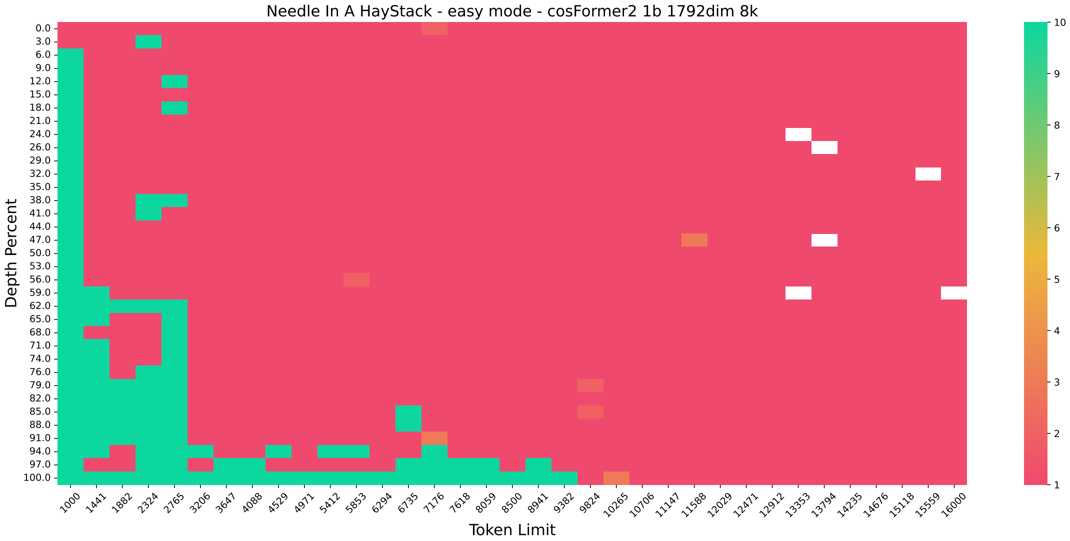

Needle In A HayStack - easy mode - cosFormer2 1b 1536dim 16k
Depth Percent
Token Limit
Score


Needle In A HayStack - easy mode - cosFormer2 1b 1536dim 8k
Depth Percent
Token Limit
Score


Needle In A HayStack - easy mode - cosFormer2 1b 1792dim 8k
Depth Percent
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9.0
12.0
15.0
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21.0
24.0
26.0
29.0
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74.0
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85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
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6735
7176
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Score
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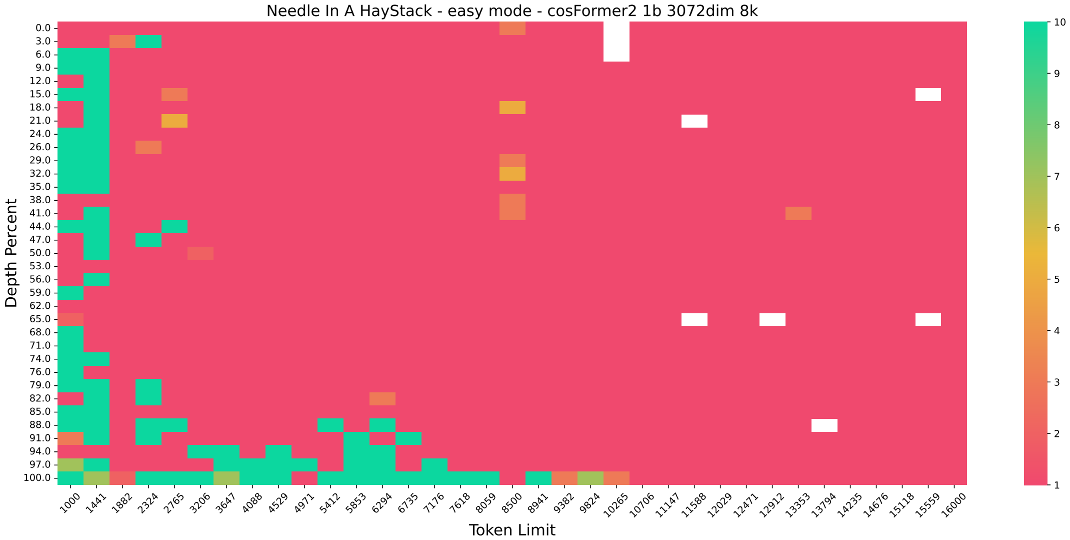
Needle In A HayStack - easy mode - cosFormer2 1b 3072dim 8k
Depth Percent
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9.0
12.0
15.0
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24.0
26.0
29.0
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65.0
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71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
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2765
3206
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4971
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6294
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7618
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10706
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12029
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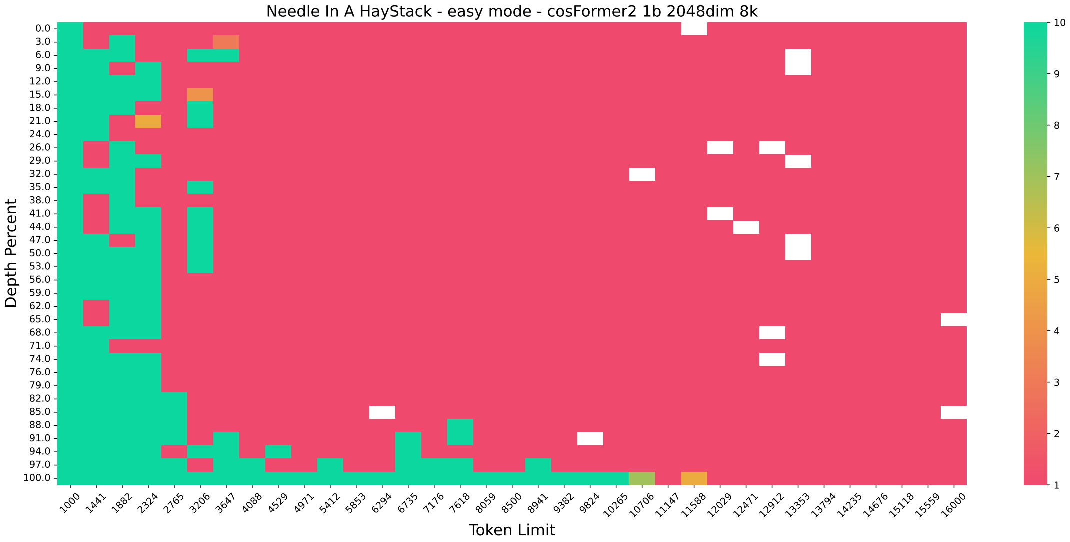
Needle In A HayStack - easy mode - cosFormer2 1b 2048dim 8k
Depth Percent
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6.0
9.0
12.0
15.0
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21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
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62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
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2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
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9824
10265
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11147
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12471
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Score
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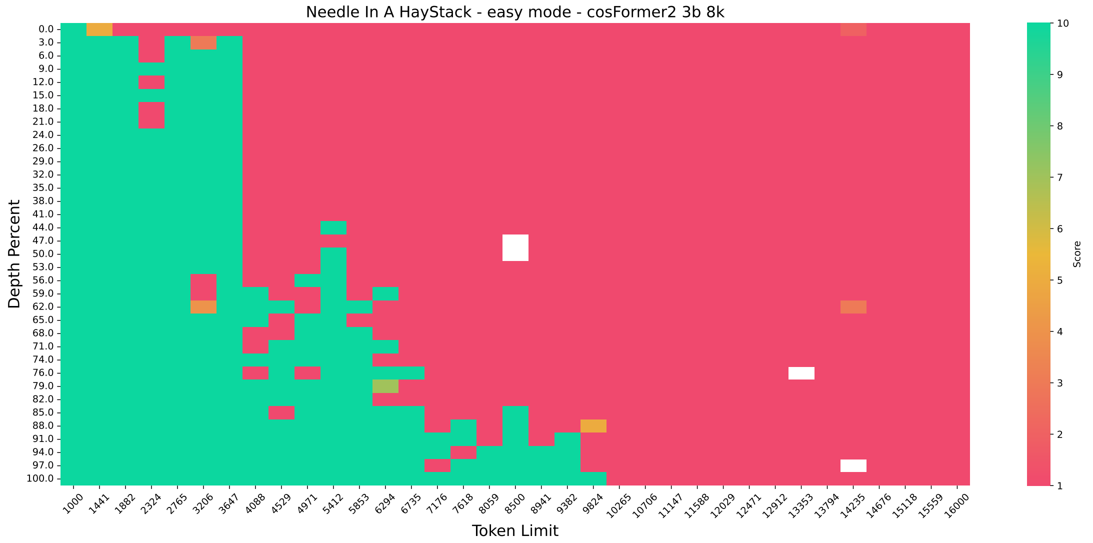
Needle In A HayStack - easy mode - cosFormer2 3b 8k
Depth Percent
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6.0
9.0
12.0
15.0
18.0
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24.0
26.0
29.0
32.0
35.0
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71.0
74.0
76.0
79.0
82.0
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88.0
91.0
94.0
97.0
100.0
Token Limit
1000
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2765
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4971
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5853
6294
6735
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7618
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9824
10265
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Score
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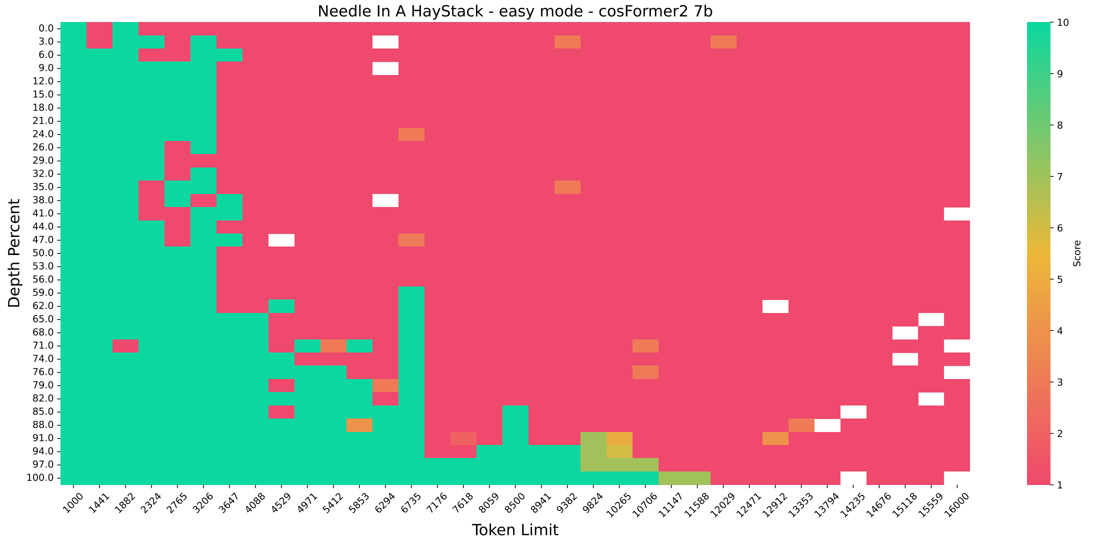
Needle In A HayStack - easy mode - cosFormer2 7b
Depth Percent
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9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
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4971
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6294
6735
7176
7618
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9824
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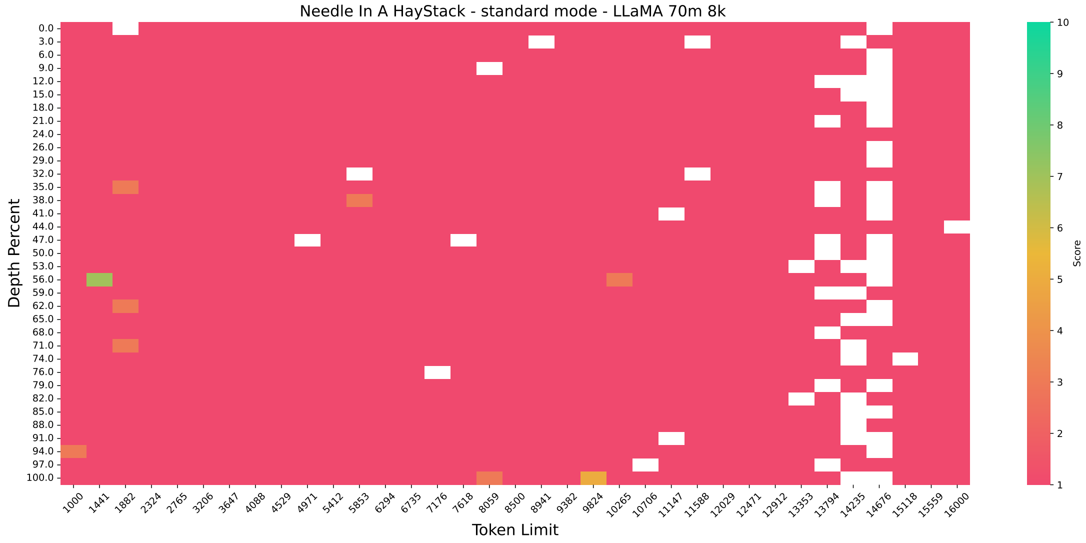
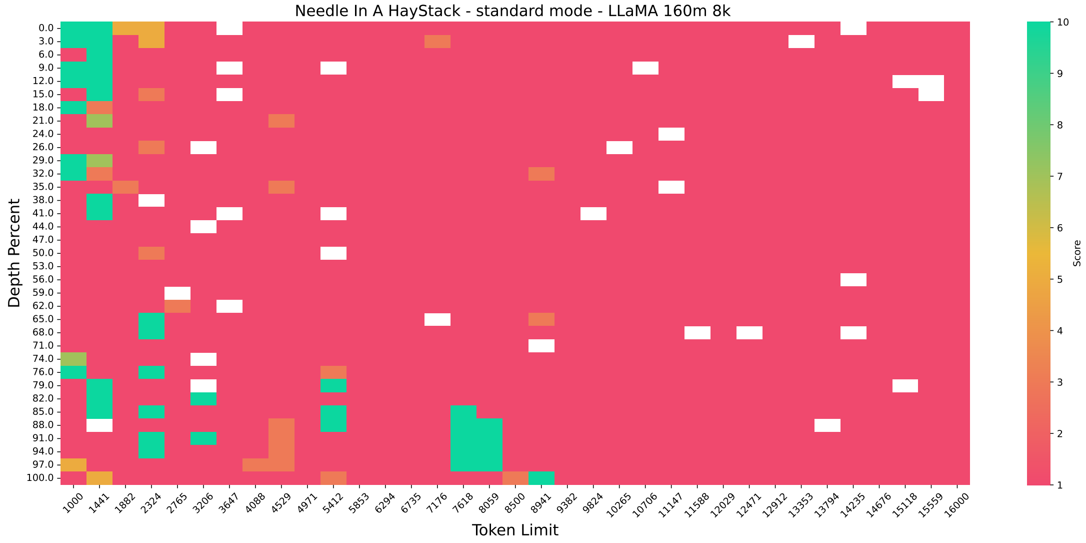
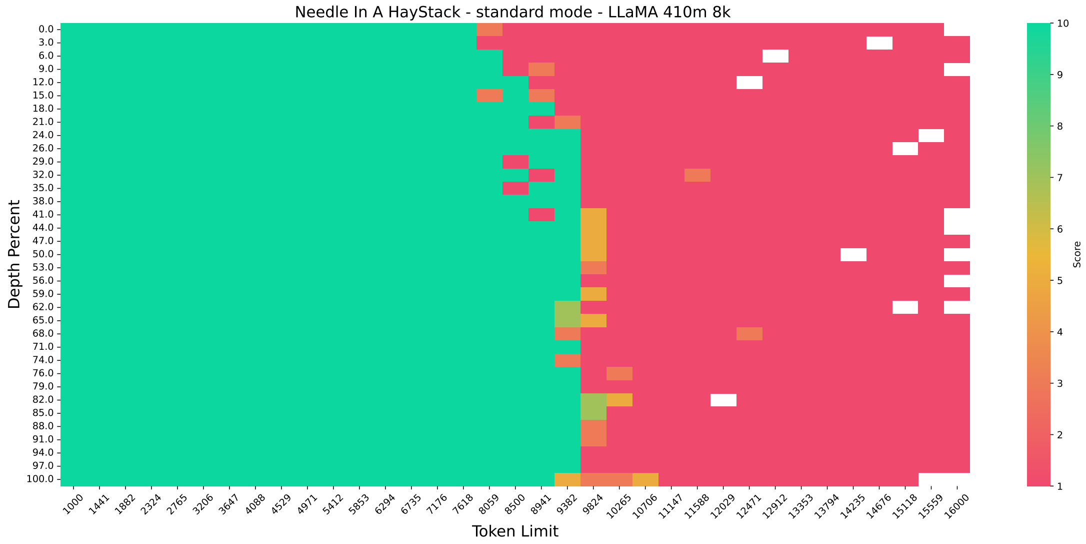

Needle In A HayStack - standard mode - LLaMA 70m 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1


Needle In A HayStack - standard mode - LLaMA 160m 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1


Needle In A HayStack - standard mode - LLaMA 410m 8k
Depth Percent
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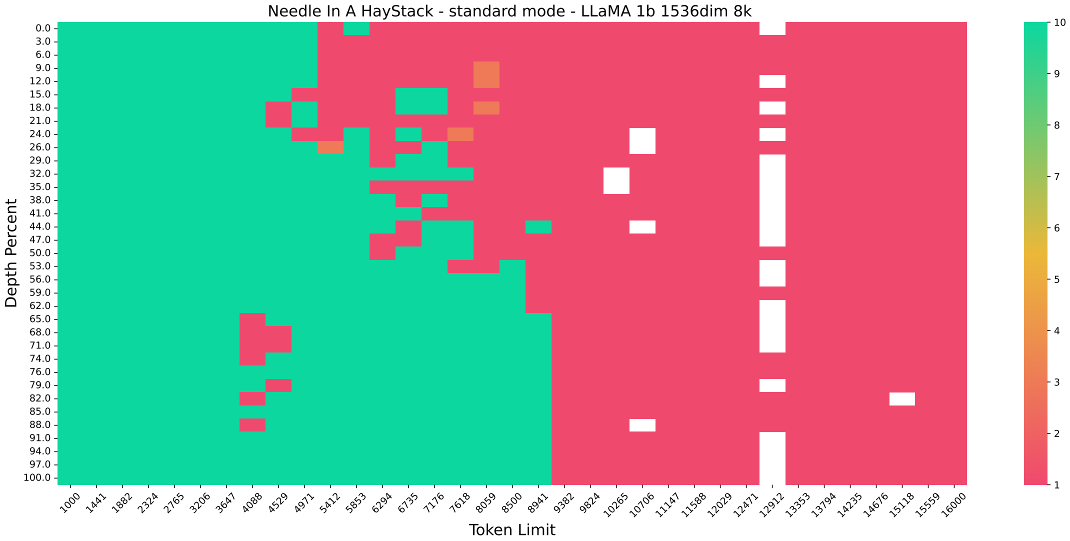
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1


Needle In A HayStack - standard mode - LLaMA 1b 1536dim 8k
Depth Percent
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
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53.0
56.0
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62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
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Score
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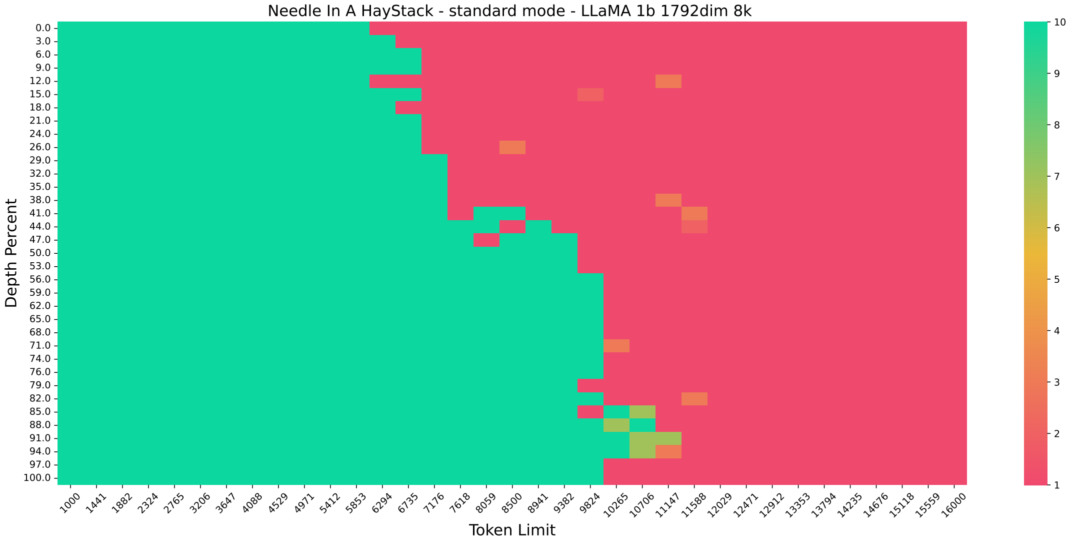
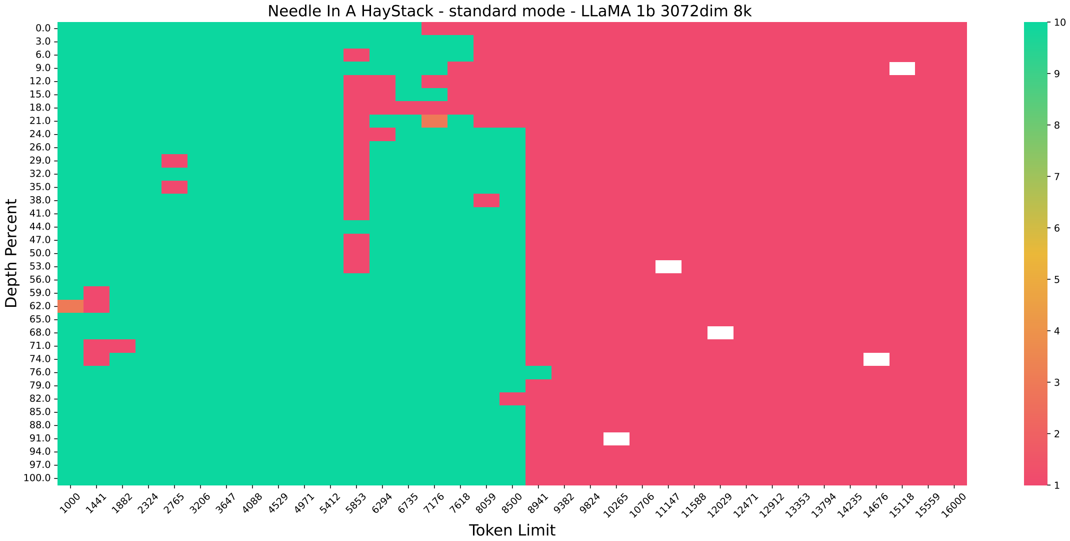
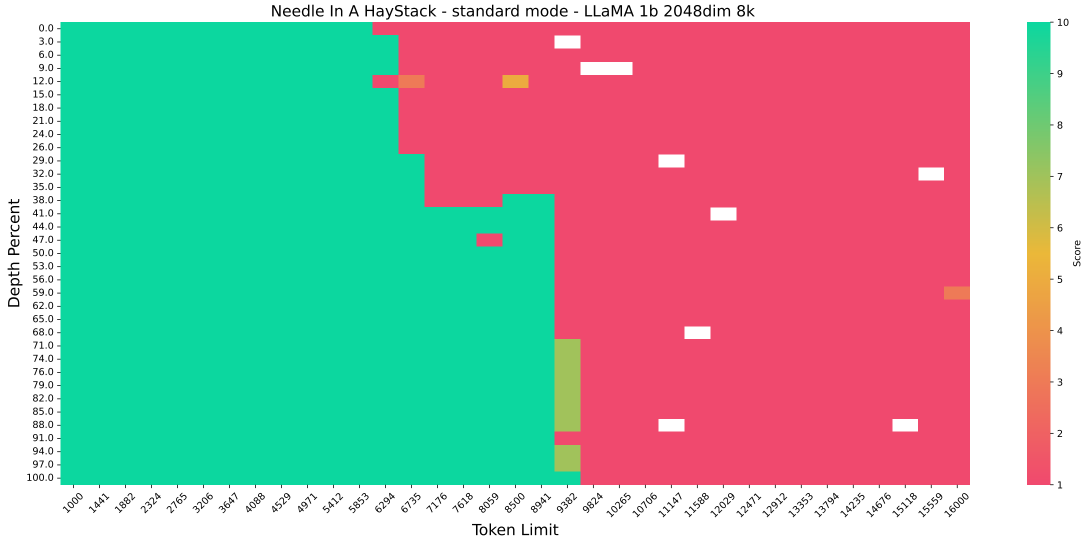
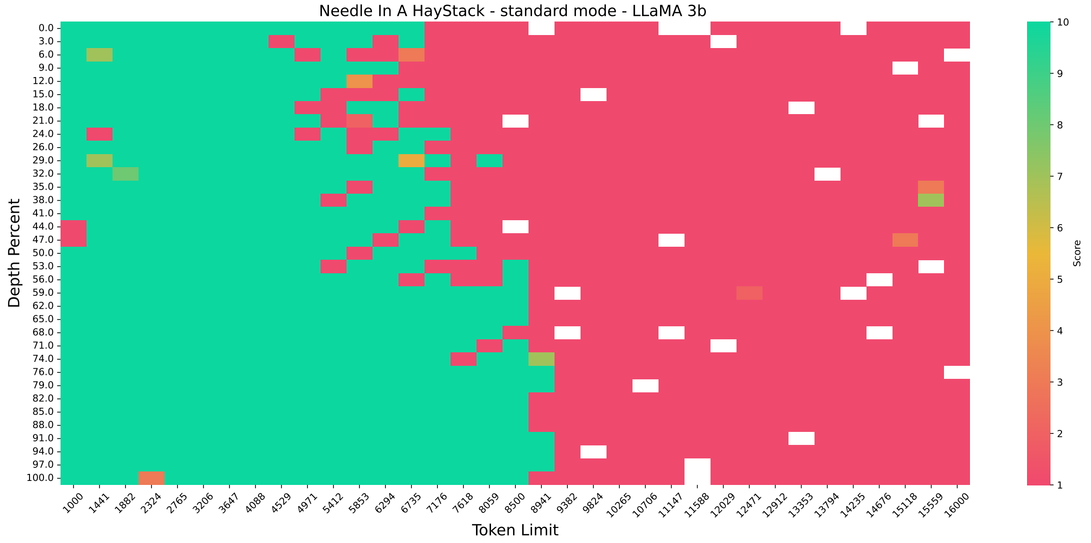
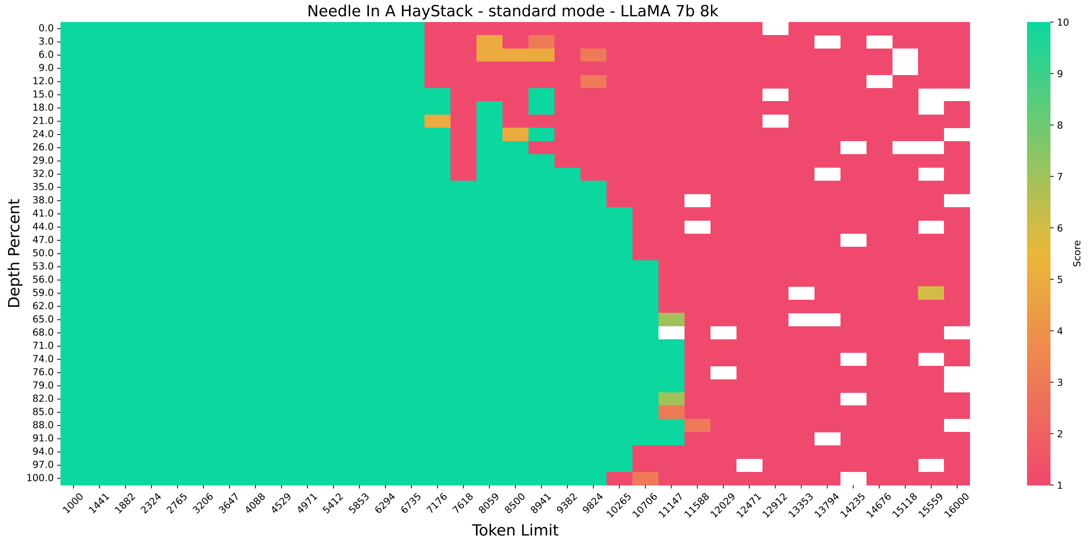
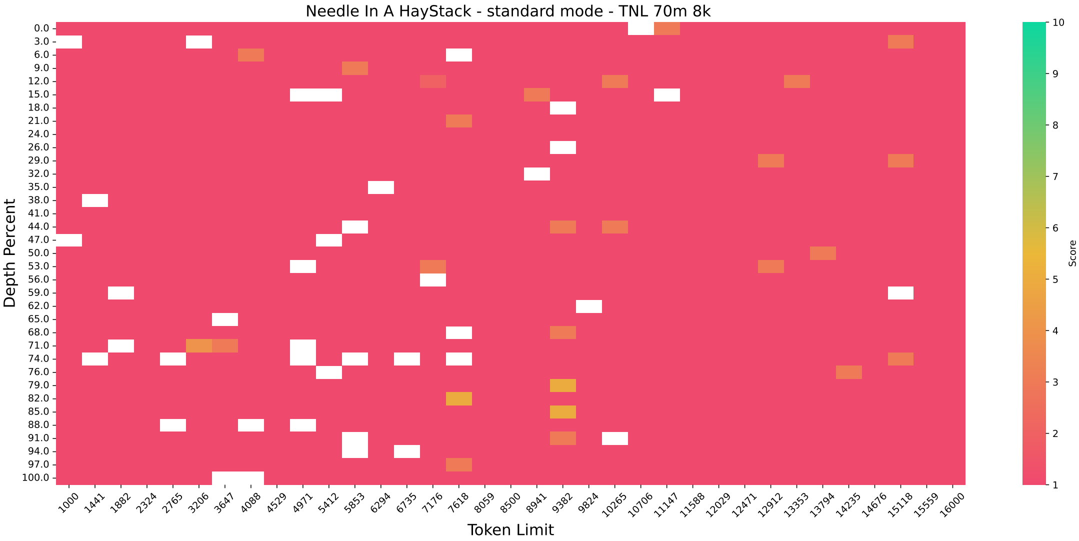

Needle In A HayStack - standard mode - LLaMA 1b 1792dim 8k
Depth Percent
Token Limit
Score


Needle In A HayStack - standard mode - LLaMA 1b 3072dim 8k
Depth Percent
Token Limit
Score


Needle In A HayStack - standard mode - LLaMA 1b 2048dim 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1


Needle In A HayStack - standard mode - LLaMA 3b
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1


Needle In A HayStack - standard mode - LLaMA 7b 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
10 9 8 7 6 5 4 3 2 1


Needle In A HayStack - standard mode - TNL 70m 8k
Depth Percent
0.0
3.0
6.0
9.0
12.0
15.0
18.0
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24.0
26.0
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65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
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13353
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16000
Score
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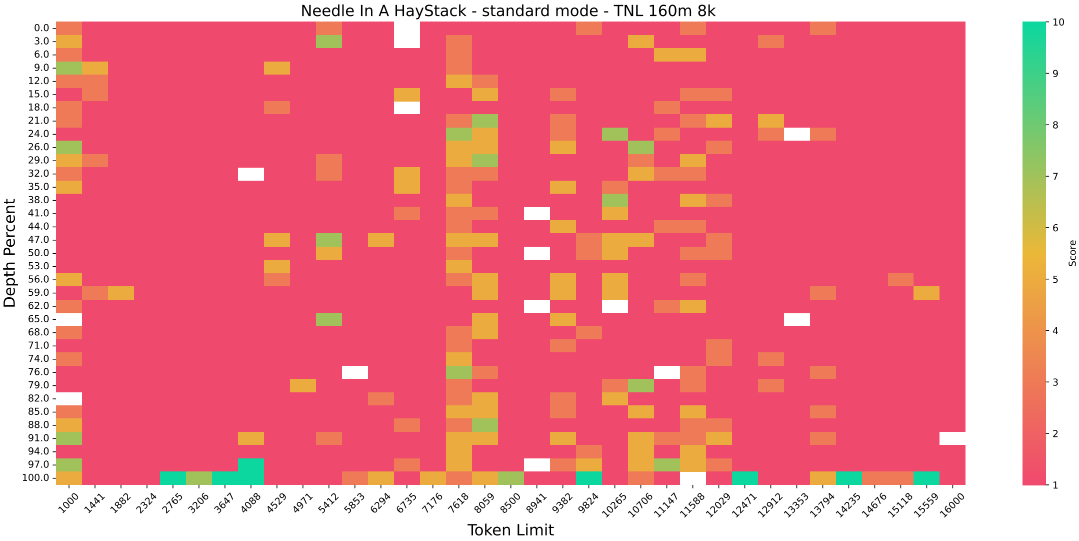
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9.0
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15.0
18.0
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65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
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4971
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6294
6735
7176
7618
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Score
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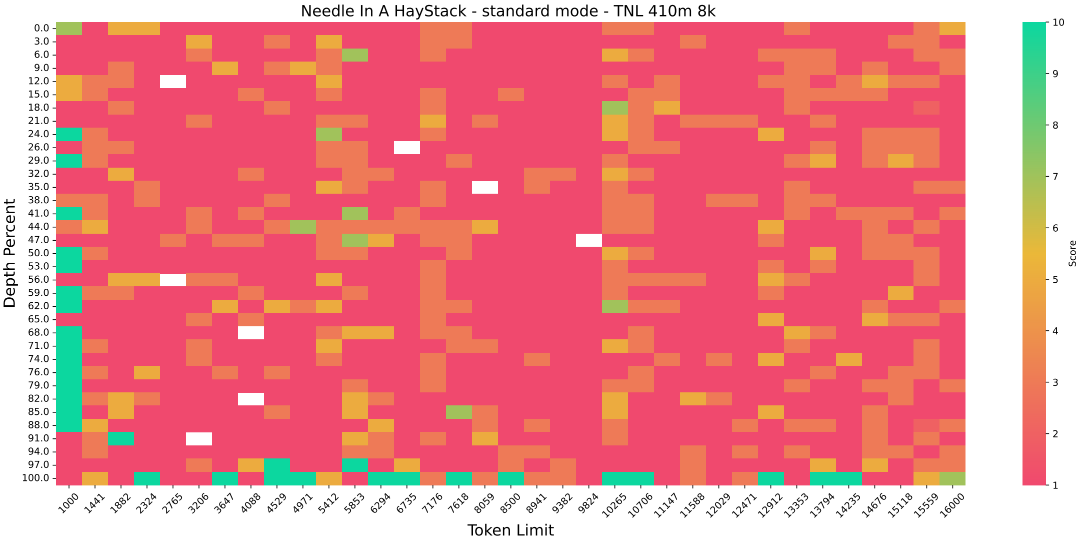
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100.0
Token Limit
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1882
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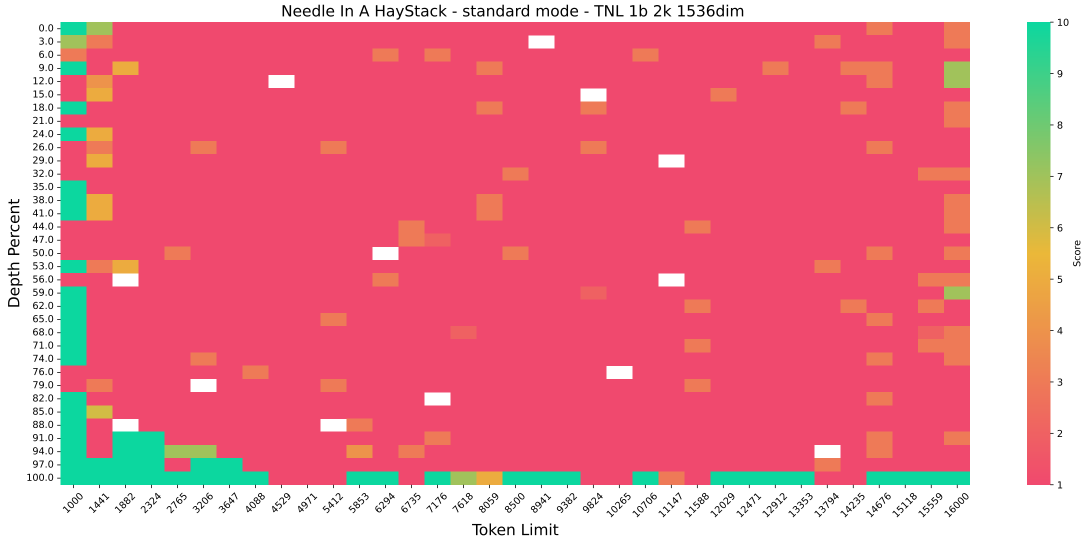
Needle In A HayStack - standard mode - TNL 1b 2k 1536dim
Depth Percent
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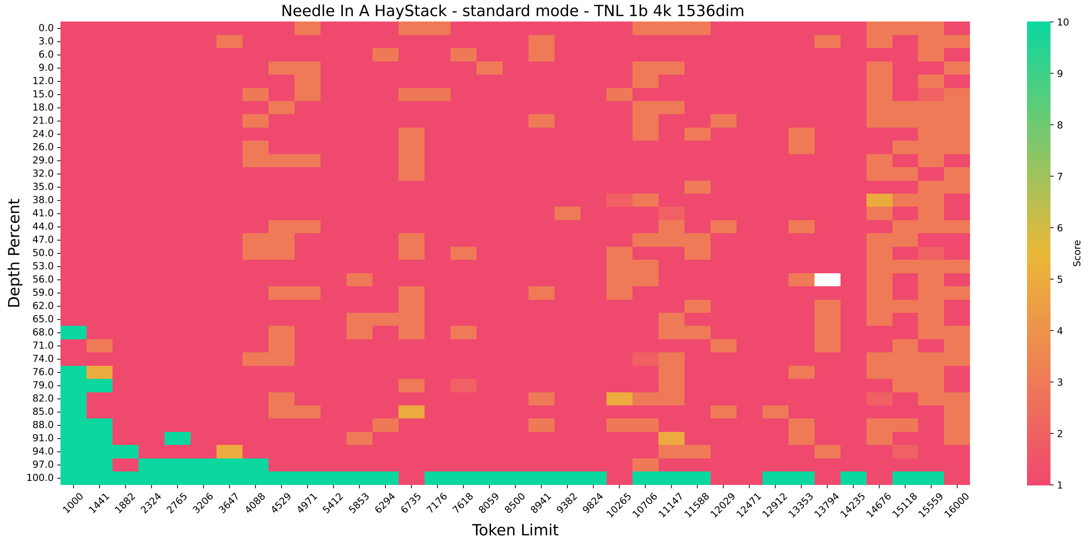
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - TNL 1b 4k 1536dim
Depth Percent
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Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
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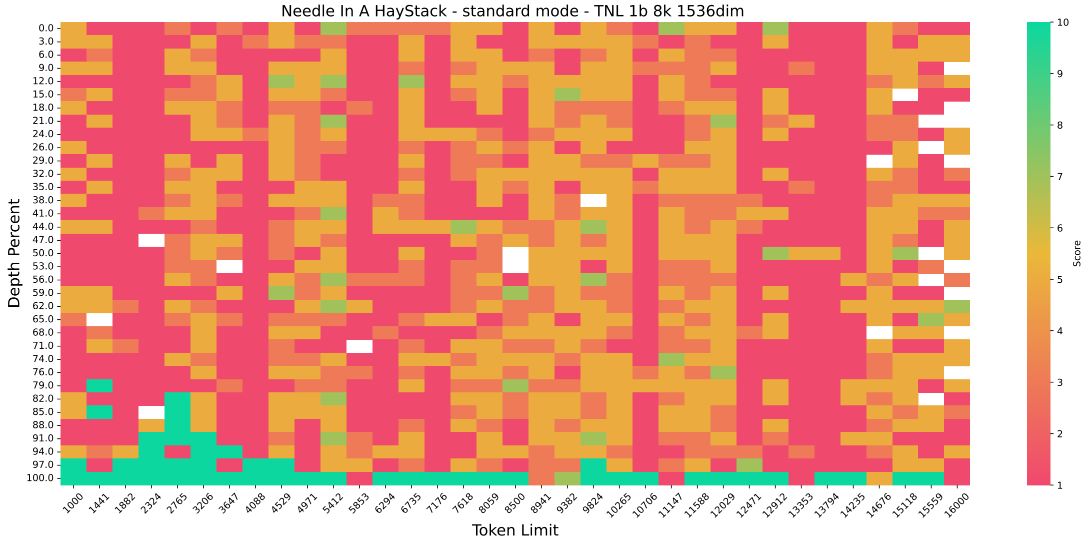
Needle In A HayStack - standard mode - TNL 1b 8k 1536dim
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Token Limit
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Score
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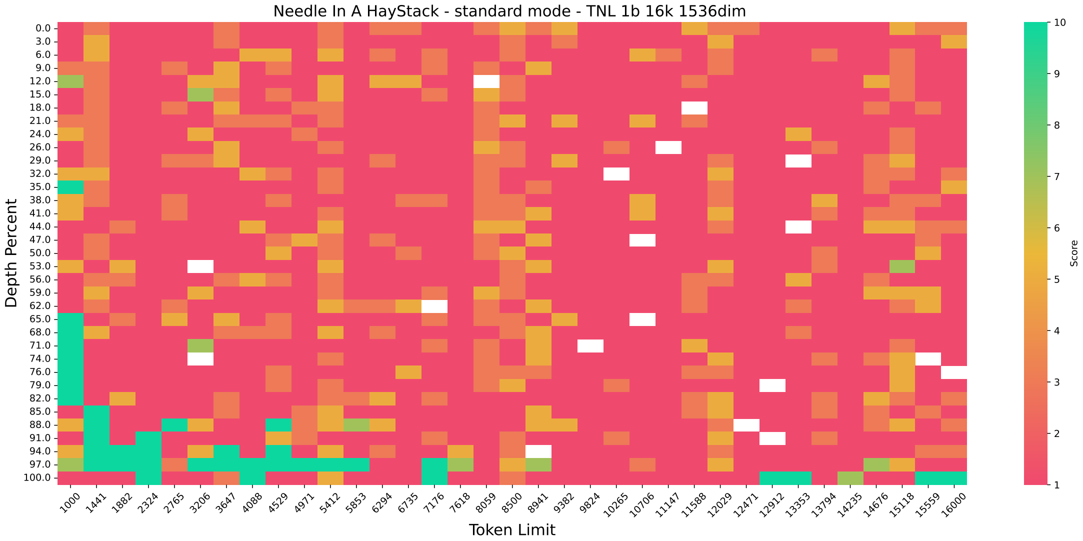
Needle In A HayStack - standard mode - TNL 1b 16k 1536dim
Depth Percent
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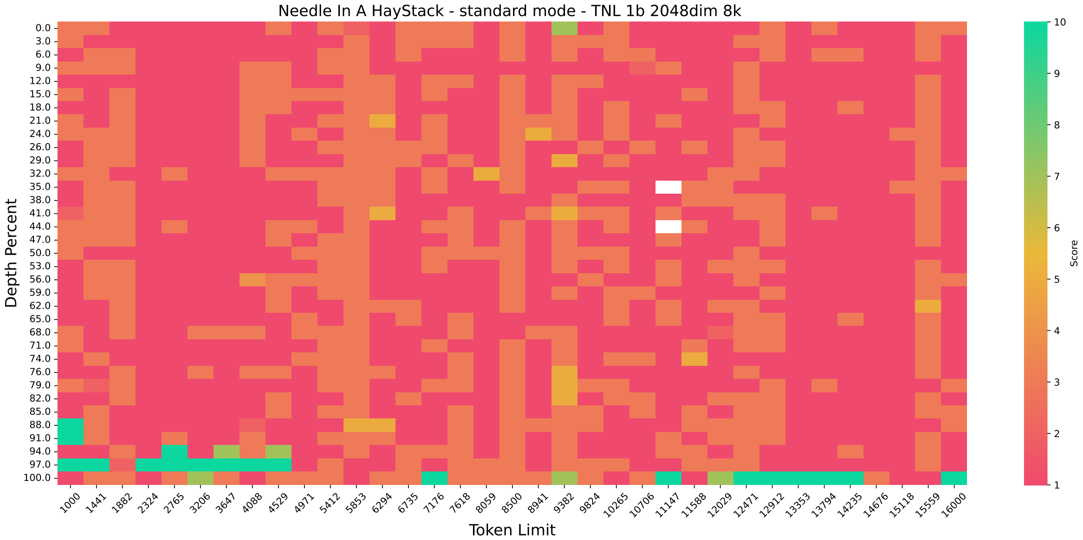
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - TNL 1b 2048dim 8k
Depth Percent
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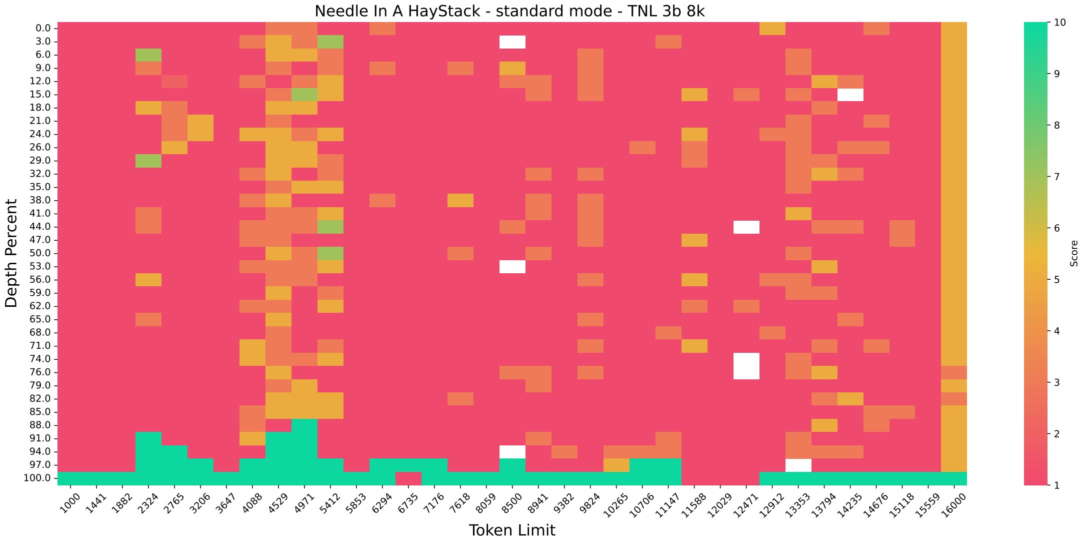
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - TNL 3b 8k
Depth Percent
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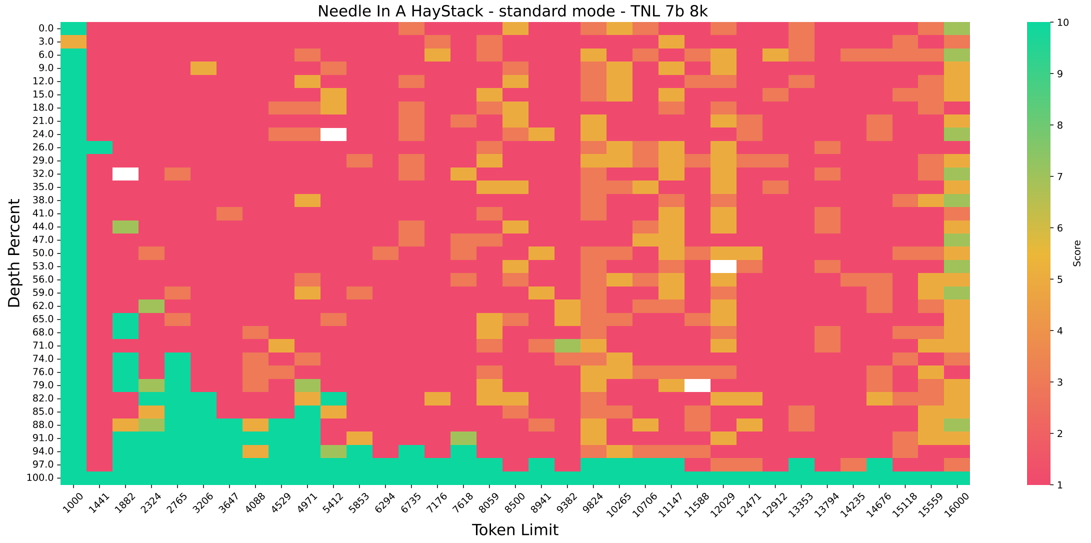
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - TNL 7b 8k
Depth Percent
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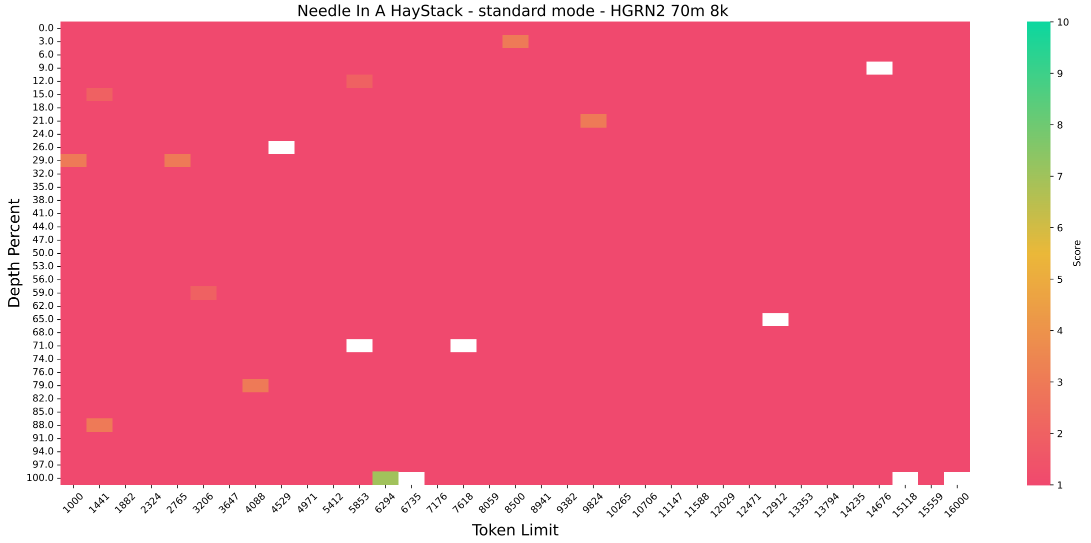
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - HGRN2 70m 8k
Depth Percent
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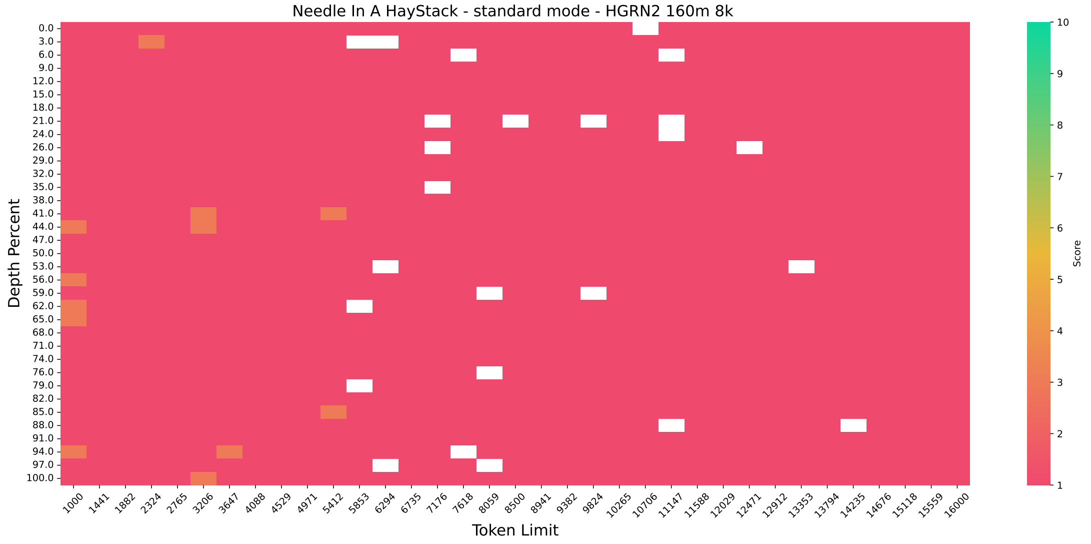
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - HGRN2 160m 8k
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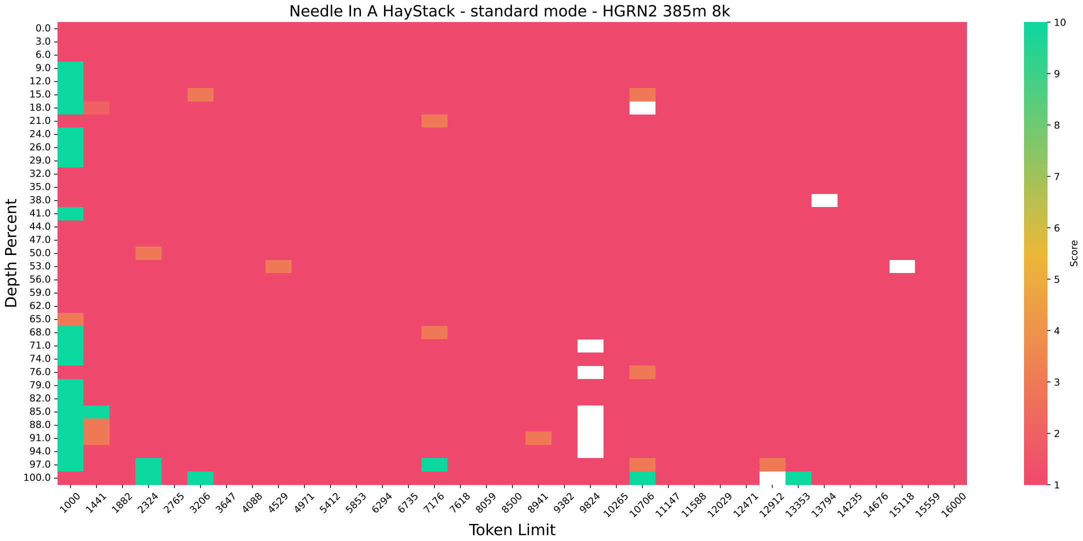
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - HGRN2 385m 8k
Depth Percent
Token Limit
Score
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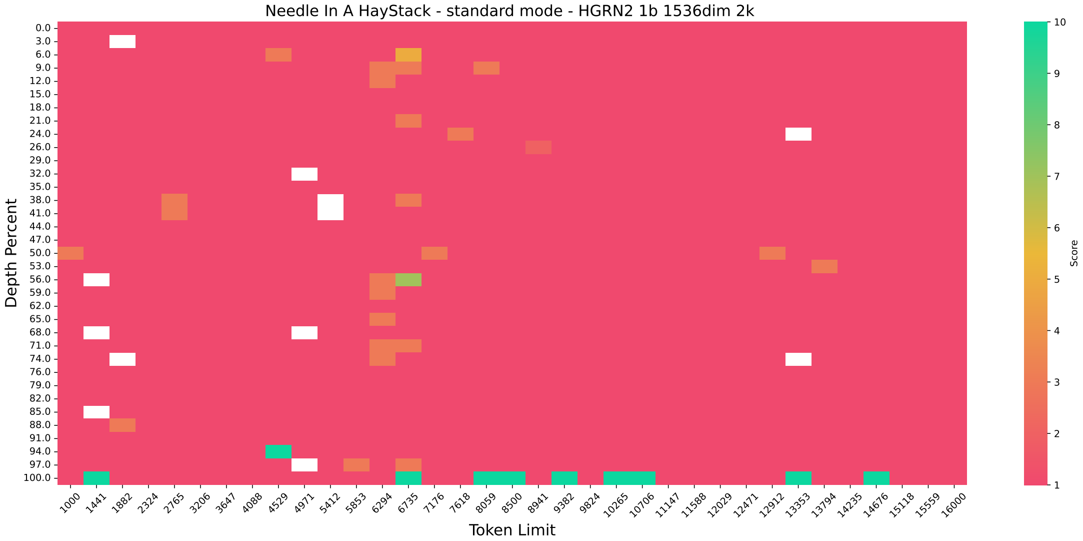
Needle In A HayStack - standard mode - HGRN2 1b 1536dim 2k
Depth Percent
Token Limit
Score
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1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
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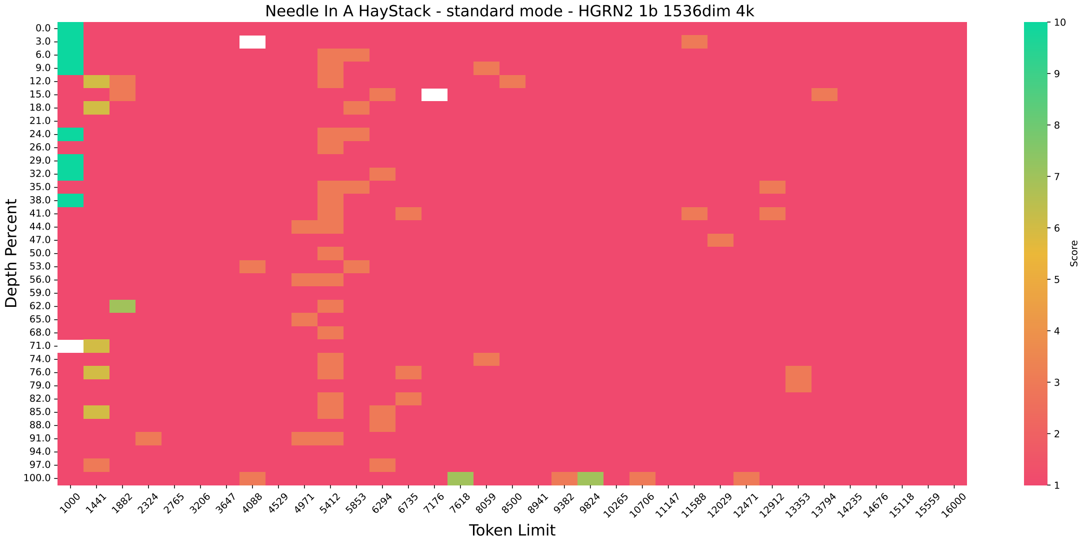
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Depth Percent
Token Limit
Score
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1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
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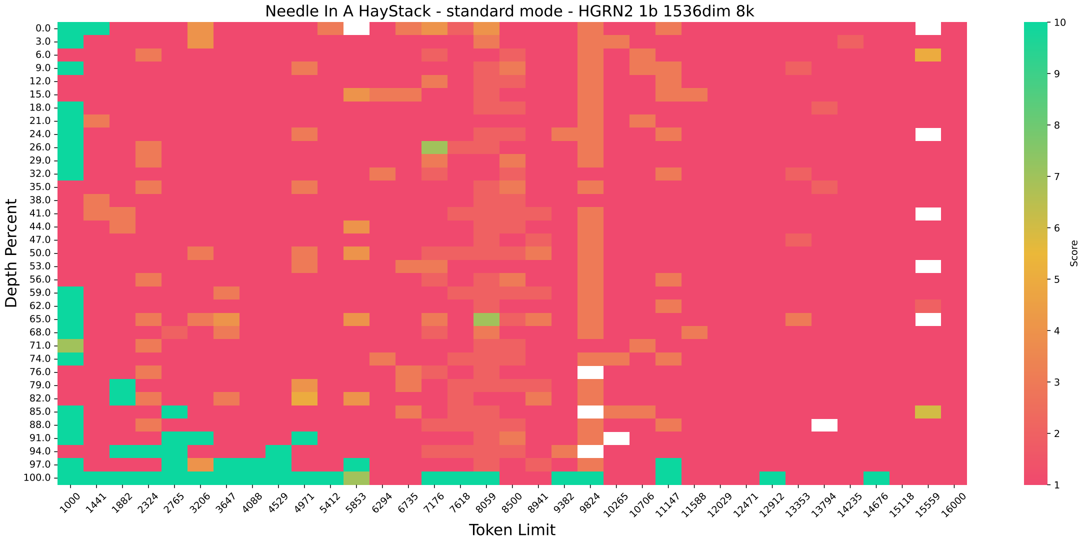
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Token Limit
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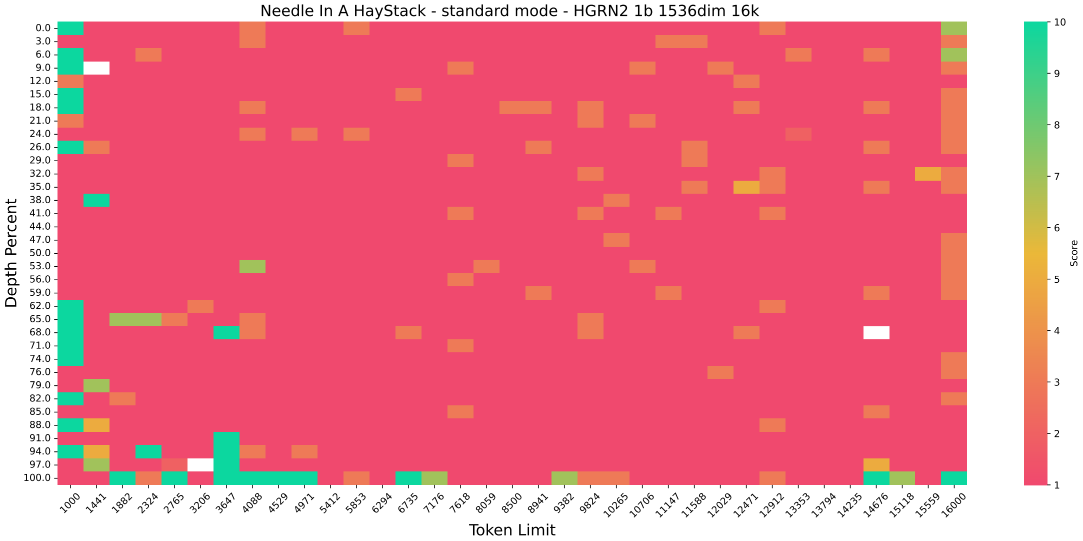
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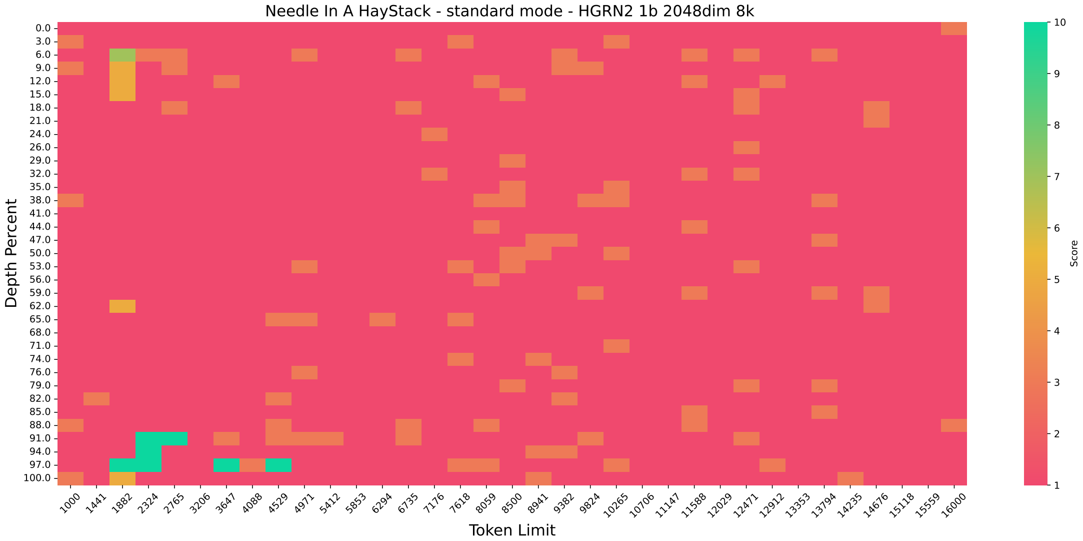
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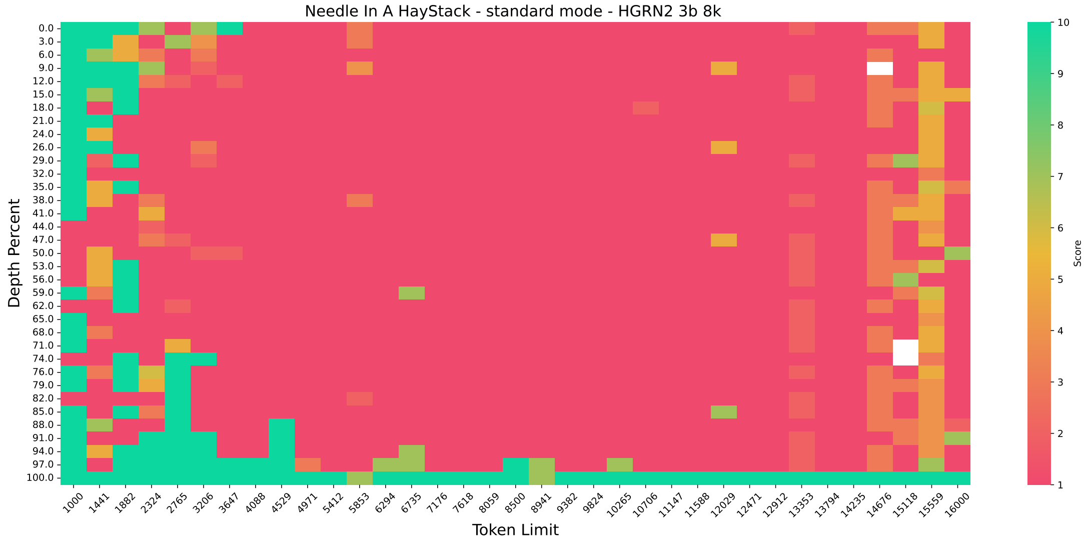
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Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
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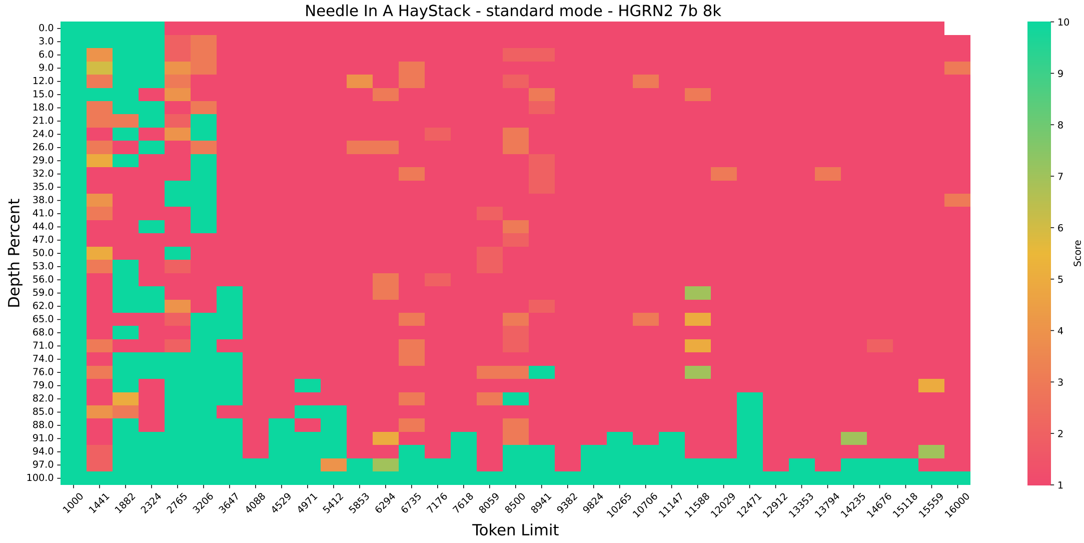
Needle In A HayStack - standard mode - HGRN2 7b 8k
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Token Limit
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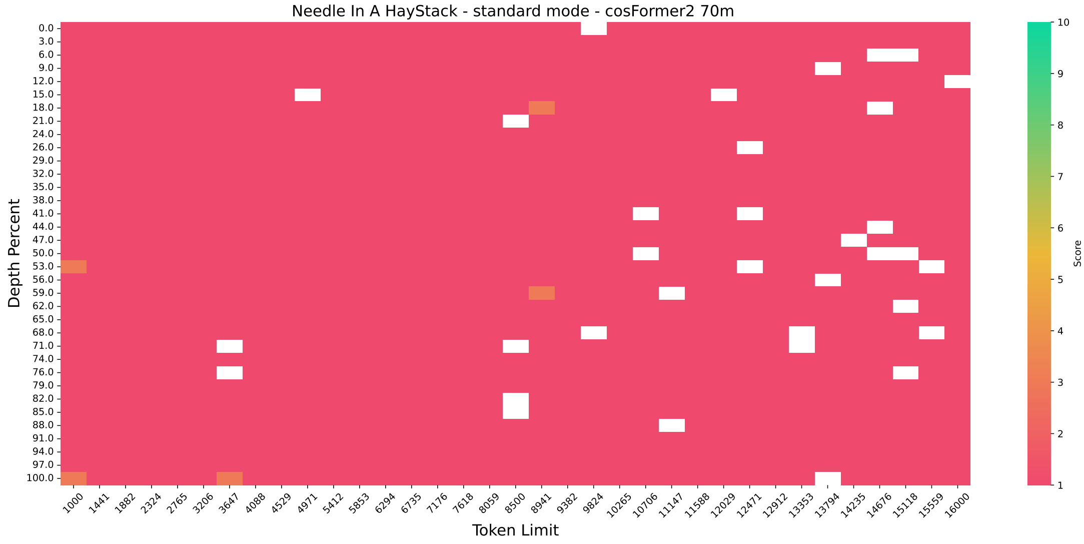
Needle In A HayStack - standard mode - cosFormer2 70m
Depth Percent
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Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
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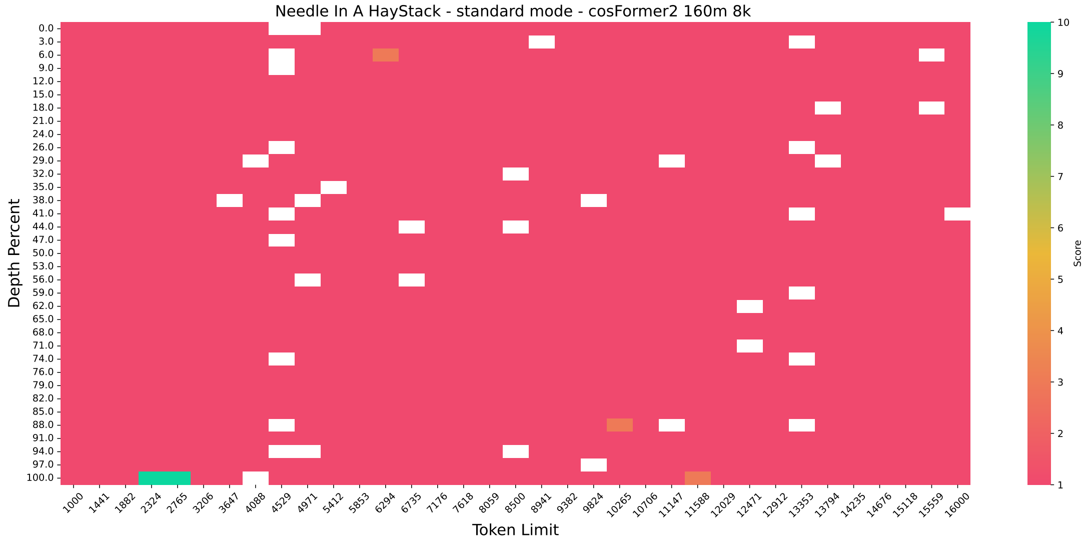
Needle In A HayStack - standard mode - cosFormer2 160m 8k
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Token Limit
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Score
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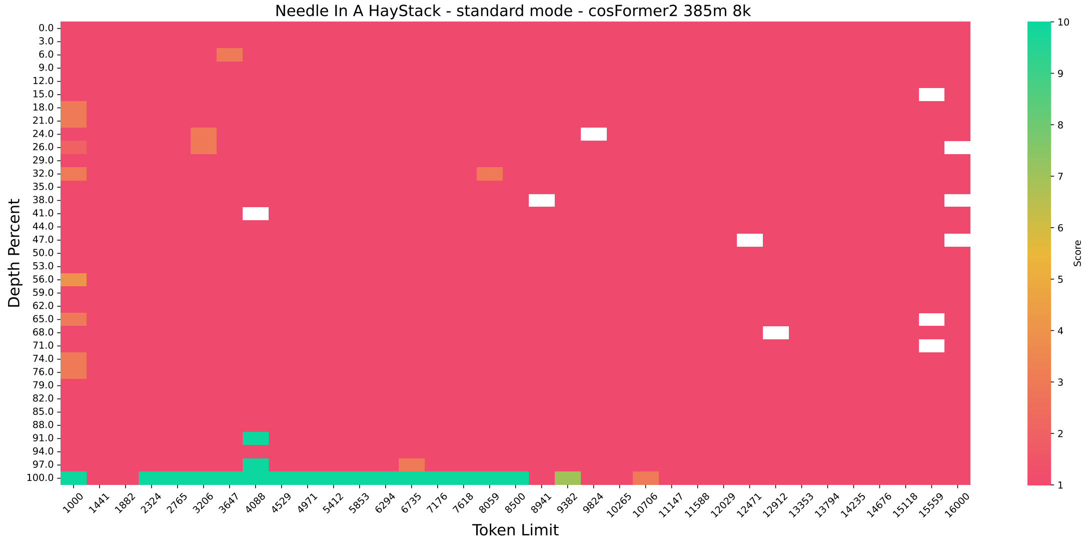
Needle In A HayStack - standard mode - cosFormer2 385m 8k
Depth Percent
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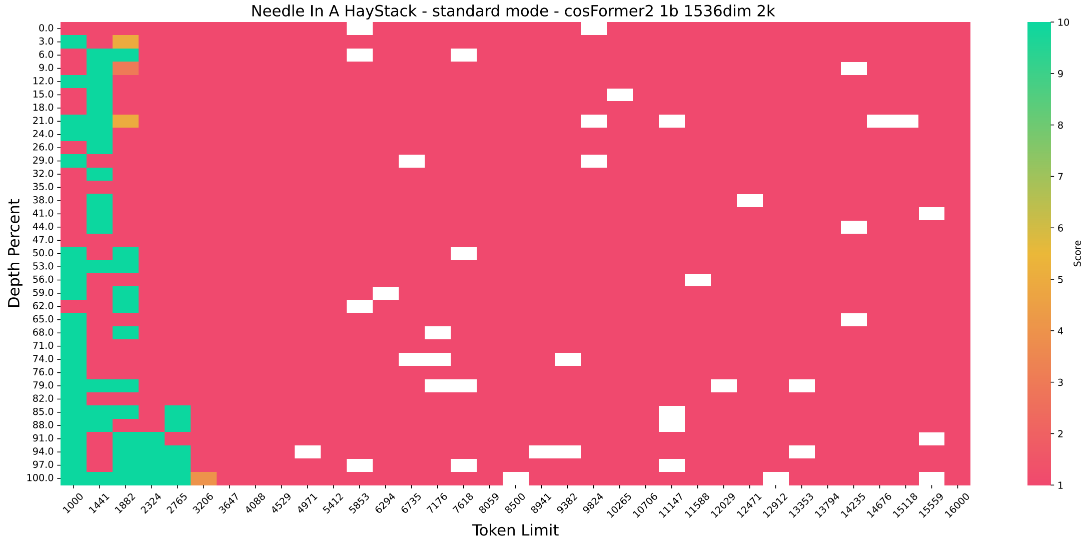
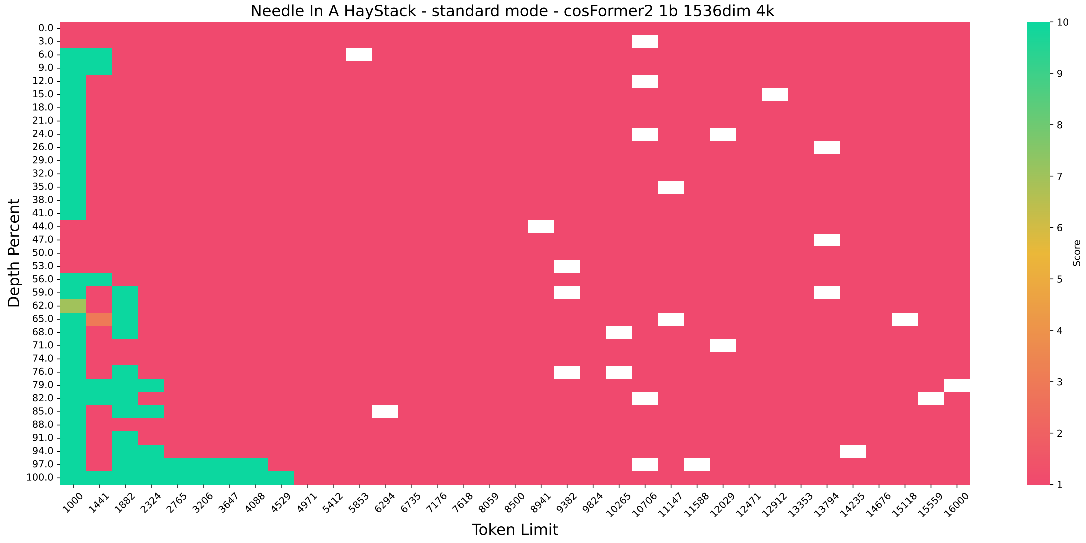
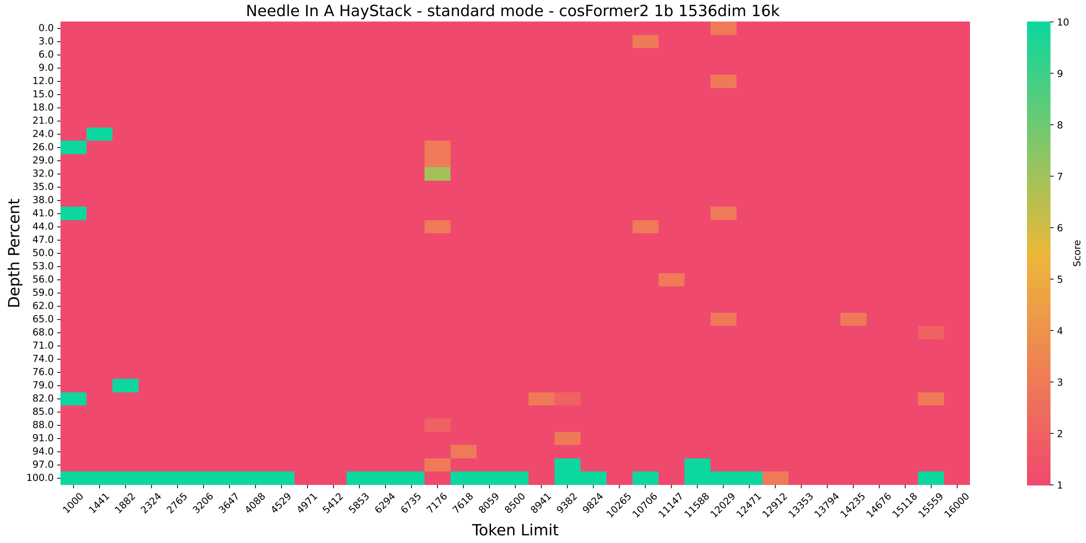
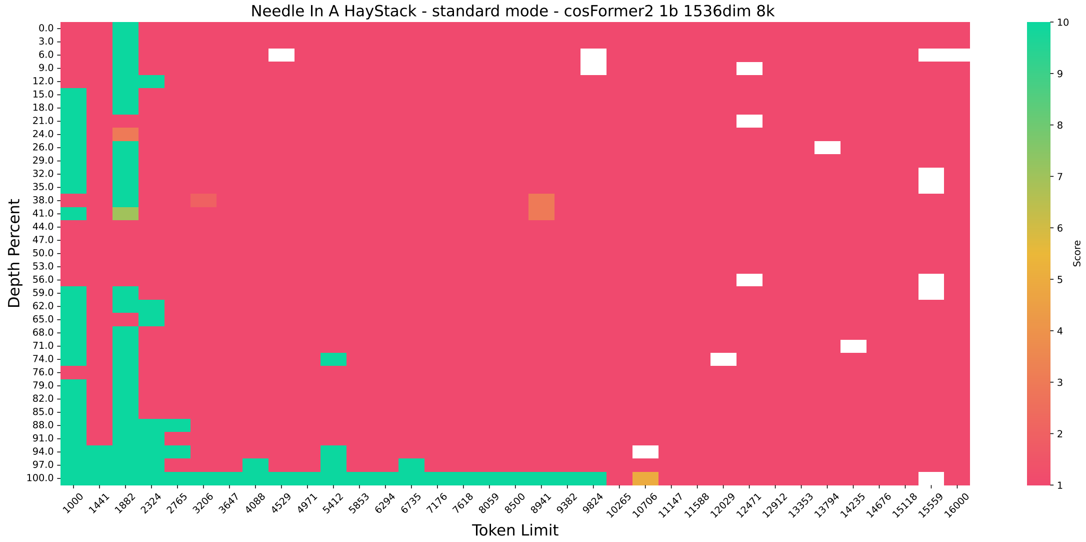
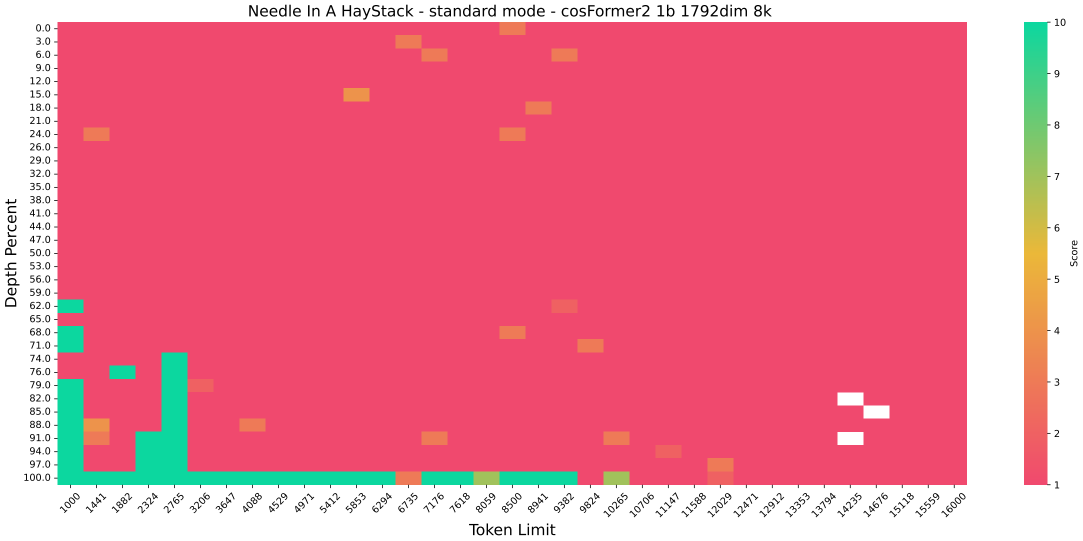
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10

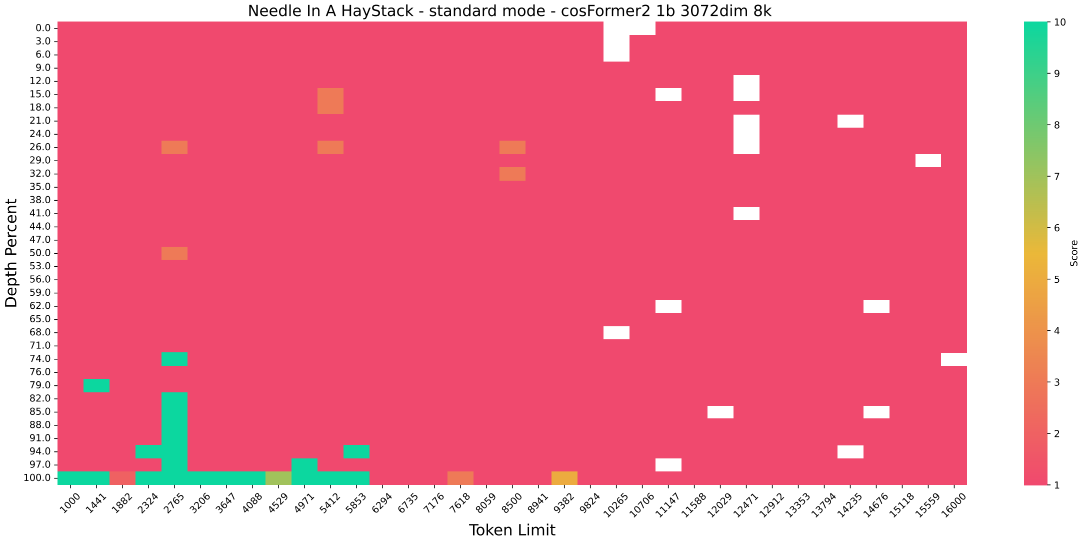

Needle In A HayStack - standard mode - cosFormer2 1b 1536dim 2k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10

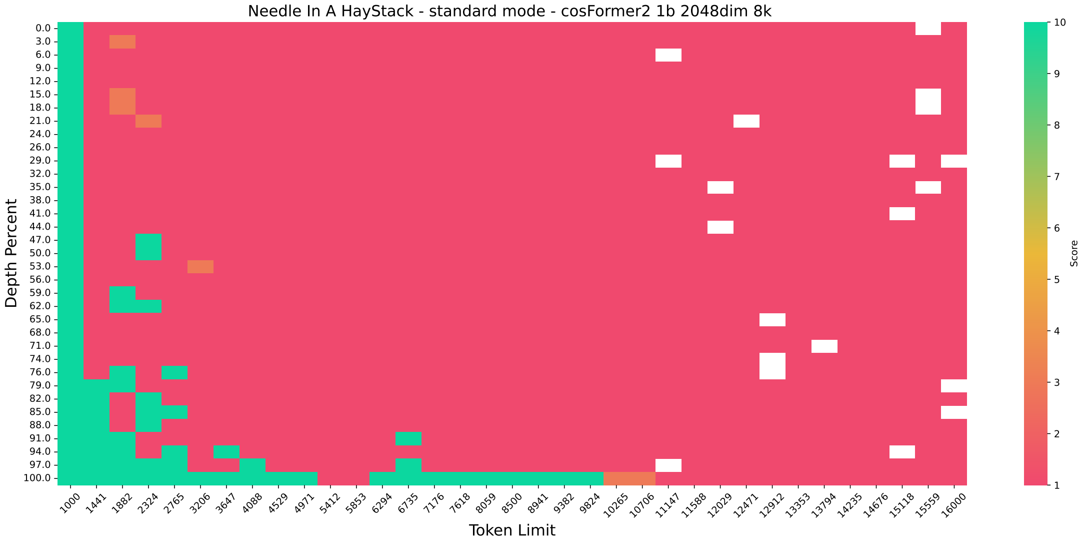

Needle In A HayStack - standard mode - cosFormer2 1b 1536dim 4k

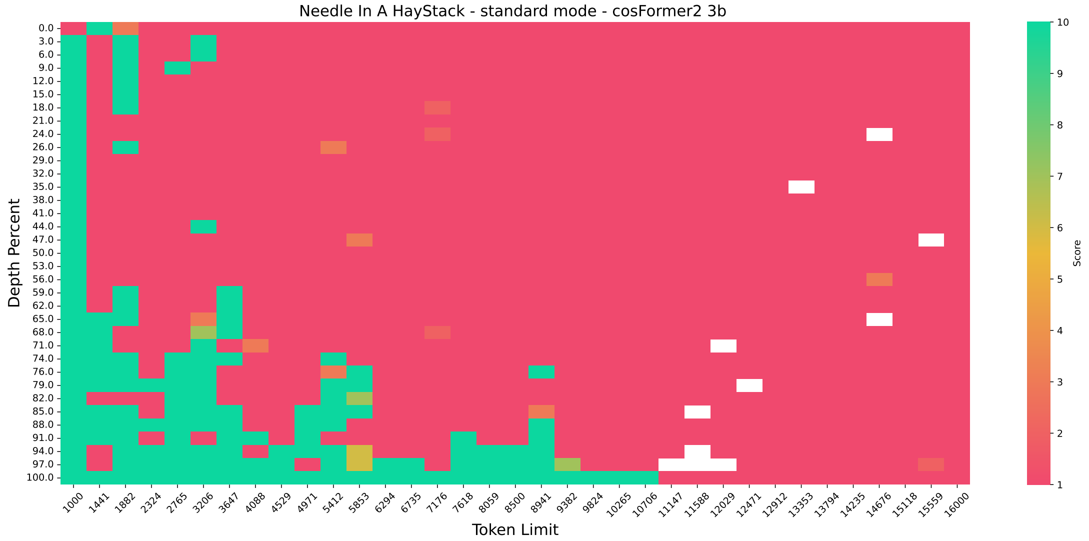

Needle In A HayStack - standard mode - cosFormer2 1b 1536dim 16k

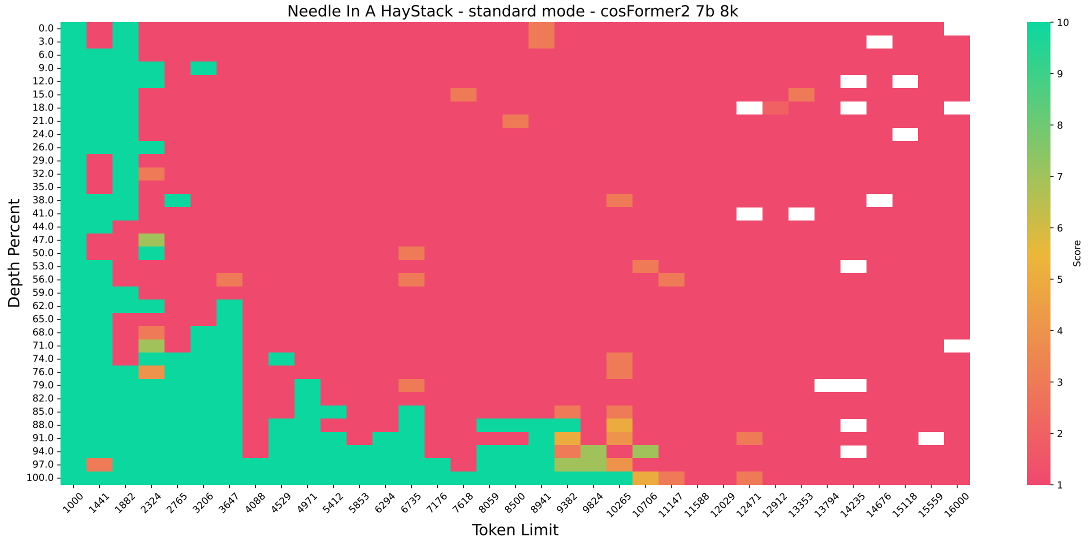

Needle In A HayStack - standard mode - cosFormer2 1b 1536dim 8k


Needle In A HayStack - standard mode - cosFormer2 1b 1792dim 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - cosFormer2 1b 3072dim 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - cosFormer2 1b 2048dim 8k
Depth Percent
0.0 3.0 6.0 9.0 12.0 15.0 18.0 21.0 24.0 26.0 29.0 32.0 35.0 38.0 41.0 44.0 47.0 50.0 53.0 56.0 59.0 62.0 65.0 68.0 71.0 74.0 76.0 79.0 82.0 85.0 88.0 91.0 94.0 97.0 100.0
Token Limit
1000 1441 1882 2324 2765 3206 3647 4088 4529 4971 5412 5853 6294 6735 7176 7618 8059 8500 8941 9382 9824 10265 10706 11147 11588 12029 12471 12912 13353 13794 14235 14676 15118 15559 16000
Score
1 2 3 4 5 6 7 8 9 10


Needle In A HayStack - standard mode - cosFormer2 3b
Depth Percent
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
8059
8500
8941
9382
9824
10265
10706
11147
11588
12029
12471
12912
13353
13794
14235
14676
15118
15559
16000
Score
10
9
8
7
6
5
4
3
2
1


Needle In A HayStack - standard mode - cosFormer2 7b 8k
Depth Percent
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
26.0
29.0
32.0
35.0
38.0
41.0
44.0
47.0
50.0
53.0
56.0
59.0
62.0
65.0
68.0
71.0
74.0
76.0
79.0
82.0
85.0
88.0
91.0
94.0
97.0
100.0
Token Limit
1000
1441
1882
2324
2765
3206
3647
4088
4529
4971
5412
5853
6294
6735
7176
7618
8059
8500
8941
9382
9824
10265
10706
11147
11588
12029
12471
12912
13353
13794
14235
14676
15118
15559
16000
Score
10
9
8
7
6
5
4
3
2
1