LS-SVM based solutions to differential equations

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From a kernel-based modeling point of view, one can consider the given differential equations together with its initial or boundary conditions as prior knowledge and seek the solution by means of Least Squares Support Vector Machines (LSSVMs) whose parameters are adjusted to minimize an appropriate error function. In particular, here we introduce a LS-SVM based framework for learning the solutions of dynamical systems governed by Ordinary Differential Equations, Differential Algebraic Equations (DAEs) as well as Partial Differential Equations (PDEs) [1, 2, 3]. The problem is formulated as an optimization problem in the primal-dual setting. The approximate solution in the primal is expressed in terms of the feature map and is forced to satisfy the system dynamics, initial/boundary conditions using a constrained optimization problem. The optimal representation of the solution is then obtained in the dual. For the linear and nonlinear cases, the model parameters are obtained by solving a system of linear and nonlinear equations, respectively. The proposed model utilizes few training points to learn a closed form approximate solution. Furthermore, it does not require index reduction techniques and can directly be applied to learn the state trajectories of the underlying implicit systems. The experimental results on different ODE systems, DEA systems with index from 0 to 3, as well as PDEs are presented and compared with analytic solutions to confirm the validity and applicability of the proposed method. Similarly, analogous approaches based on the introduced LS-SVM framework can be used to address inverse problems where given the observational data one in particular aims at estimating the unknown parameters of the system [4, 5]. Here we present an extended formulation using LS-SVM core model to estimate the unknown constant or time-varying parameters of the given dynamical system described by ordinary (delay) differential equations. Finally, some possibilities and future directions for considering deeper architectures will be discussed.

Références

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