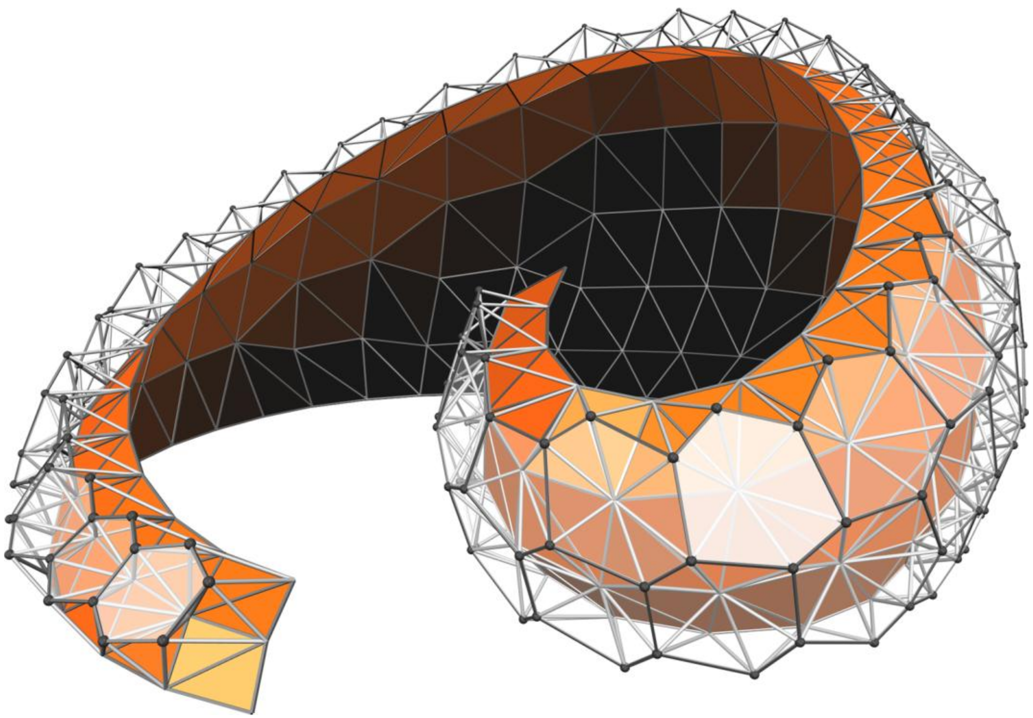
Variational Tangent Plane Intersection for Planar Polygonal Meshing

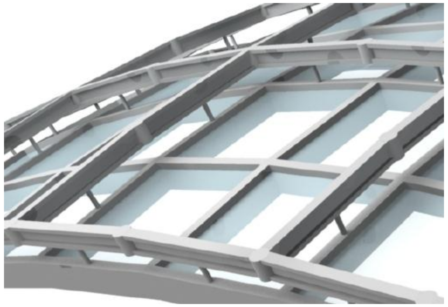


Henrik Zimmer, Marcel Campen, Ralf Herkrath, Leif Kobbelt

Motivation

Motivation

- Multi-Layer Support Structures



Discrete Surfaces for Architectural Design. Pottmann, Brell-Cokcan, Wallner. *2006*

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Geometry of multi-layer freeform structures for architecture. **Pottmann**, Liu, Wallner, Bobenko, Wang. *2007*

Motivation

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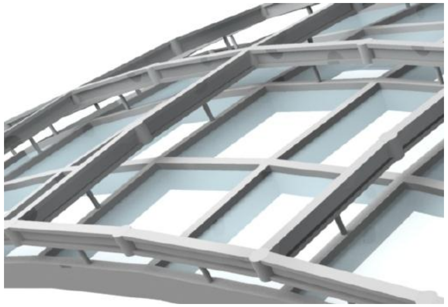

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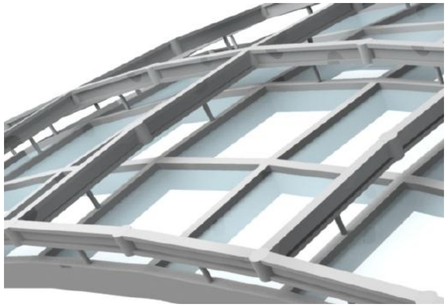

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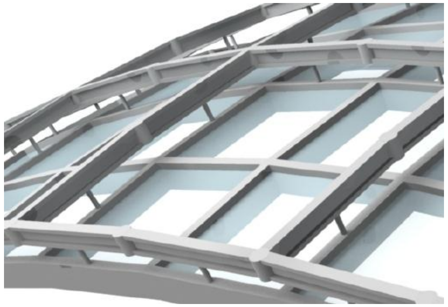

Supporting + Covering Layer

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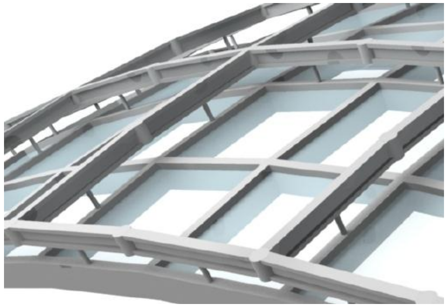
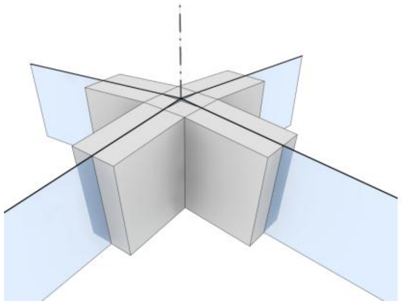


Supporting + Covering Layer



Node/edge simplicity

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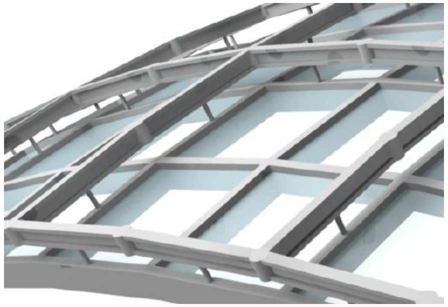
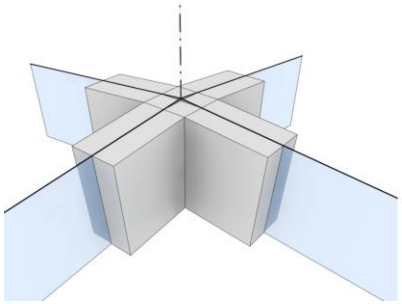
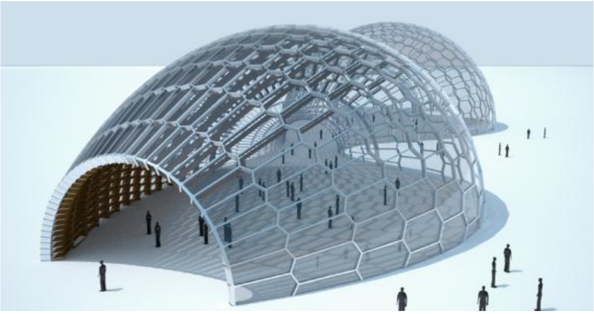


Supporting + Covering Layer



Node/edge simplicity



Planar panels

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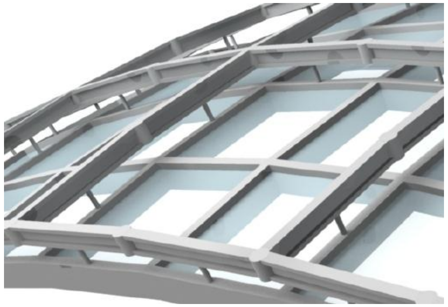
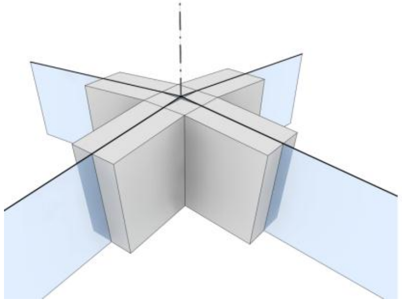
Motivation

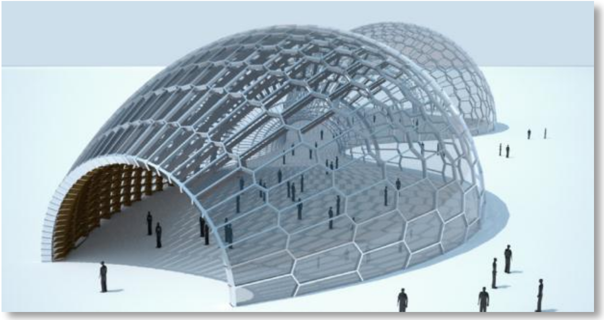
- Multi-Layer Support Structures



Supporting + Covering Layer



Node/edge simplicity



Planar panels

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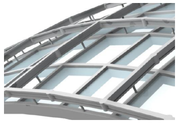
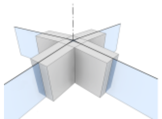
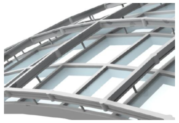
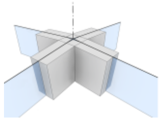
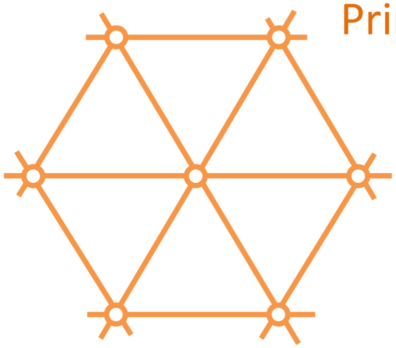
Motivation

- Multi-Layer Support Structures   
- Multi-Layer Dual Structures

Motivation

- Multi-Layer Support Structures   
- Multi-Layer Dual Structures



Motivation

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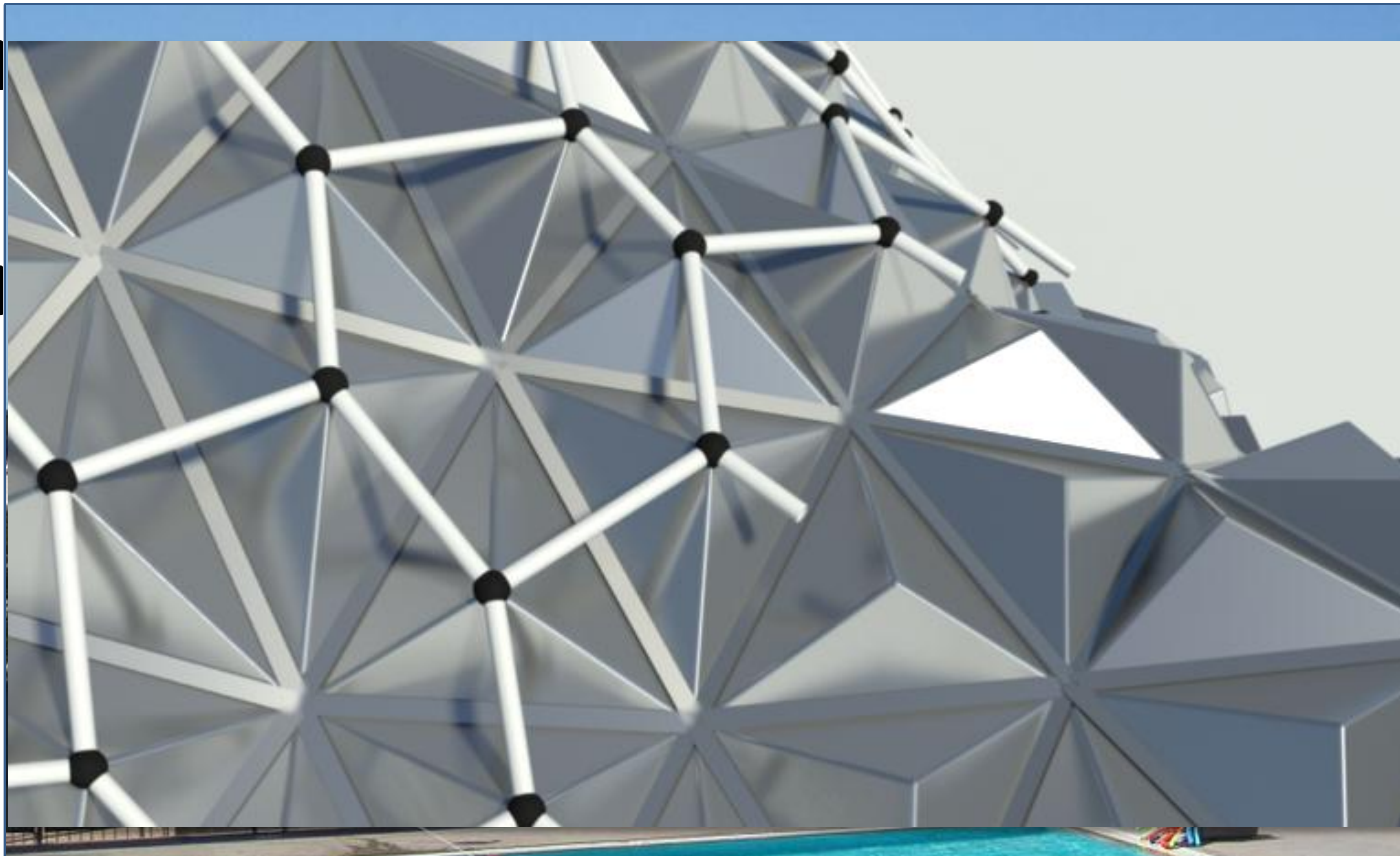
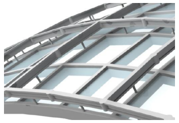
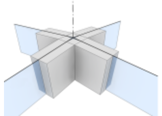
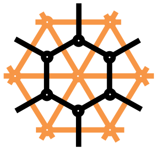
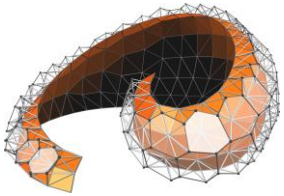


Motivation

- Multi-Layer Support Structures

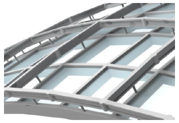
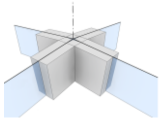




- Multi-Layer Dual Structures

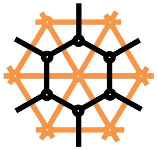
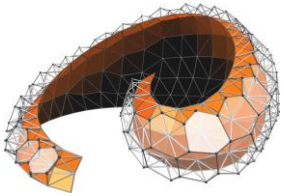



Motivation

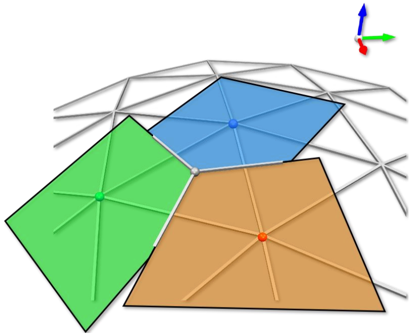
- Multi-Layer Support Structures


- Multi-Layer Dual Structures

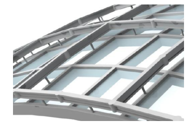
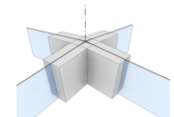
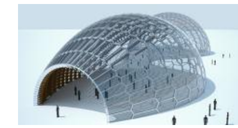



- Tangent Plane Intersection (TPI)

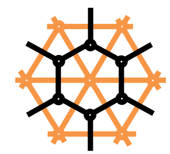
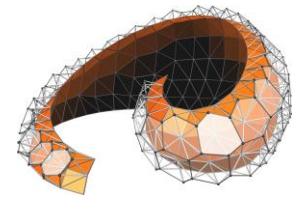


Motivation

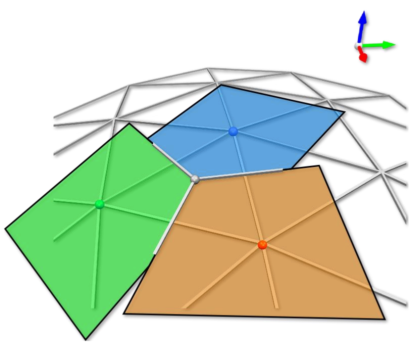
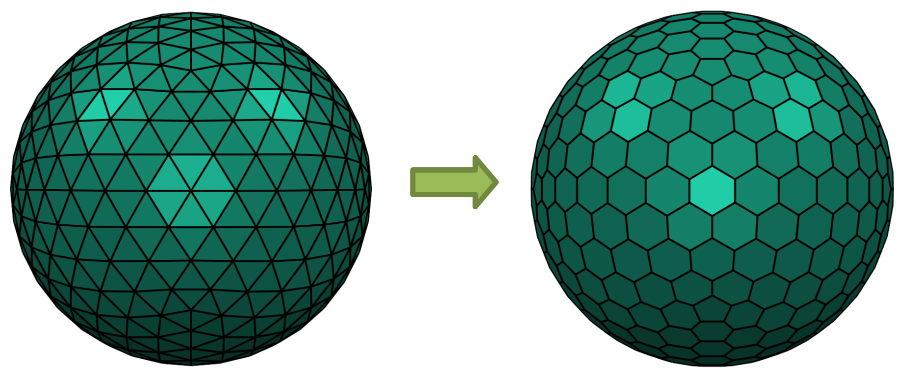
- Multi-Layer Support Structures

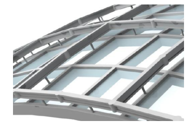
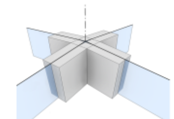
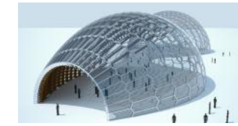
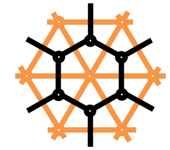
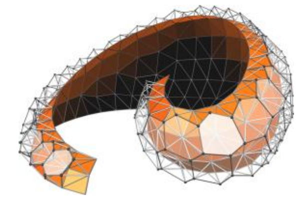
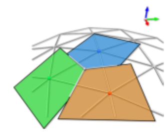
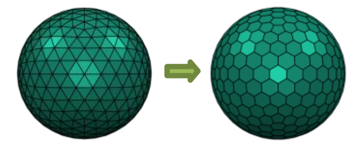
- Multi-Layer Dual Structures

- Tangent Plane Intersection (TPI)

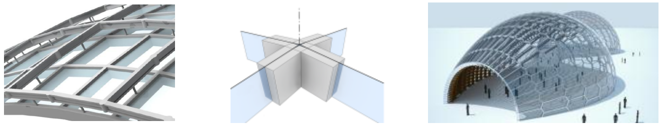



Motivation

- Multi-Layer Support Structures   
- Multi-Layer Dual Structures  
- Tangent Plane Intersection (TPI)  
- *Variational* TPI

Motivation

- Multi-Layer Support Structures



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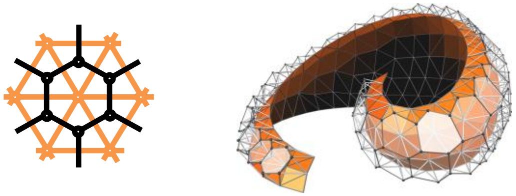

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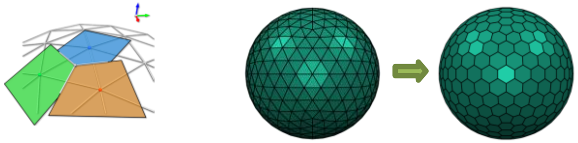
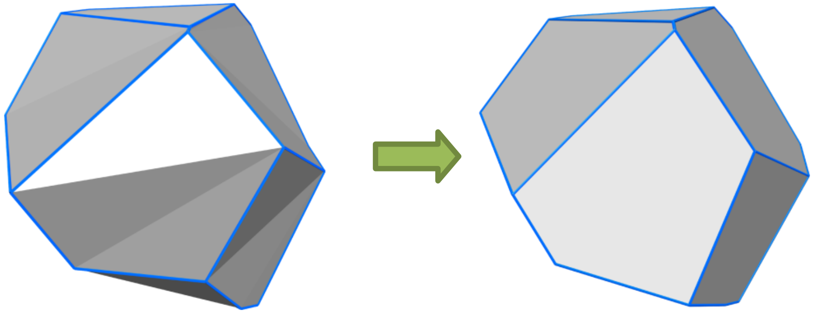


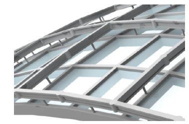
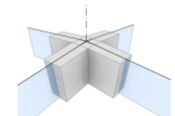
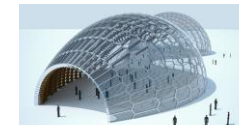
- *Variational* TPI
  - Polygon Mesh Planarization



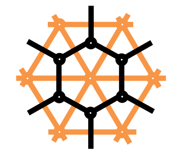
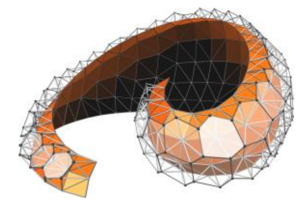
Computer Graphics Group
Henrik Zimmer
RWTH AACHEN UNIVERSITY

Motivation

- Multi-Layer Support Structures

- Multi-Layer Dual Structures

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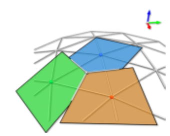
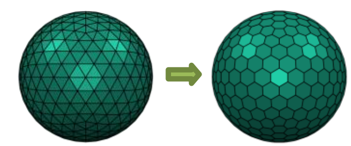
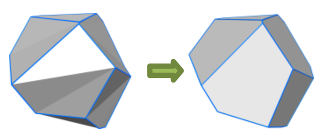
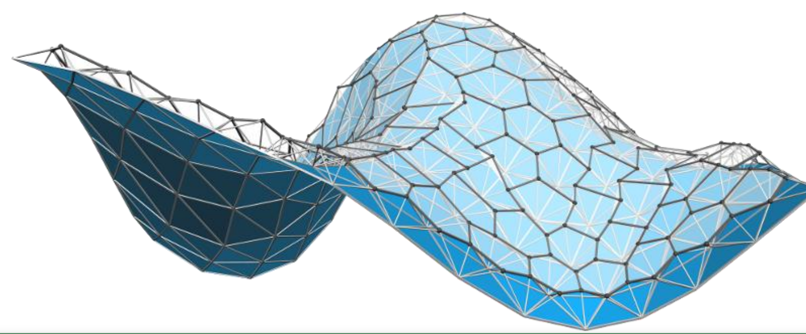
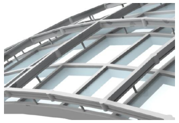
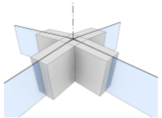
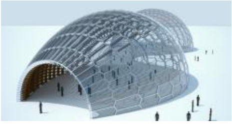
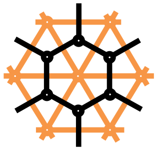
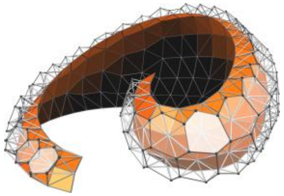
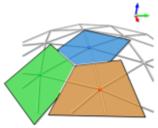
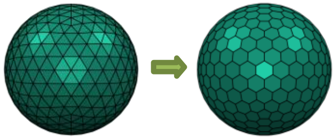
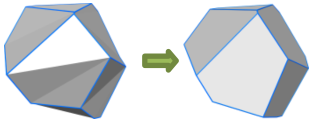
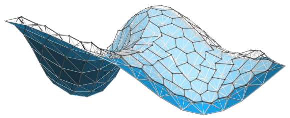



- *Variational* TPI
  - Dual Support Structures

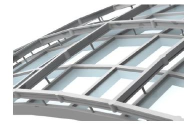
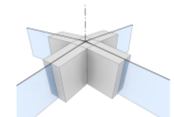
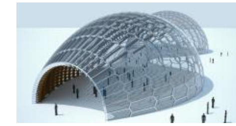



Motivation

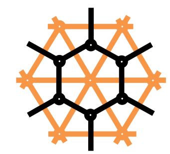
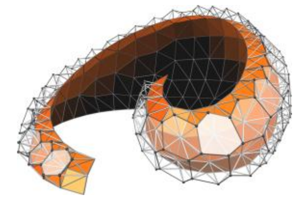
- Multi-Layer Support Structures   
- Multi-Layer Dual Structures  
- Tangent Plane Intersection (TPI)  
- *Variational* TPI  
  - Variational Shape Approximation

Motivation

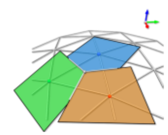
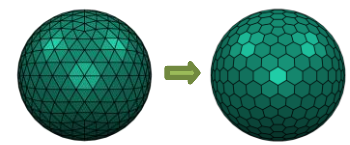
- Multi-Layer Support Structures

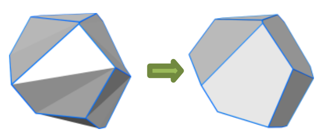
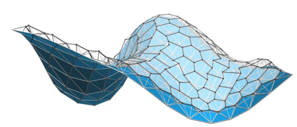
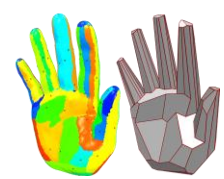
- Multi-Layer Dual Structures

- Tangent Plane Intersection (TPI)

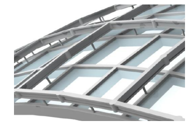
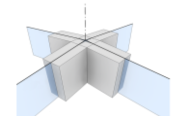
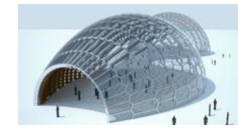
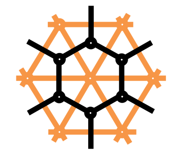
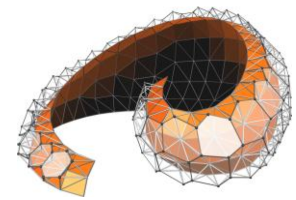
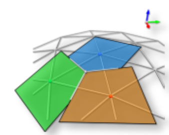
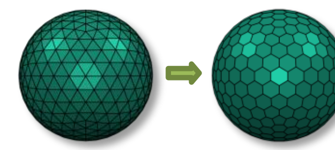
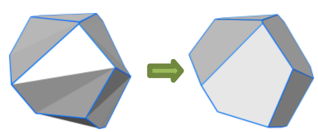
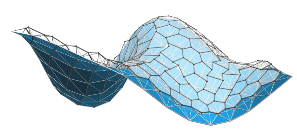
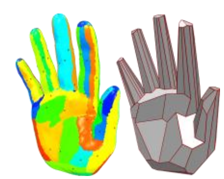



- *Variational* TPI

Planarization, Dual Support Structures, Shape Approximation

Motivation

- Multi-Layer Support Structures   
- Multi-Layer Dual Structures  
- Tangent Plane Intersection (TPI)  
- *Variational* TPI   

Planarization, Dual Support Structures, Shape Approximation

Motivation

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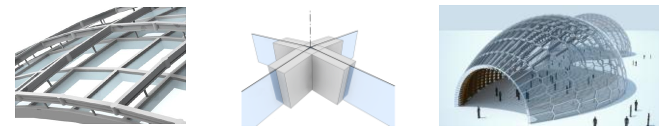

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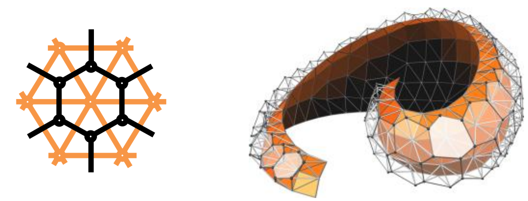

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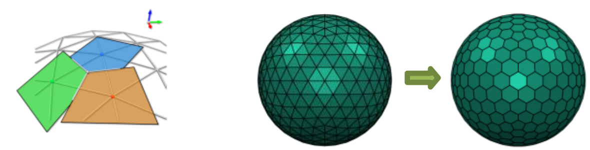

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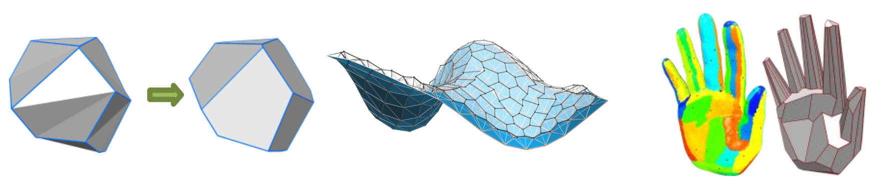
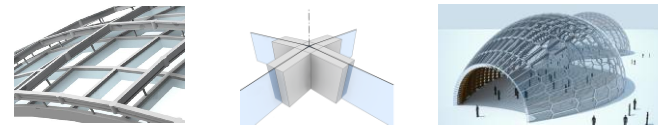


Planarization, Dual Support Structures, Shape Approximation

Motivation

- Multi-Layer Support Structures



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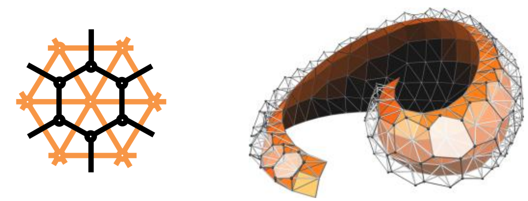

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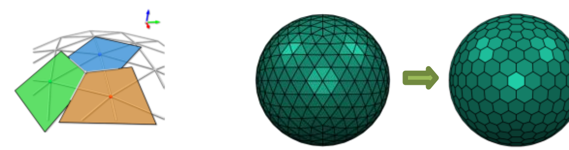

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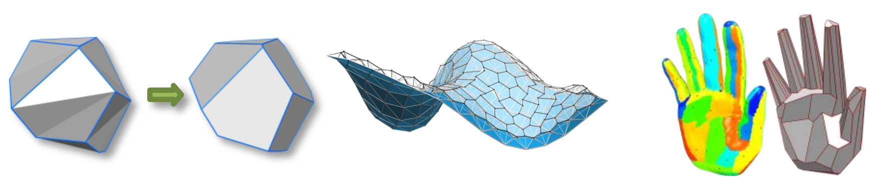
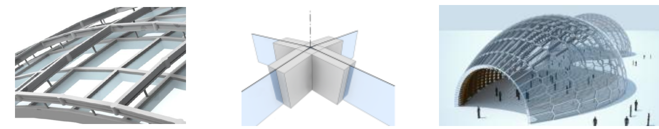


Planarization, Dual Support Structures, Shape Approximation

Motivation

- Multi-Layer Support Structures



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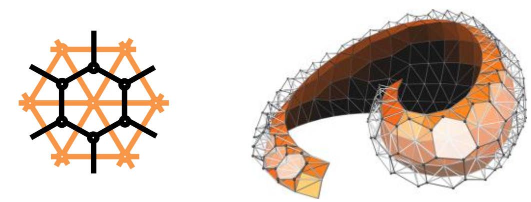

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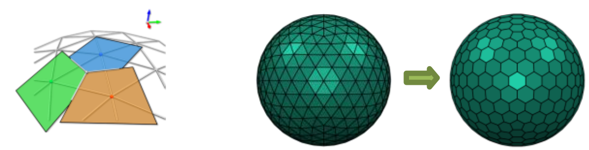
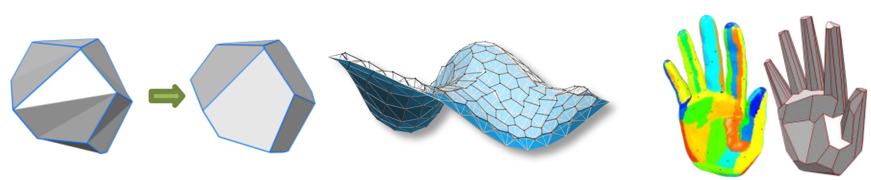


- *Variational* TPI



Planarization, Dual Support Structures, Shape Approximation

Tangent Plane Intersection

- 3 planes necessary for unique intersection point

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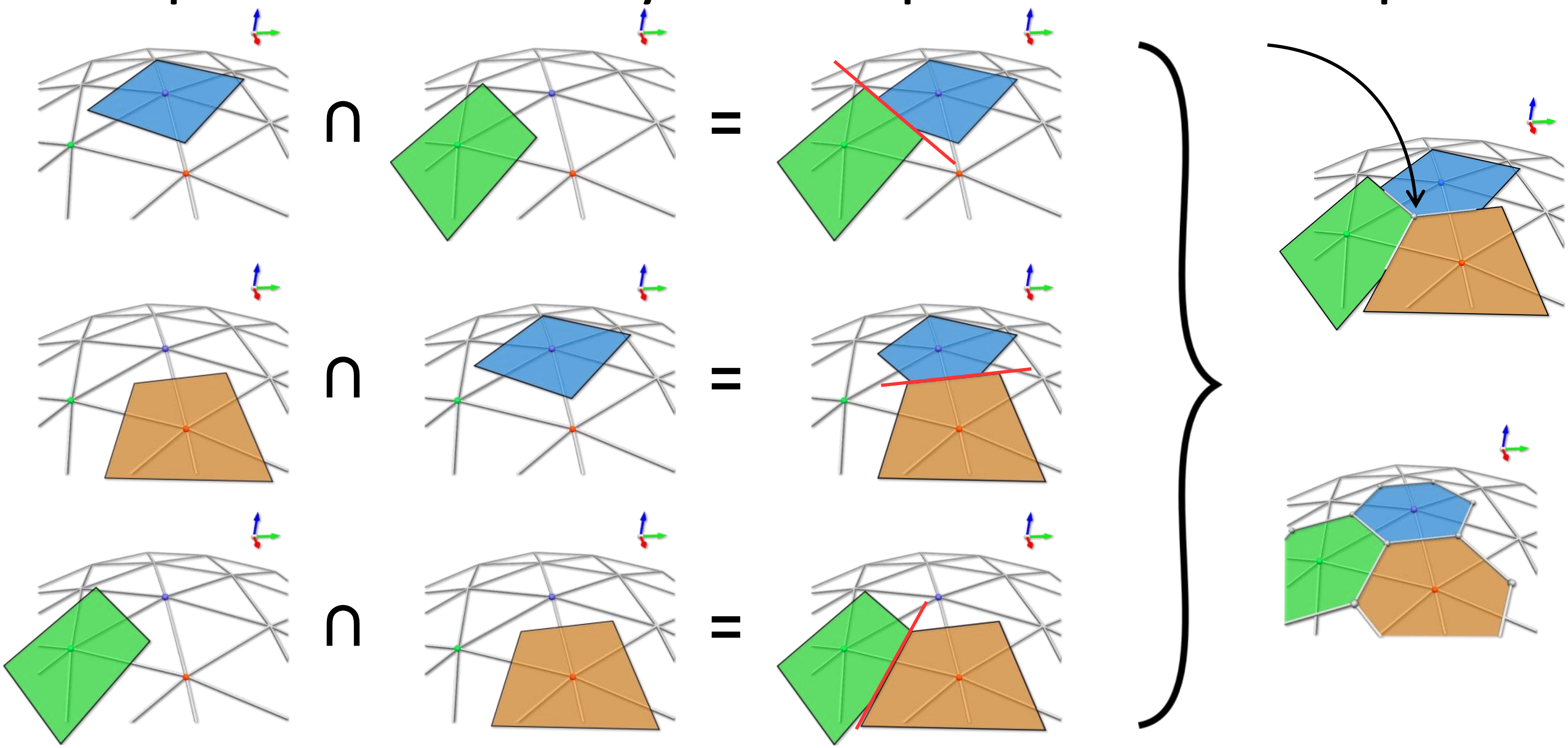

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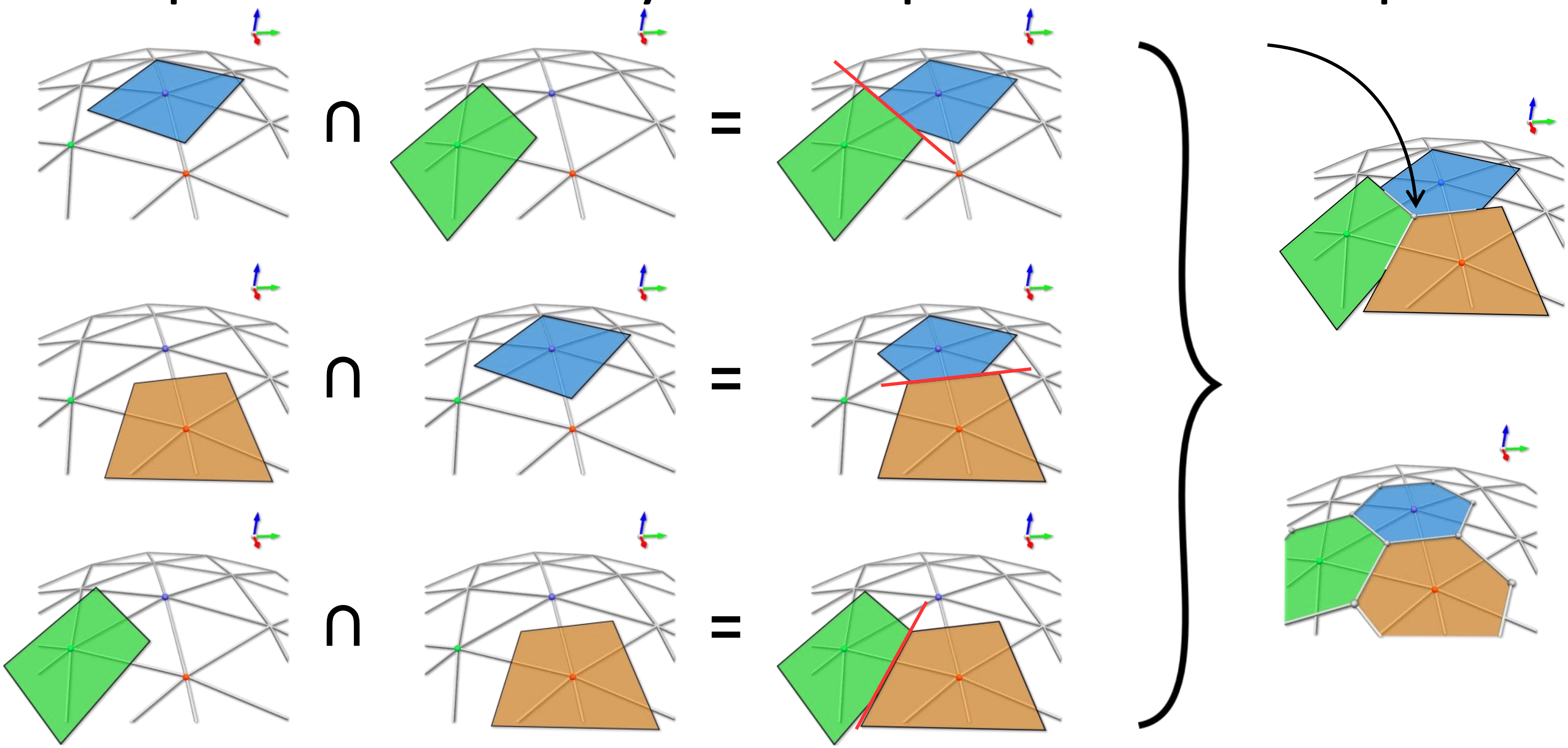

[*Planar Hexagonal Meshes by Tangent Plane Intersection*, Troche 2008]

Tangent Plane Intersection

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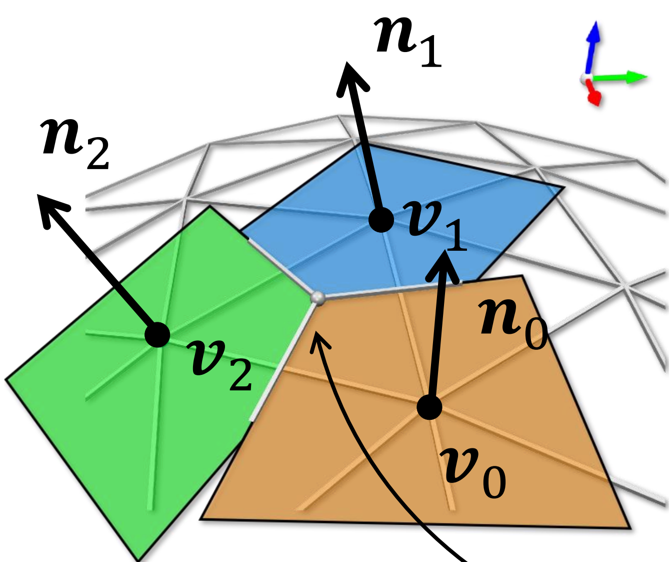


$$\begin{bmatrix} \boldsymbol{n}_0^T \\ \boldsymbol{n}_1^T \\ \boldsymbol{n}_2^T \end{bmatrix} \boldsymbol{x} = \begin{pmatrix} \boldsymbol{n}_0^T \boldsymbol{v}_0 \\ \boldsymbol{n}_1^T \boldsymbol{v}_1 \\ \boldsymbol{n}_2^T \boldsymbol{v}_2 \end{pmatrix} \Leftrightarrow N\boldsymbol{x} = \boldsymbol{b}$$

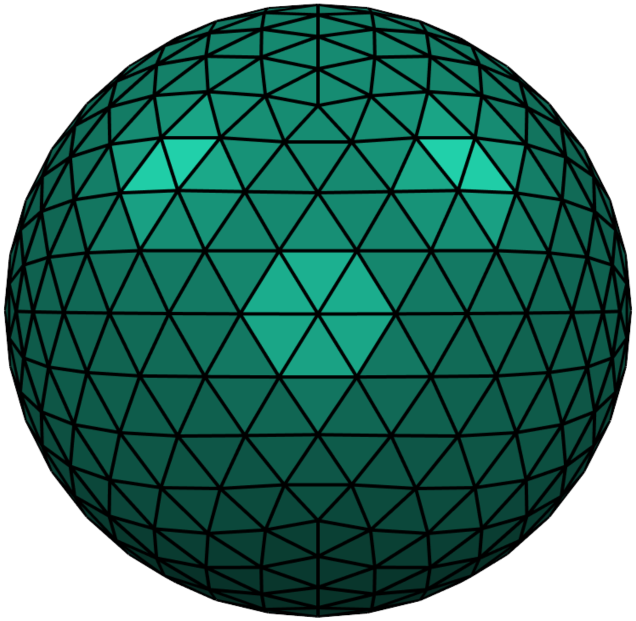
Intersection point $\boldsymbol{x}$ obtained by inversion

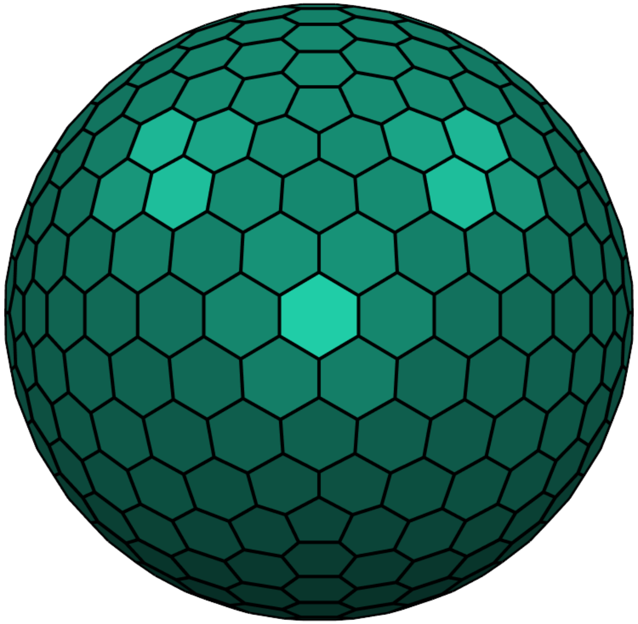
$$\boldsymbol{x} = N^{-1}\boldsymbol{b}$$

[*Planar Hexagonal Meshes by Tangent Plane Intersection*, Troche 2008]

Tangent Plane Intersection

- 3 planes necessary for unique intersection point
  - Positive Curvature: OK

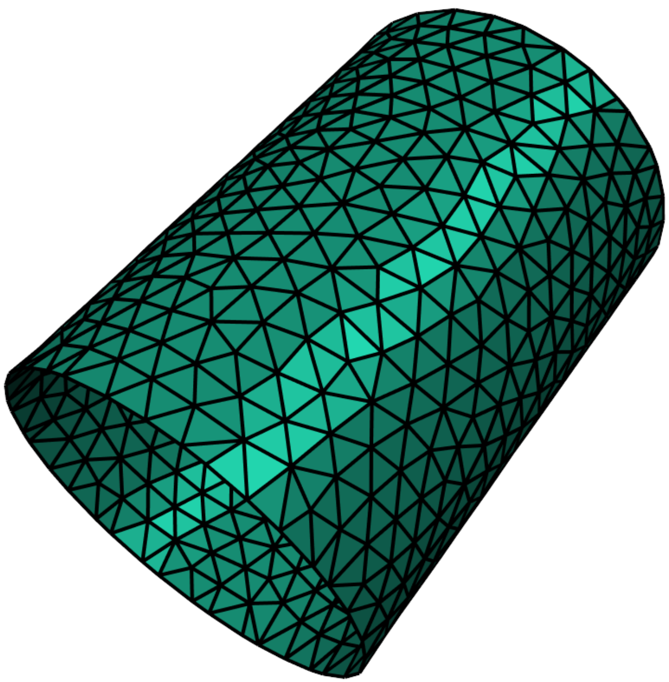


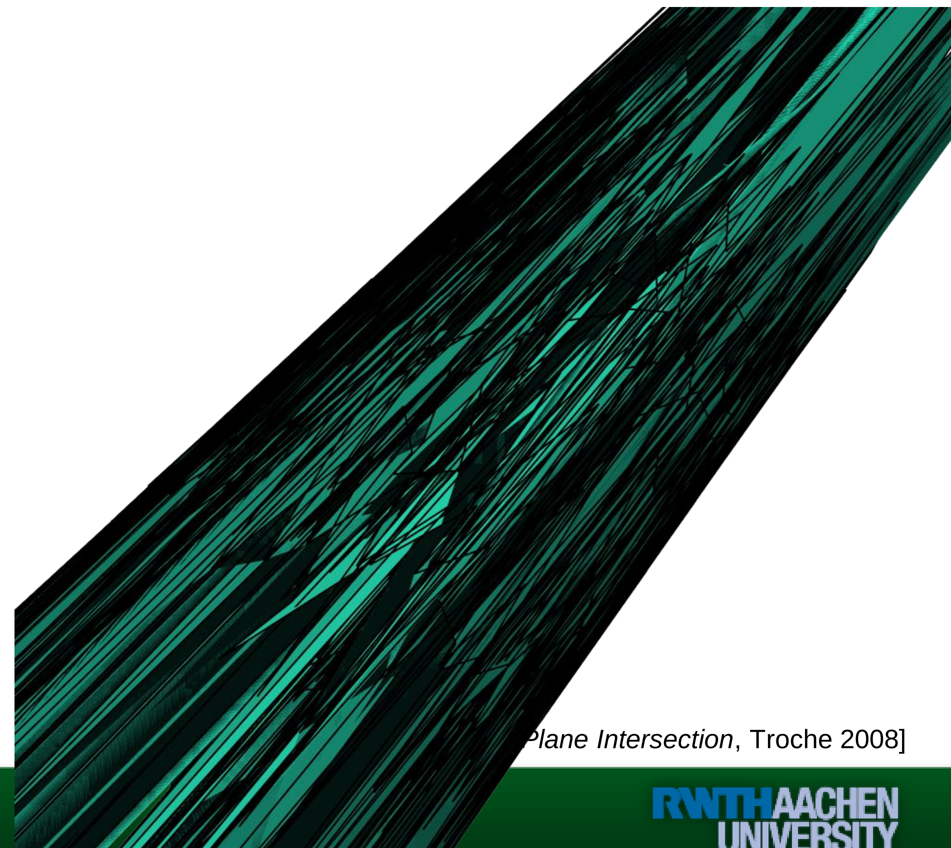


[*Planar Hexagonal Meshes by Tangent Plane Intersection*, Troche 2008]

Tangent Plane Intersection

- 3 planes necessary for unique intersection point
  - Low Gaussian Curvature: UNSTABLE





Plane Intersection, Troche 2008]

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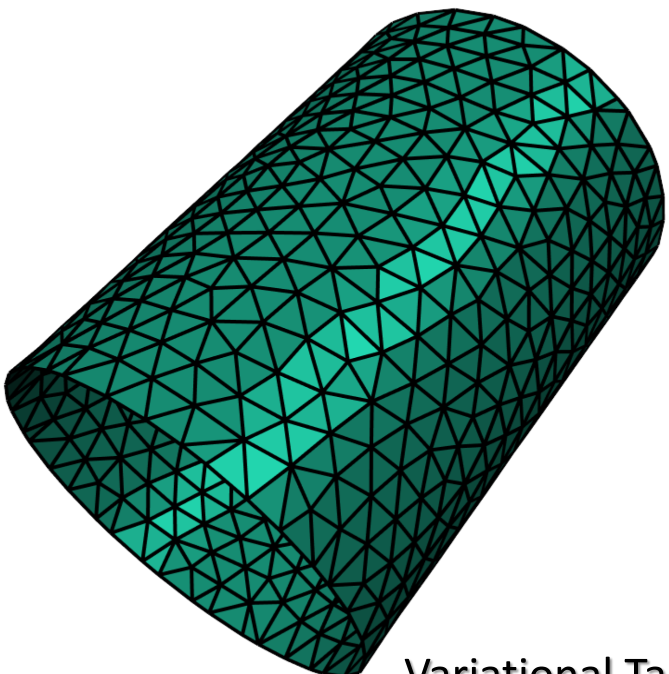

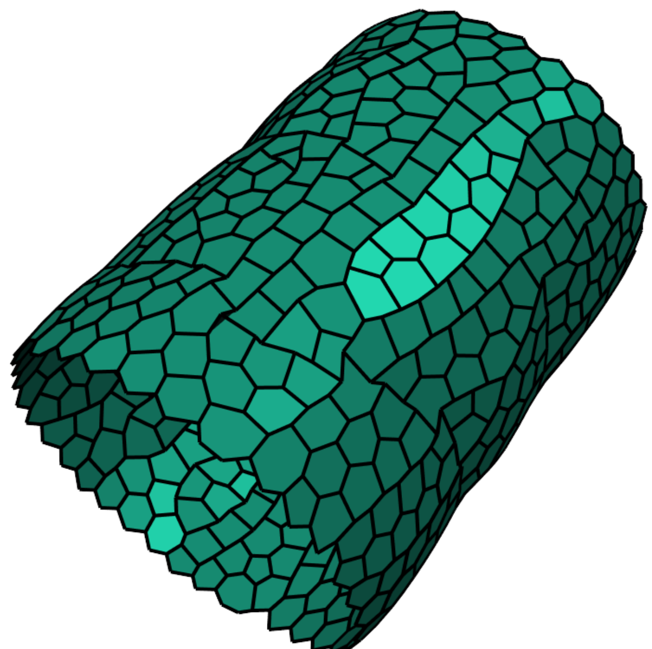

Variational Tangent Plane Intersection

[*Planar Hexagonal Meshes by Tangent Plane Intersection*, Troche 2008]

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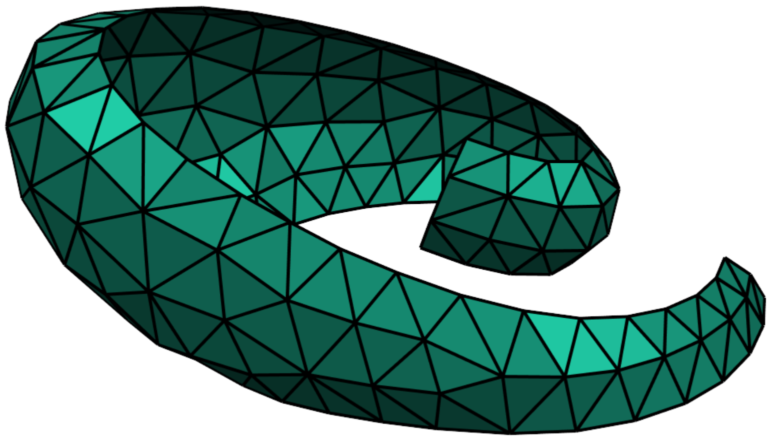
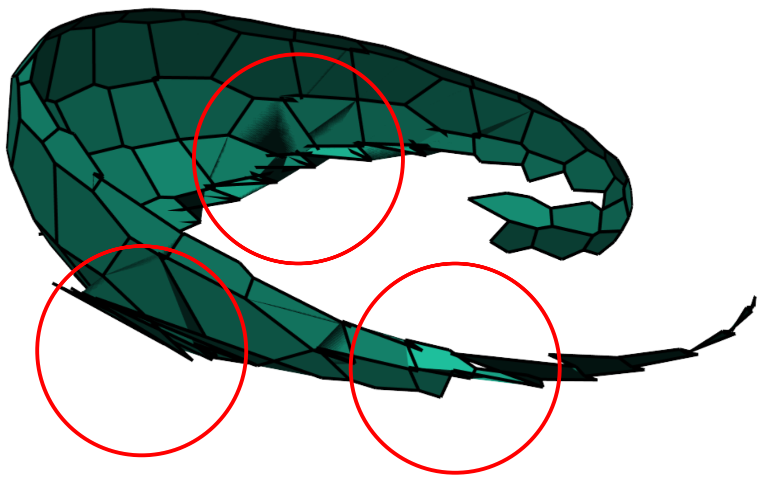
Tangent Plane Intersection

- 3 planes necessary for unique intersection point
  - Intersections are predetermined: No design DoFs





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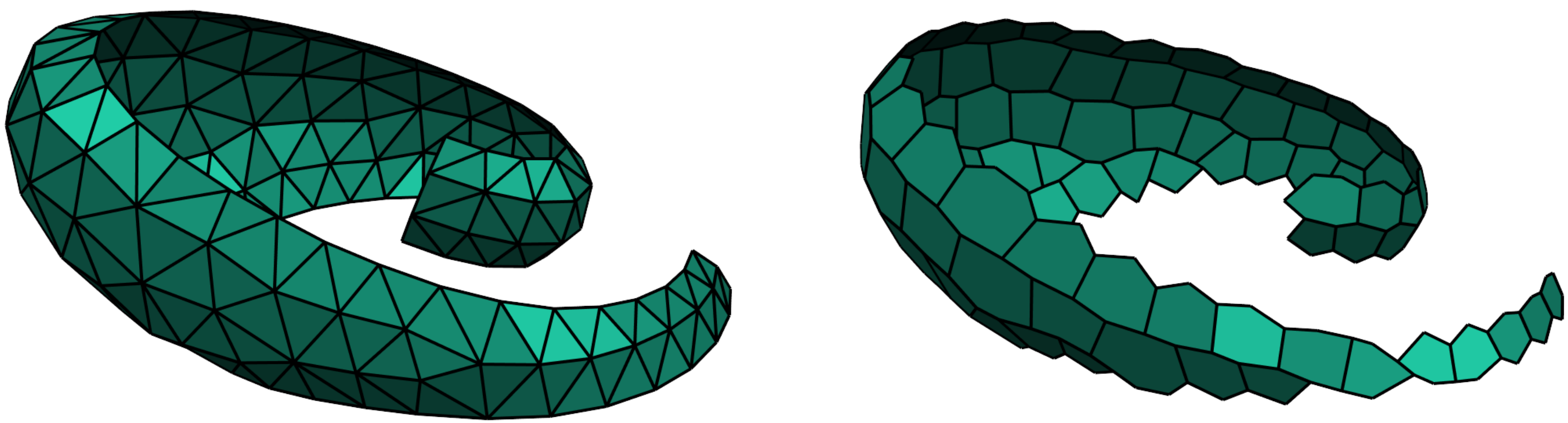
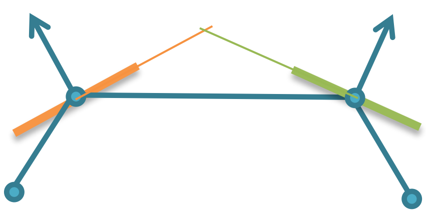


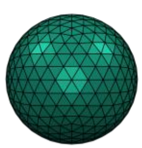
Variational Tangent Plane Intersection

[*Planar Hexagonal Meshes by Tangent Plane Intersection*, Troche 2008]

Tangent Plane Intersection

- <u>Problem 1</u>: *Instability*, co-planar tangent planes





Tangent Plane Intersection

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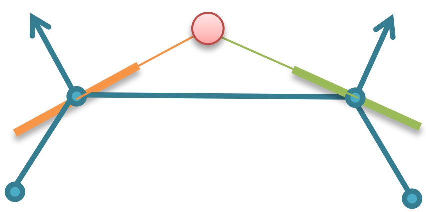

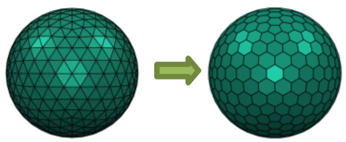

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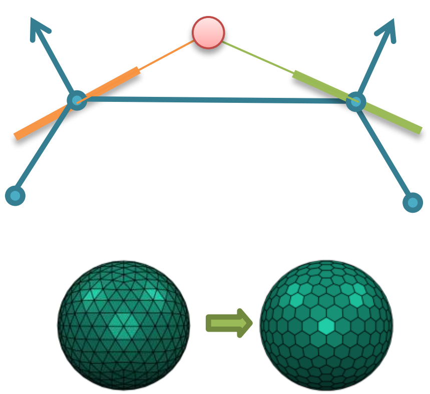
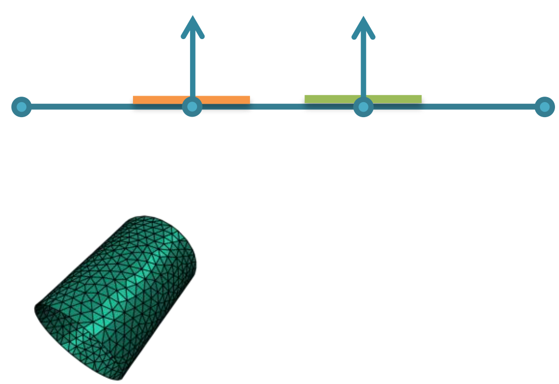
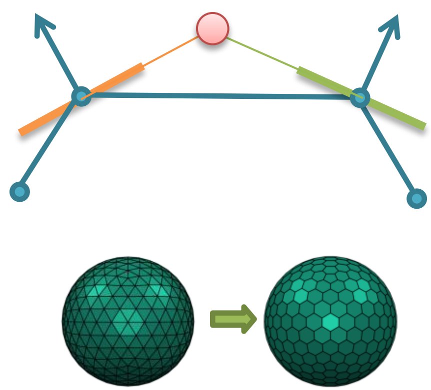
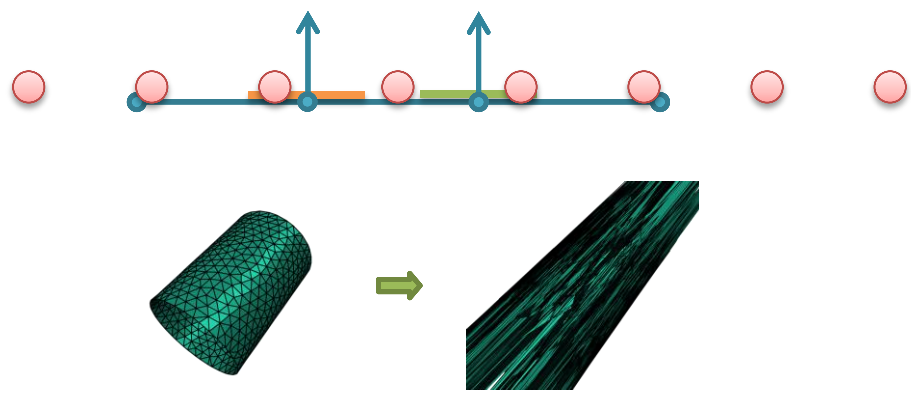




Tangent Plane Intersection

- Problem 1: *Instability*, co-planar tangent planes
  - Intersection point $\boldsymbol{x} = N^{-1}\boldsymbol{b}$ is not well-defined





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Tangent Plane Intersection

- Problem 1: *Instability*, co-planar tangent planes
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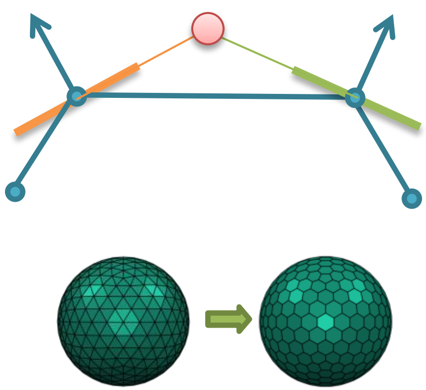

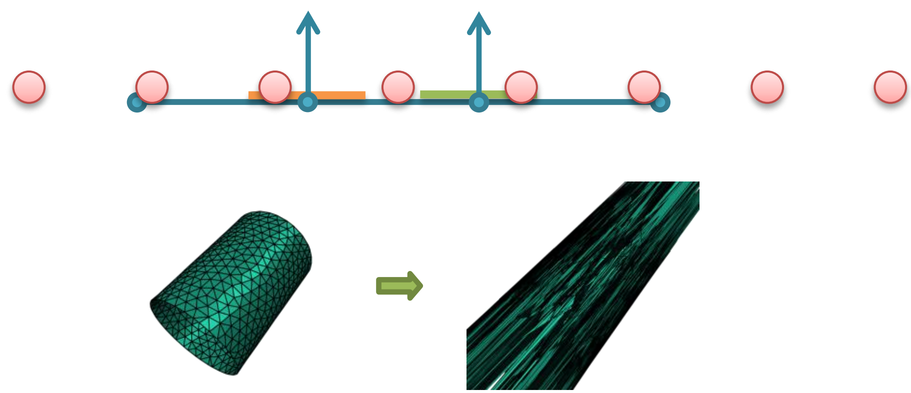

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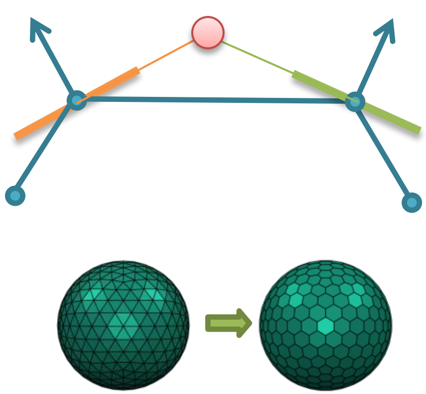
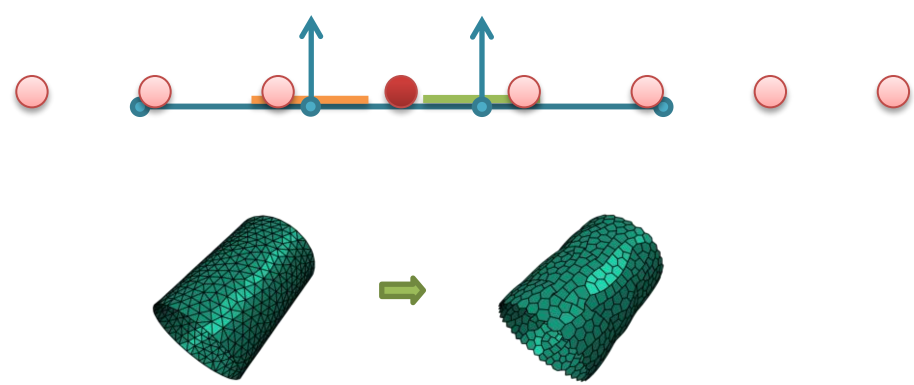
Tangent Plane Intersection

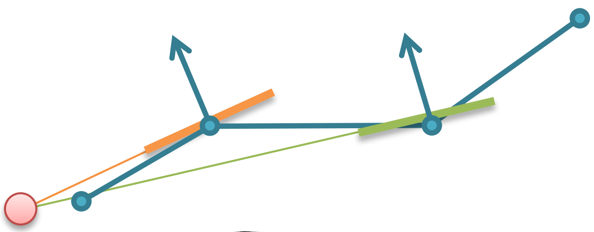
- Problem 1: *Instability*, co-planar tangent planes
  - – Intersection point $\boldsymbol{x} = N^{-1}\boldsymbol{b}$ is not well-defined
  - – However, $N\boldsymbol{x} = \boldsymbol{b}$ still holds for all points
    - → from all points we would like to choose our favorite

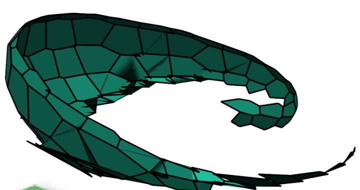




Tangent Plane Intersection

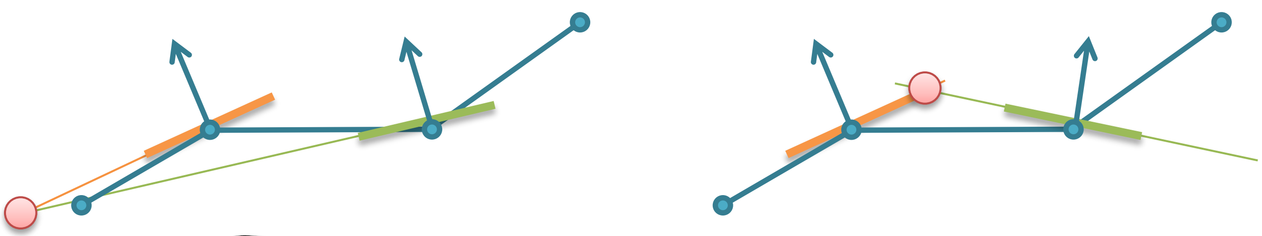
- <u>Problem 2</u>: *bad (predetermined) intersections*
  - Intersection is well-defined but position unwanted

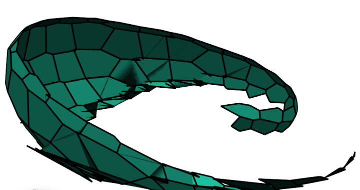




Tangent Plane Intersection

- Problem 2: *bad (predetermined) intersections*
  - Intersection is well-defined but position unwanted
  - Could obtain more degrees of freedom by:
    - rotating tangent planes





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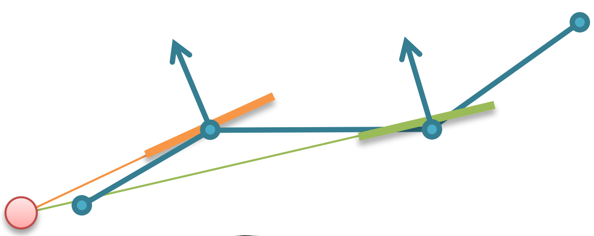
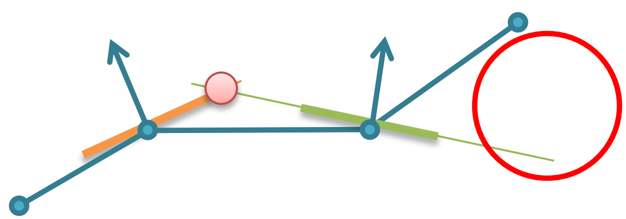
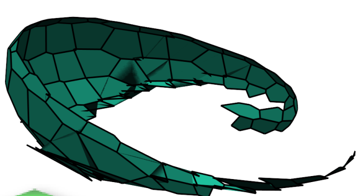
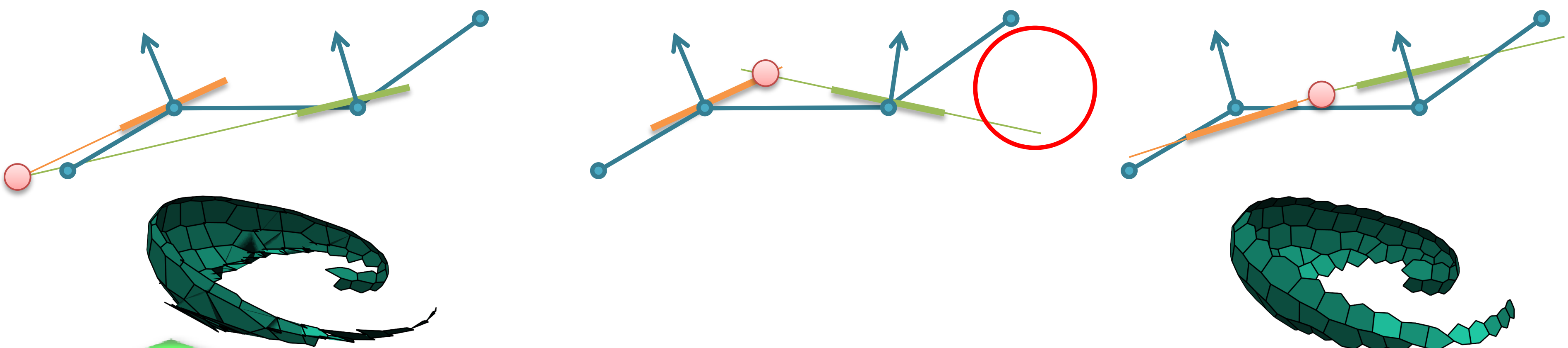






Tangent Plane Intersection

- Problem 2: *bad (predetermined) intersections*
  - Intersection is well-defined but position unwanted
  - Could obtain more degrees of freedom by:
    - rotating tangent planes
    - offsetting tangent planes



Variational Tangent Plane Intersection

Variational Tangent Plane Intersection

- Let $M = (V, F)$ be a triangle mesh

Variational Tangent Plane Intersection

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- Formulate TPI as a constrained optimization:

$$\text{minimize } E \quad \text{s.t.} \quad C_{\text{int}}: \quad N_f \boldsymbol{x}_f = \boldsymbol{b}_f \quad \forall f \in F$$

where $\boldsymbol{x}_f$ are the unknown intersection points

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  where $\boldsymbol{x}_f$ are the unknown intersection points

- In non-degenerate configurations the solution is equivalent to the explicit TPI approach

Variational Tangent Plane Intersection

E s.t. C_{int}: $N_f \boldsymbol{x}_f = \boldsymbol{b}_f$ $\quad \forall f \in F$

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Variational Tangent Plane Intersection

- Solution to Problem 1: instability
  - Specify energy with preferred intersection points $\boldsymbol{p}_f$

$$E := \sum_{f \in F} \left\| \boldsymbol{x}_f - \boldsymbol{p}_f \right\|^2 \quad \text{s.t.} \quad C_{\text{int}}: \; N_f \boldsymbol{x}_f = \boldsymbol{b}_f \qquad \forall f \in F$$

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Variational Tangent Plane Intersection

- Solution to Problem 1: instability
  - Specify energy with preferred intersection points $\boldsymbol{p}_f$
- Solution to Problem 2: "bad" intersection points
  - Introduce variable offsets h_v

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Variational Tangent Plane Intersection

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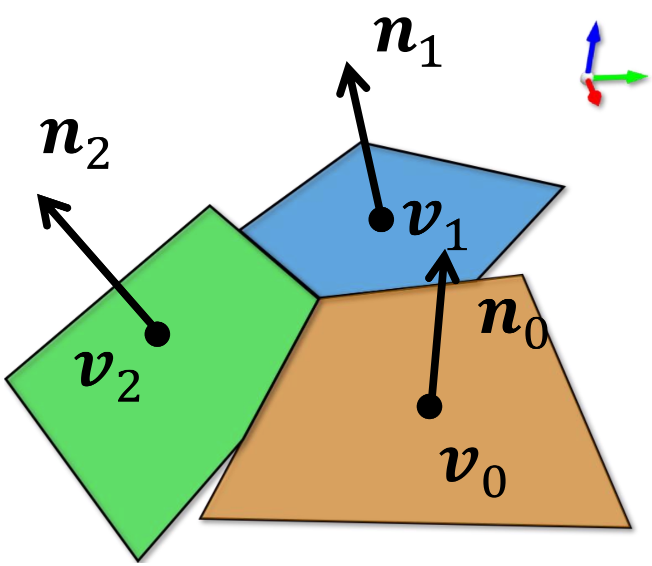
- So far VTPI is defined for triangle meshes …

Multiple Tangent Plane Intersection

Multiple Tangent Plane Intersection

- 3 planes *necessary* for unique intersection point



$$\begin{bmatrix} \boldsymbol{n}_0^T \\ \boldsymbol{n}_1^T \\ \boldsymbol{n}_2^T \end{bmatrix} \boldsymbol{x} = \begin{pmatrix} \boldsymbol{n}_0^T \boldsymbol{v}_0 \\ \boldsymbol{n}_1^T \boldsymbol{v}_1 \\ \boldsymbol{n}_2^T \boldsymbol{v}_2 \end{pmatrix} \Leftrightarrow N\boldsymbol{x} = \boldsymbol{b}$$

Multiple Tangent Plane Intersection

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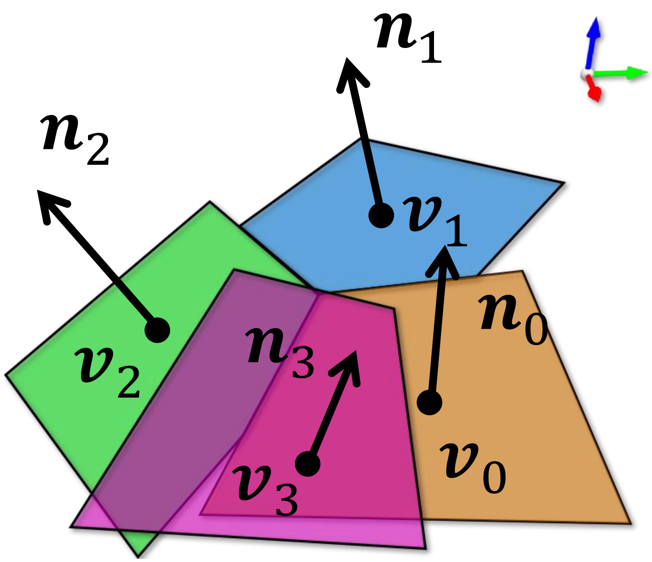

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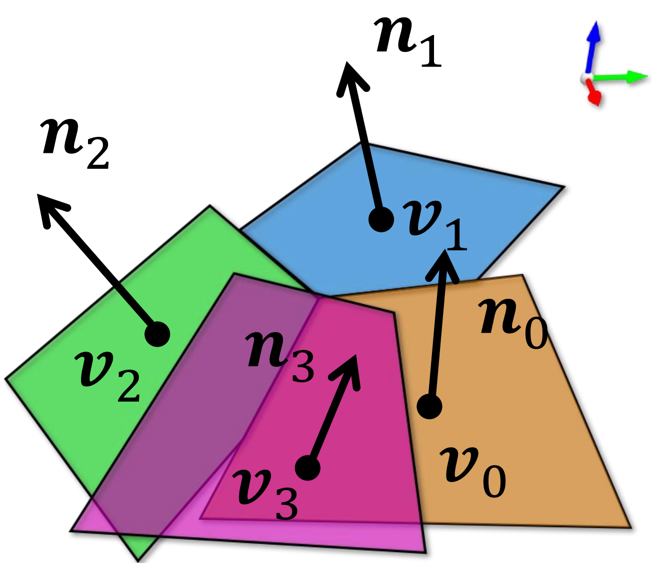
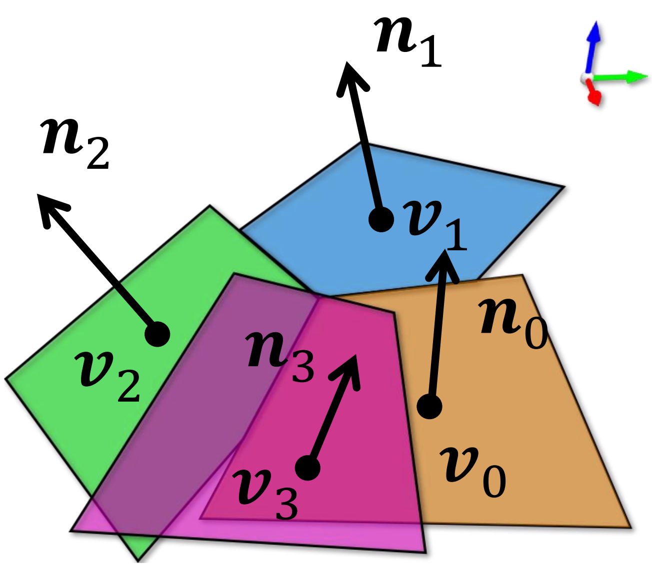


$$\begin{bmatrix} \boldsymbol{n}_0^T \\ \boldsymbol{n}_1^T \\ \boldsymbol{n}_2^T \\ \boldsymbol{n}_3^T \end{bmatrix} \boldsymbol{x} = \begin{pmatrix} \boldsymbol{n}_0^T \boldsymbol{v}_0 \\ \boldsymbol{n}_1^T \boldsymbol{v}_1 \\ \boldsymbol{n}_2^T \boldsymbol{v}_2 \\ \boldsymbol{n}_3^T \boldsymbol{v}_3 \end{pmatrix} \Leftrightarrow N\boldsymbol{x} = \boldsymbol{b}$$

- No longer limited to triangle meshes

Multiple Tangent Plane Intersection

- 3 planes *necessary* for unique intersection point



$$\begin{bmatrix} \boldsymbol{n}_0^T \\ \boldsymbol{n}_1^T \\ \boldsymbol{n}_2^T \\ \boldsymbol{n}_3^T \\ \vdots \end{bmatrix} \boldsymbol{x} = \begin{pmatrix} \boldsymbol{n}_0^T \boldsymbol{v}_0 \\ \boldsymbol{n}_1^T \boldsymbol{v}_1 \\ \boldsymbol{n}_2^T \boldsymbol{v}_2 \\ \boldsymbol{n}_3^T \boldsymbol{v}_3 \\ \vdots \end{pmatrix} \Leftrightarrow N\boldsymbol{x} = \boldsymbol{b}$$

- Works on general polygon meshes

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VTPI for Polygon Mesh Planarization

VTPI for Polygon Mesh Planarization

- $M \mapsto$ planar $\mathrm{dual}(M)$

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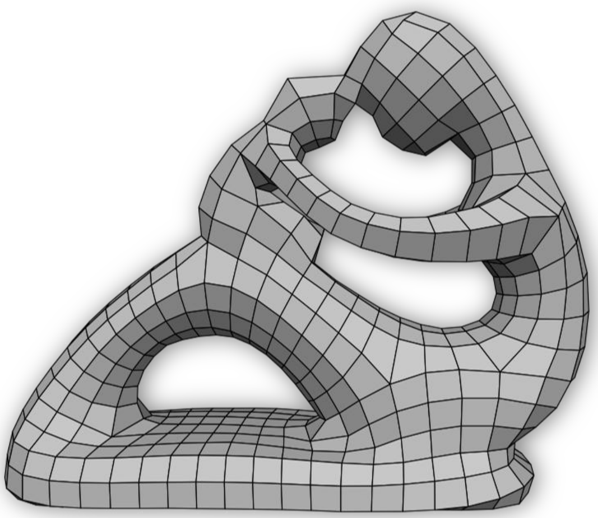

FERTILITY

VTPI for Polygon Mesh Planarization

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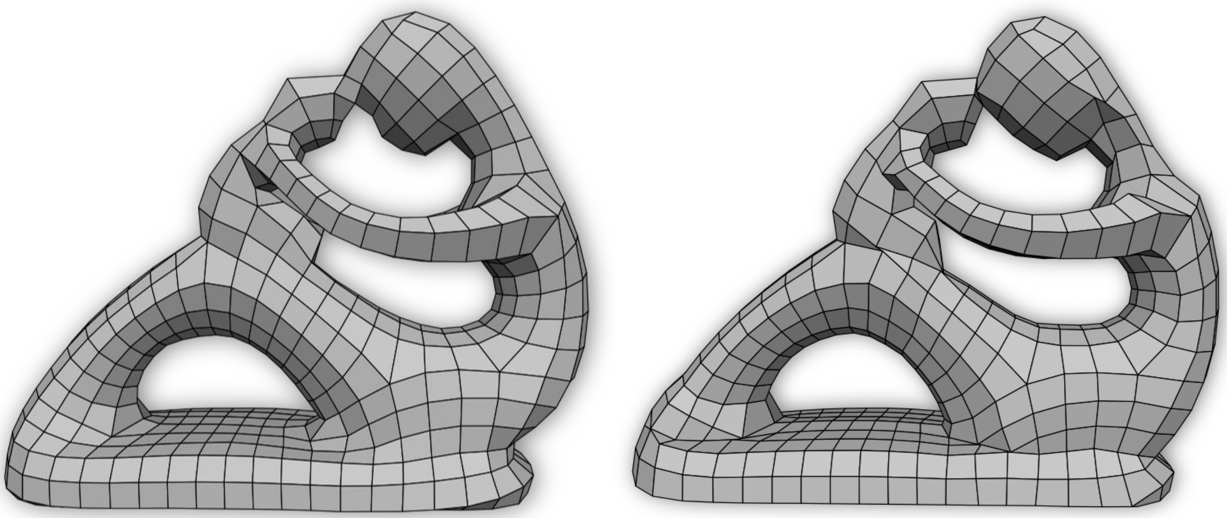

FERTILITY

[Variational Tangent Plane Intersection]

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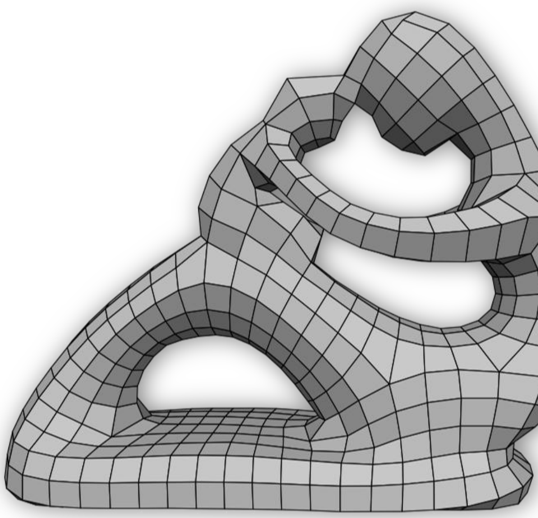
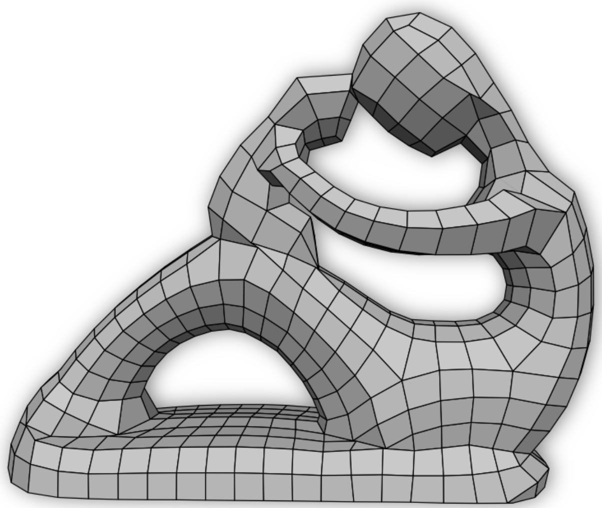
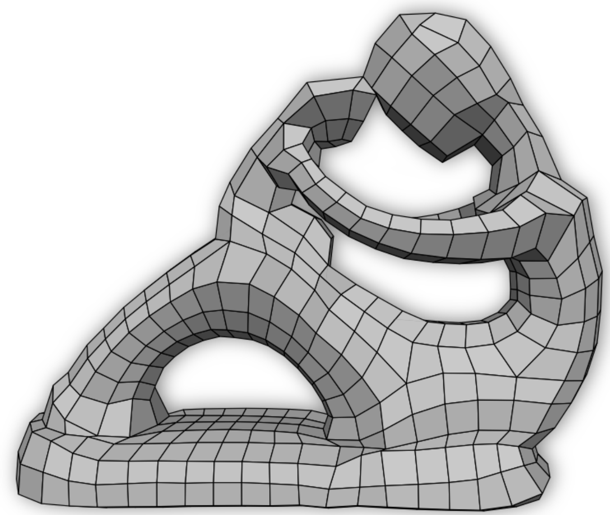


FERTILITY



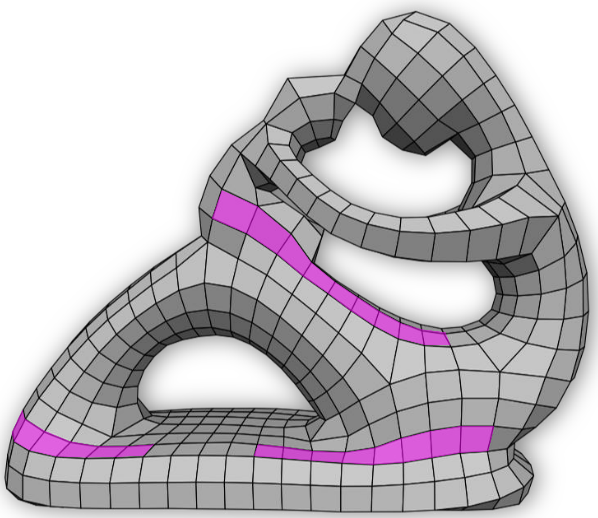
[Variational Tangent Plane Intersection]



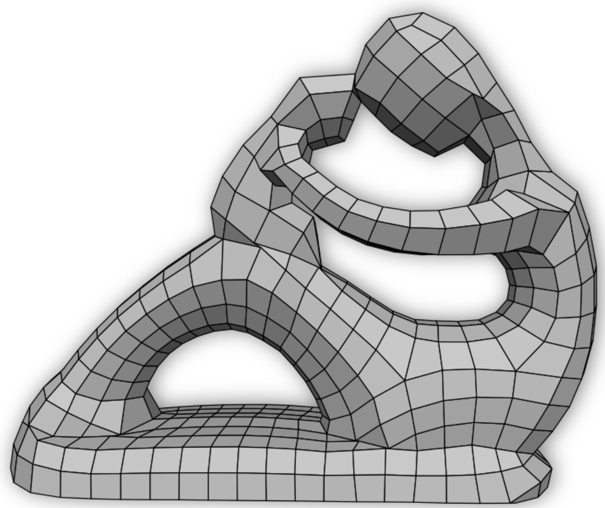
[*Discrete Laplacians on general polygonal meshes.* Alexa, Wardetzky. *2011]*

VTPI for Polygon Mesh Planarization

- $M \mapsto$ planar $\mathrm{dual}(M)$
- $\mathrm{dual}(M) \mapsto$ planar M
- Consider quad strips undergoing twists



FERTILITY



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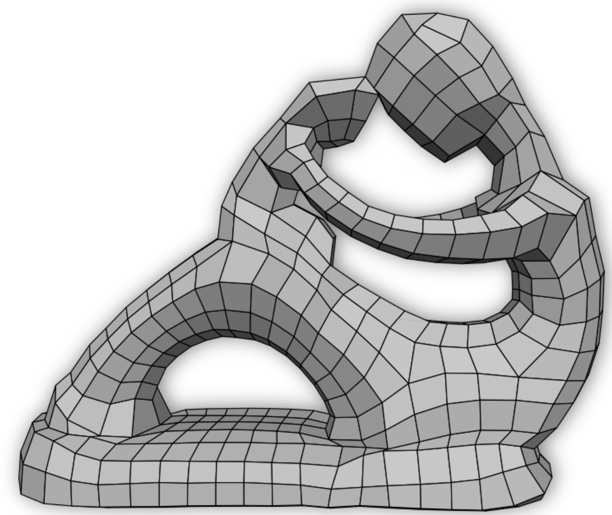

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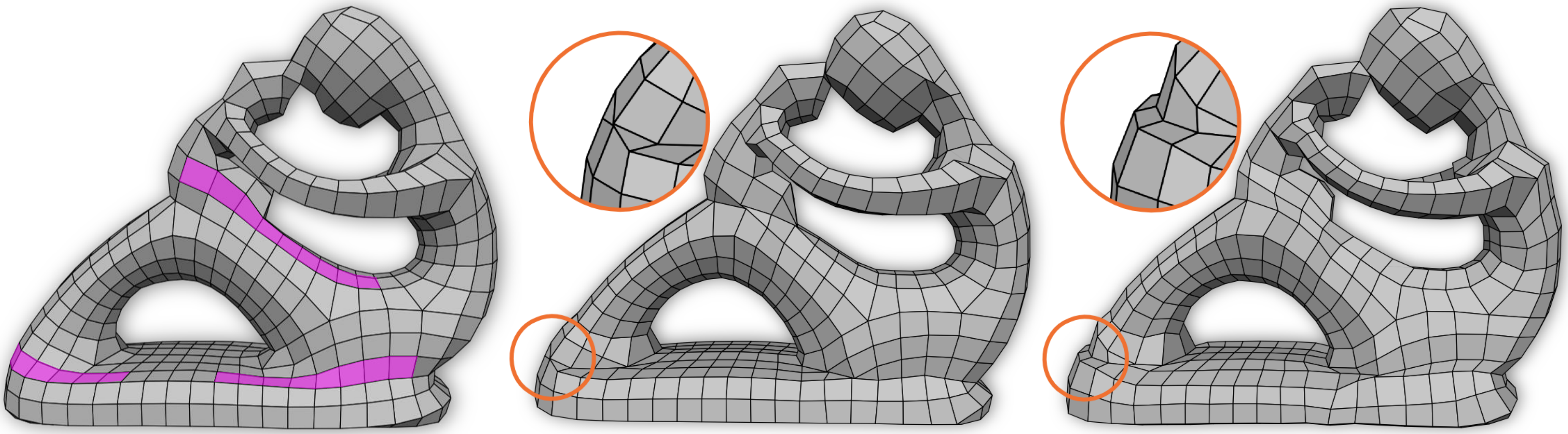

FERTILITY

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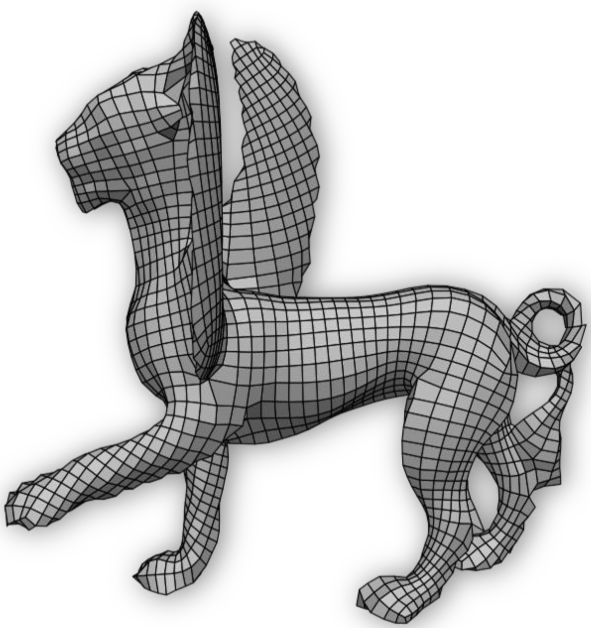
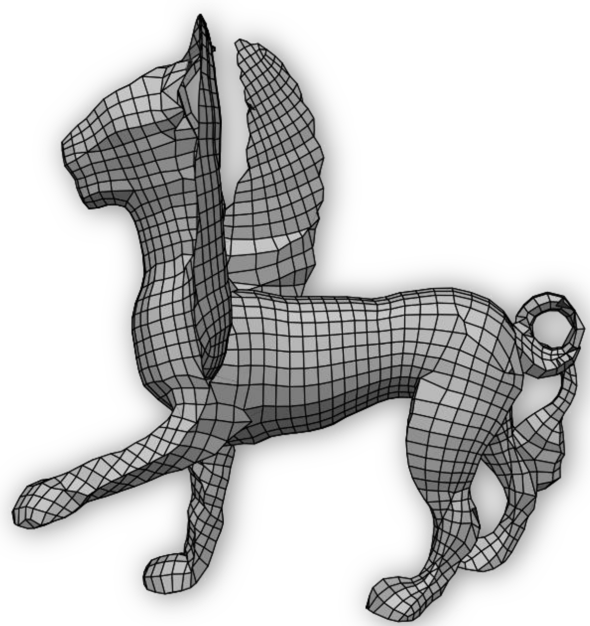


FELINE



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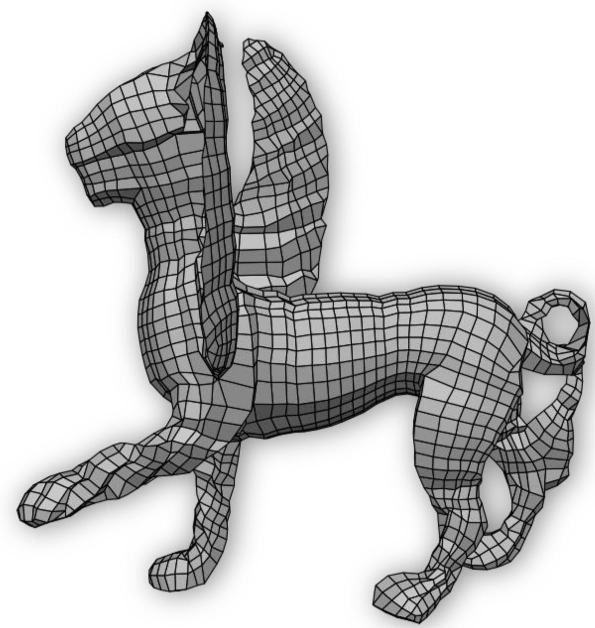

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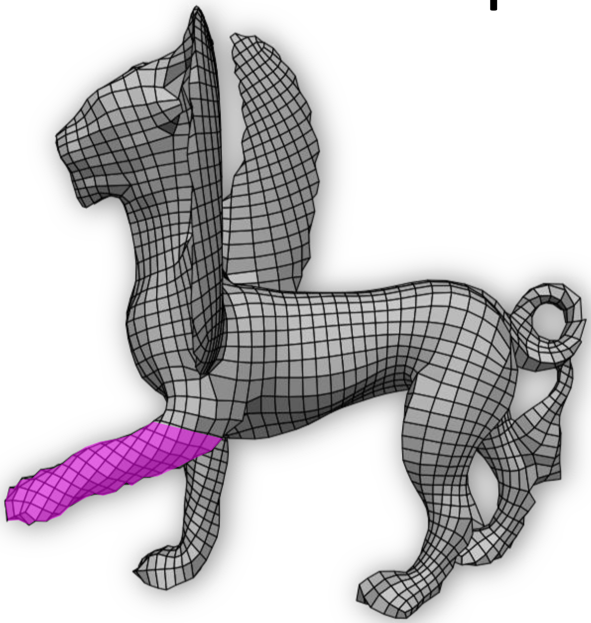
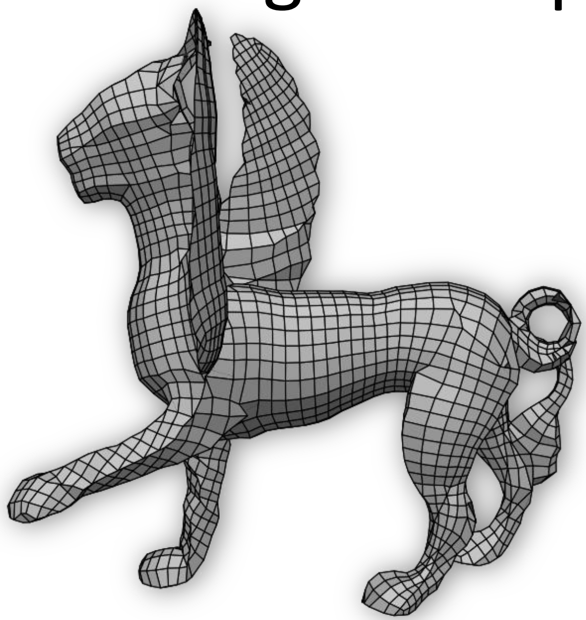


FELINE



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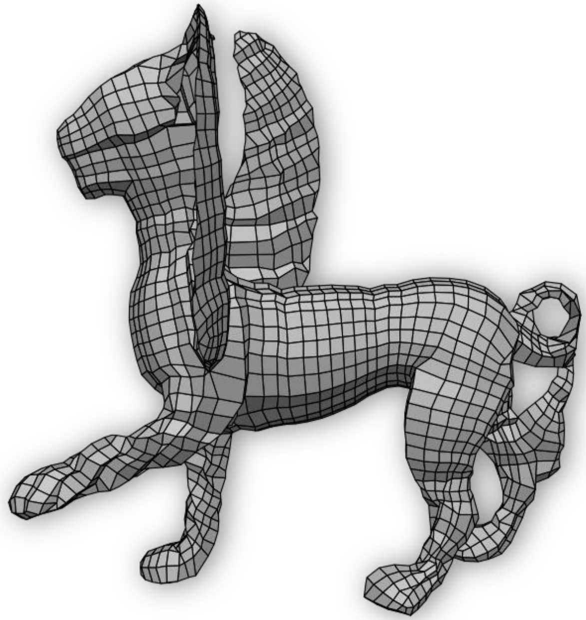

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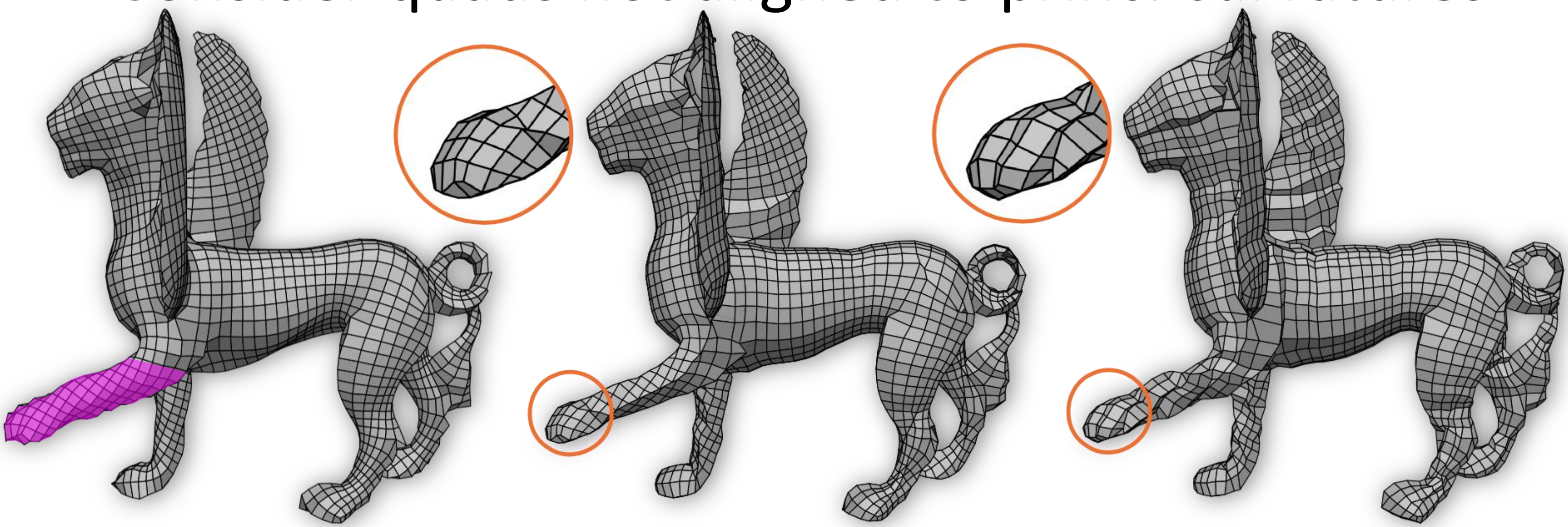

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VTPI for Polygon Mesh Planarization

- Comparison to other planarization techniques
  - Optimization (perturbation)-based methods
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    [*Geometric Modeling with Conical Meshes and Developable Surfaces.* Liu, Pottmann, Wallner, Yang, Wang. *2006]*
  - Planarizing flow
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		Method		
		VTPI	Opt.	Flow
Property	Parameters	-	-	+
	Extensions	+	+	-
	Normals	+	-	-

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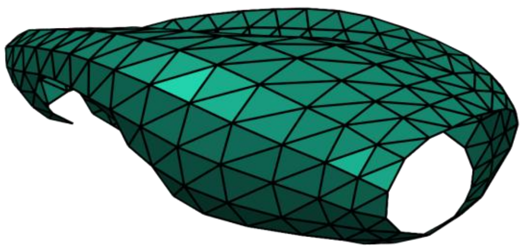
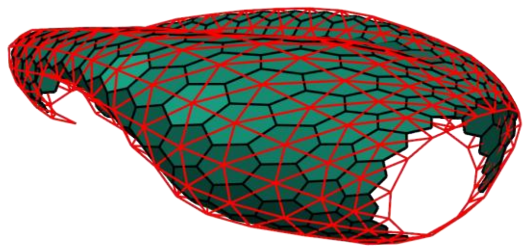
		Method		
		VTPI	Opt.	Flow
Property	Parameters	-	-	+
	Extensions	+	+	-
	Normals	+	-	-

- use normals → intersection-free dual structures

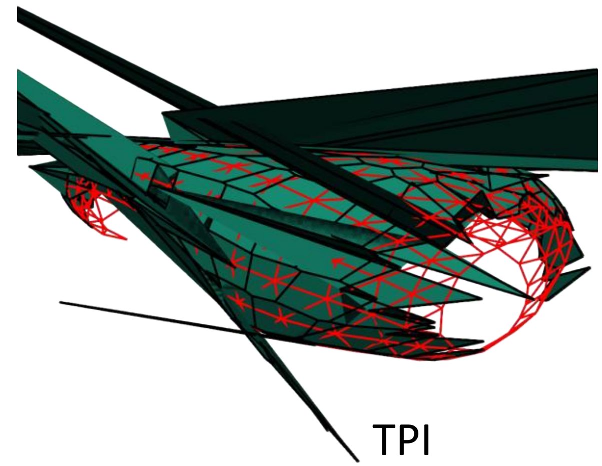
VTPI for Multi-Layer Dual Structures



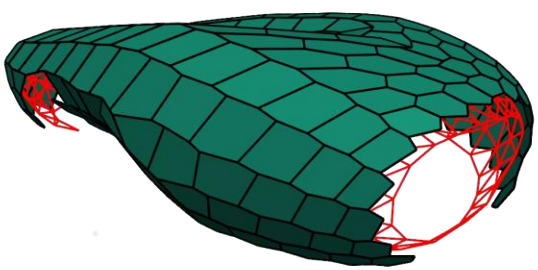
Input



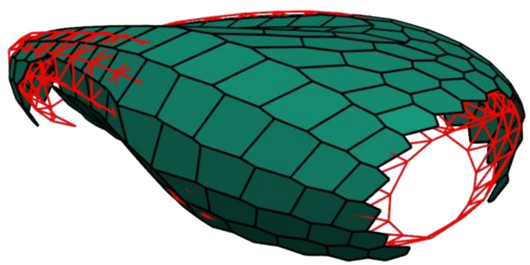
Dual



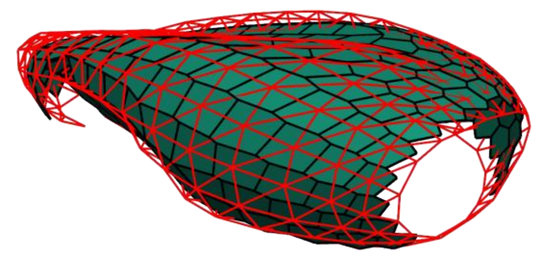
TPI



VTPI
+ preferred offsets
+ intersection hard-constraints



VTPI
+ preferred offsets

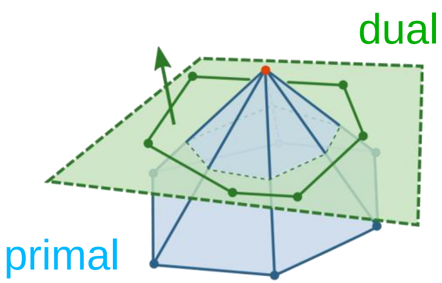


VTPI

Avoiding (local) Intersections

Problem	Solution (constraints)

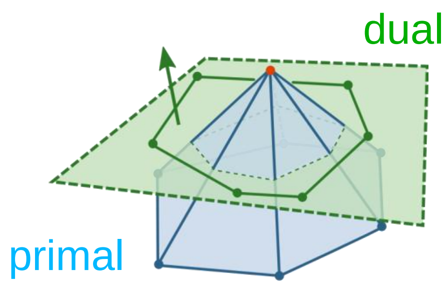
Avoiding (local) Intersections

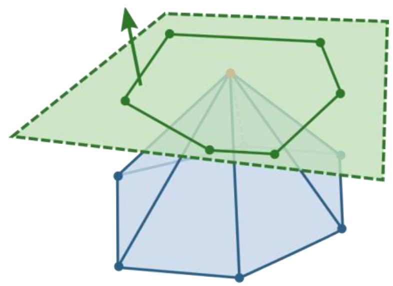
Problem	Solution (constraints)
Vertex Intersection 	

Avoiding (local) Intersections

Problem	Solution (constraints)

Vertex Intersection



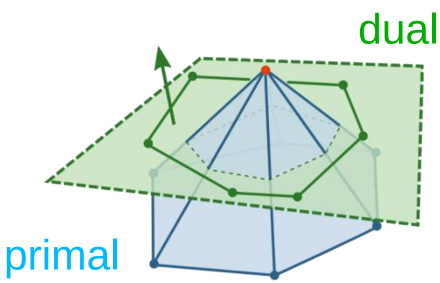


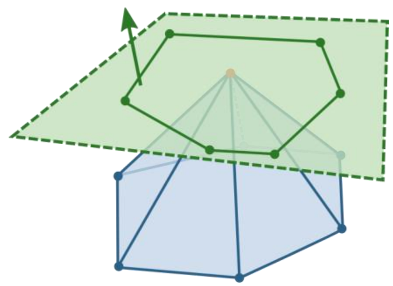
$$h > 0$$

Avoiding (local) Intersections

Problem	Solution (constraints)

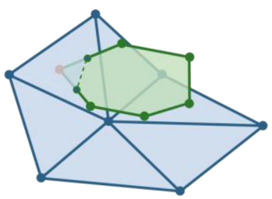
Vertex Intersection



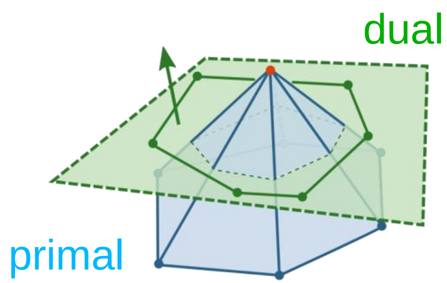
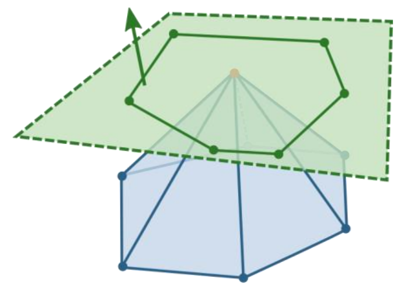
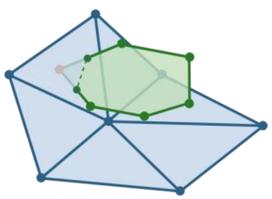
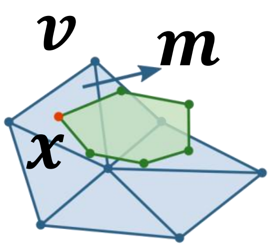


$$h > 0$$

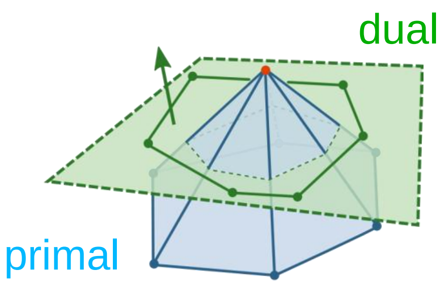
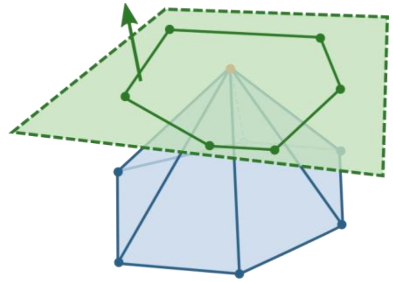
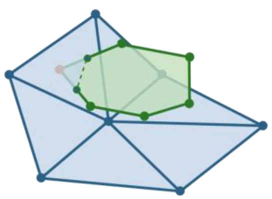
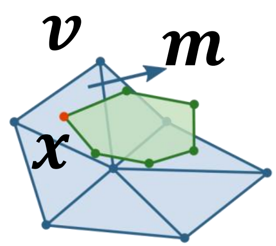
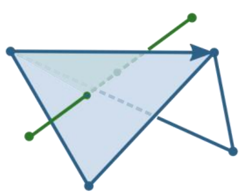
Face Intersection



Avoiding (local) Intersections

Problem		Solution (constraints)	
Vertex Intersection			$h > 0$
Face Intersection			$\boldsymbol{m}^T(\boldsymbol{x}-\boldsymbol{v}) > 0$

Avoiding (local) Intersections

Problem	Solution (constraints)	
Vertex Intersection 		$h > 0$
Face Intersection 		$\boldsymbol{m}^T(\boldsymbol{x}-\boldsymbol{v}) > 0$
Edge Intersection 		

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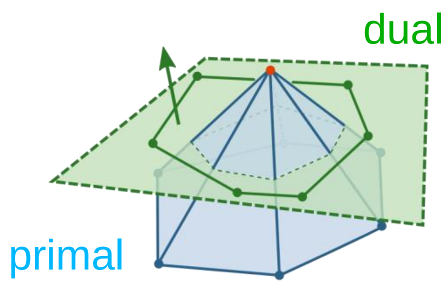
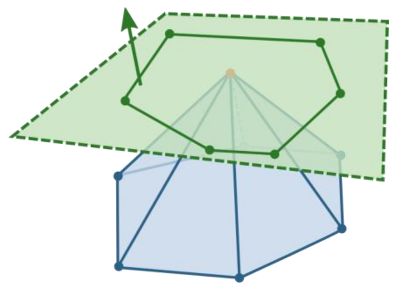
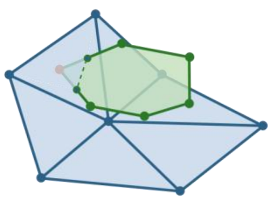
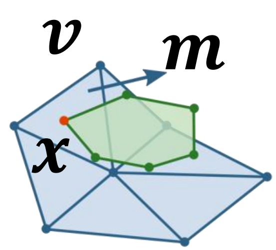
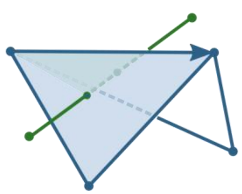
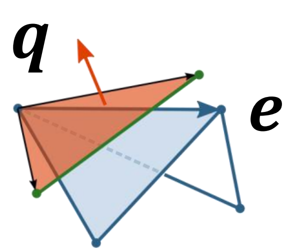
Vertex Intersection			$h > 0$
Face Intersection			$\boldsymbol{m}^T(\boldsymbol{x} - \boldsymbol{v}) > 0$
Edge Intersection			$\boldsymbol{q}^T \boldsymbol{e} < 0$

Results of Multi-Layer Dual Structures

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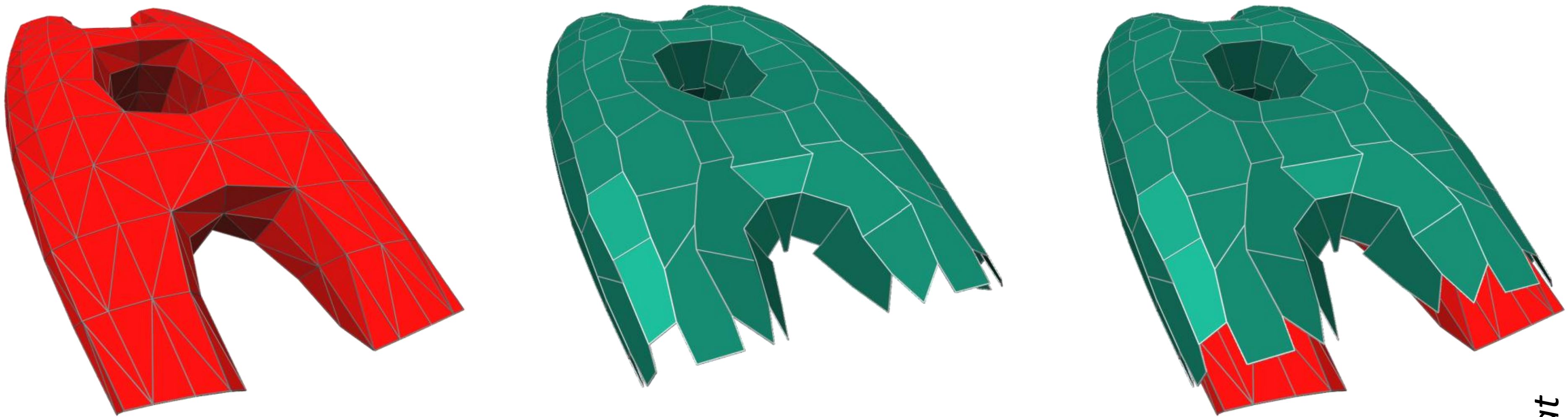
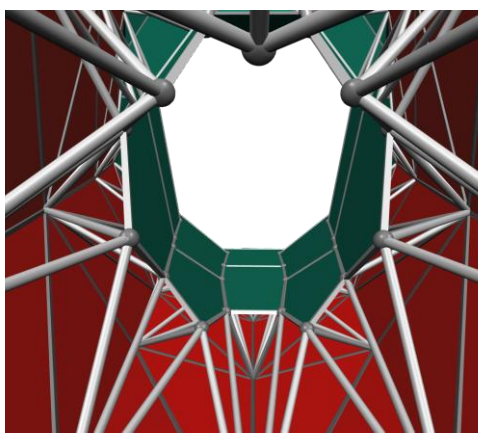
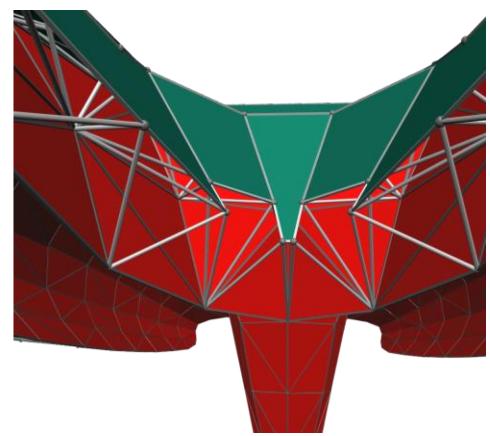


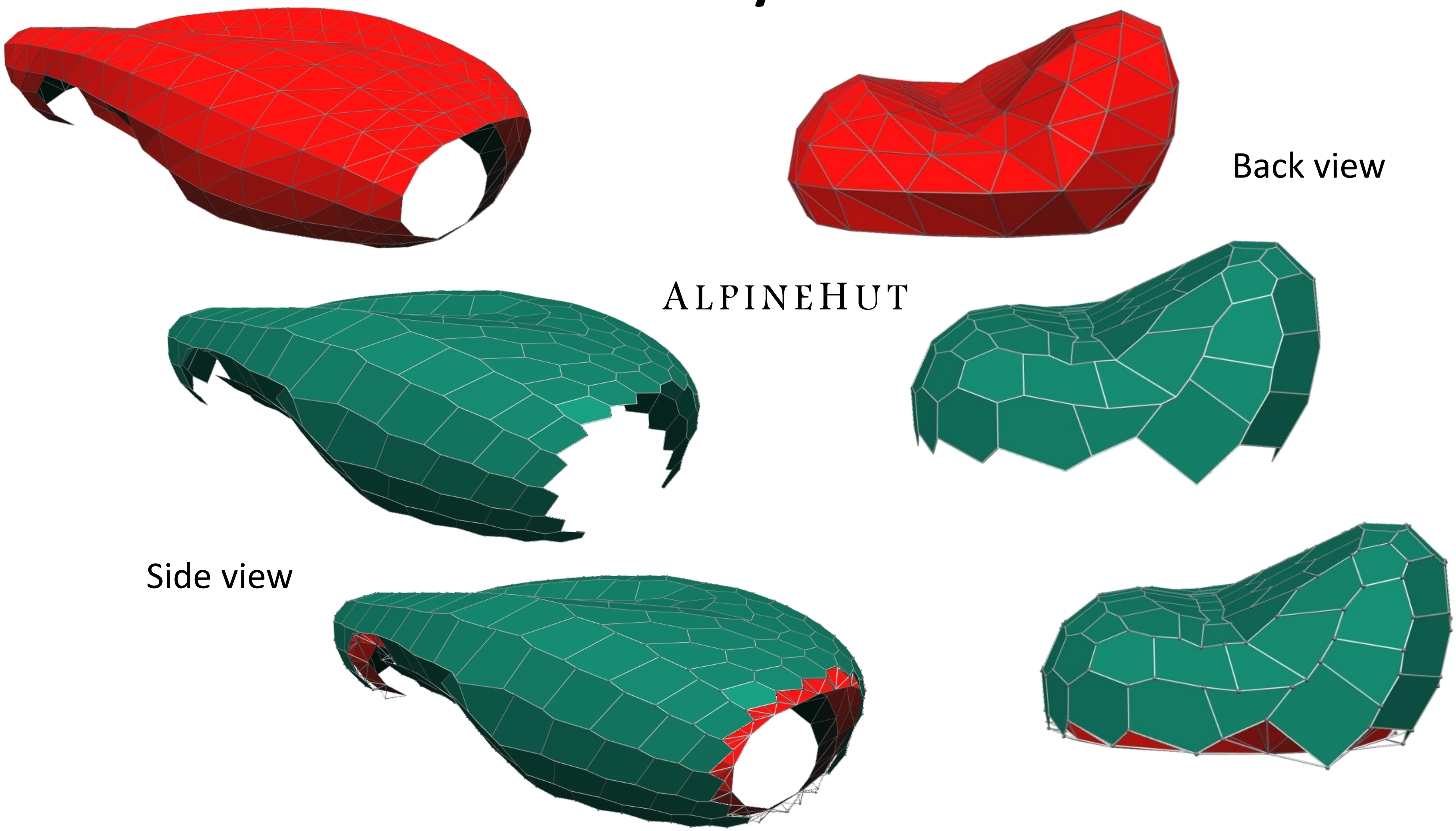
TRAINSTATION





Courtesy of *www.evolute.at*

Results of Multi-Layer Dual Structures



Conclusion

- What is VTPI?

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Conclusion

- What is VTPI?
  - A variational formulation of tangent plane intersection
    - Guided intersections of several planes
    - Useful for geometric problems (e.g. mesh planarization)
    - Solved by global optimization (freely available solvers)

Conclusion

- What is VTPI?
  - A variational formulation of tangent plane intersection
    - Guided intersections of several planes
    - Useful for geometric problems (e.g. mesh planarization)
    - Solved by global optimization (freely available solvers)
- What is VTPI not?
  - A "fix" to topological issues involved in planar meshing
    - Degeneracies will occur where necessary, e.g. concave (or degenerate) hexagons in hyperbolic surface regions
    - Energies can sometimes be used to shift such effects ...

Limitations & Discussion

- Output depends on input tessellation and energy

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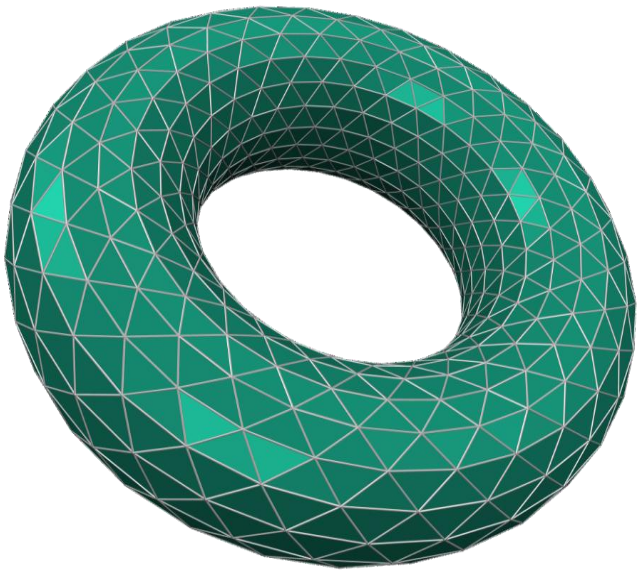
  - Different tessellations, same topology



Limitations & Discussion

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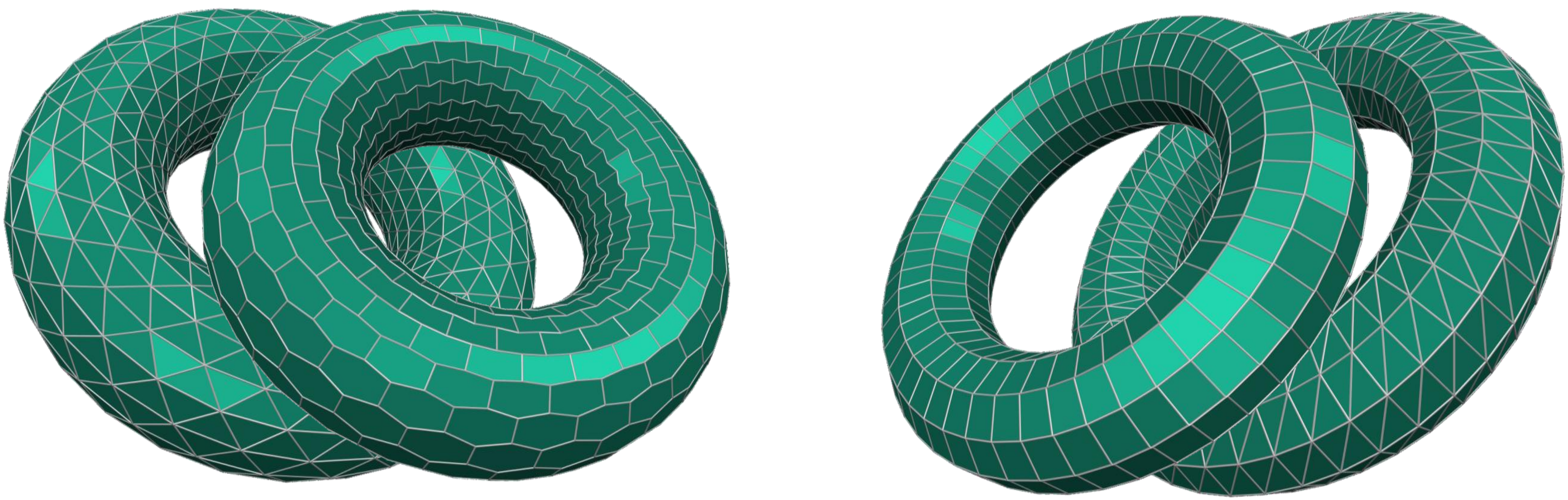

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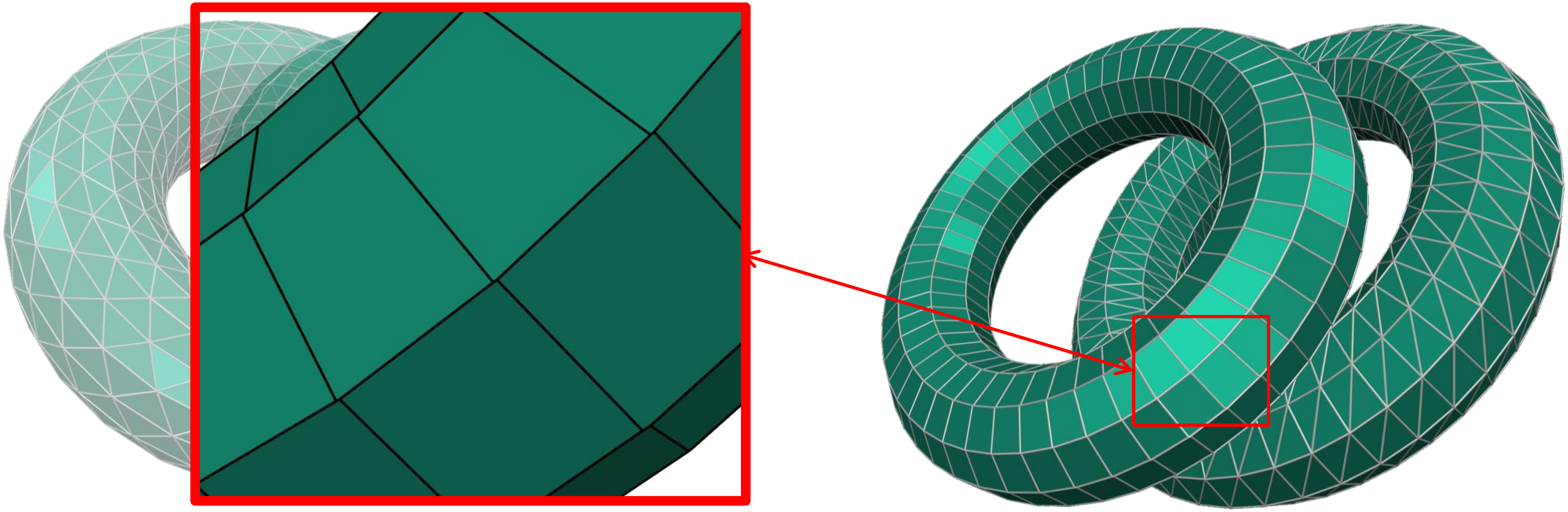

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Limitations & Discussion

- Output depends on input tessellation and energy
  - energies can partly shift some effects on the mesh

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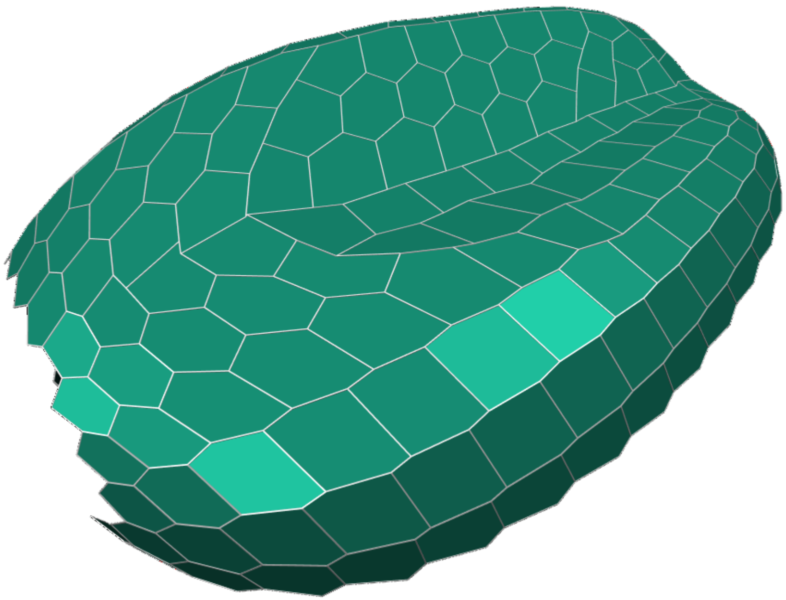
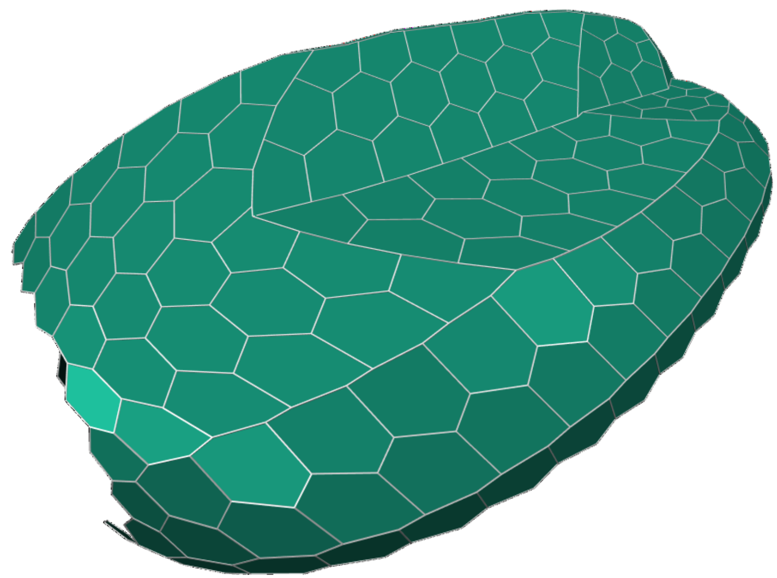
Limitations & Discussion

- Output depends on input tessellation and energy
  - Same tessellation, same topology, different functional



Normal Smoothness



Element Fairing

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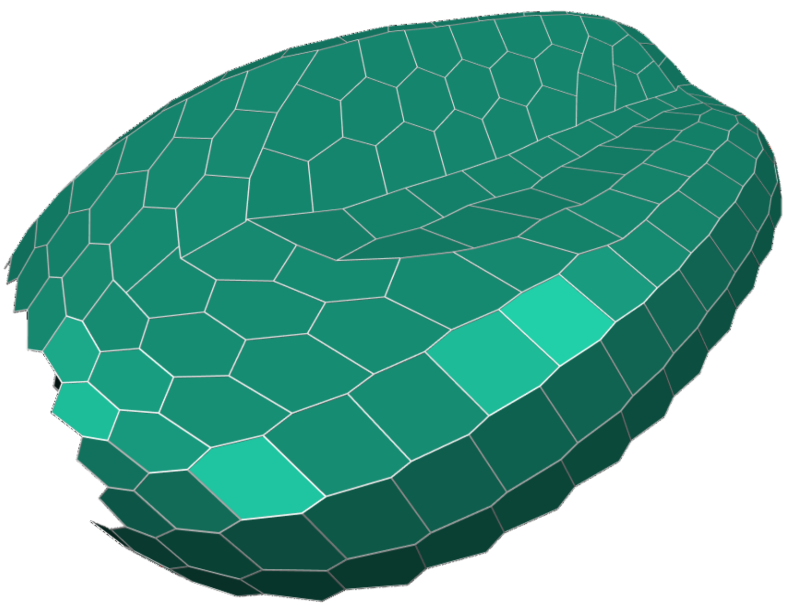
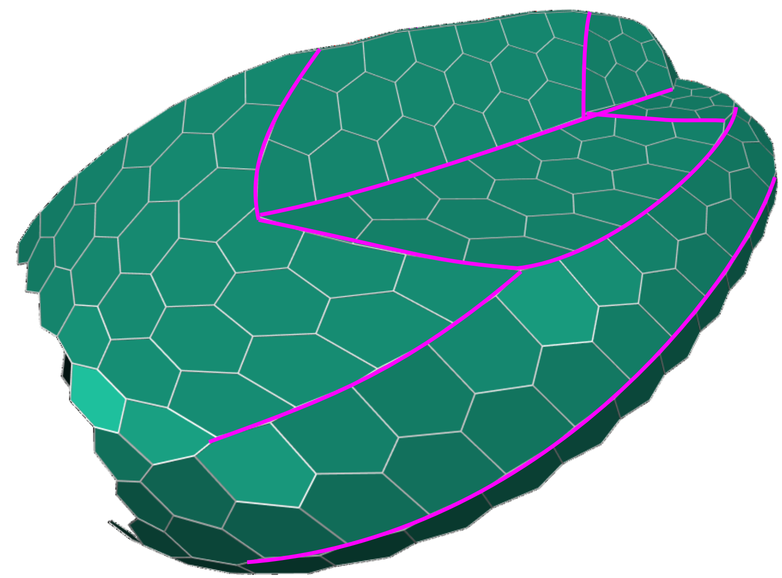


Normal Smoothness



Element Fairing

The End

Thank you for your attention!