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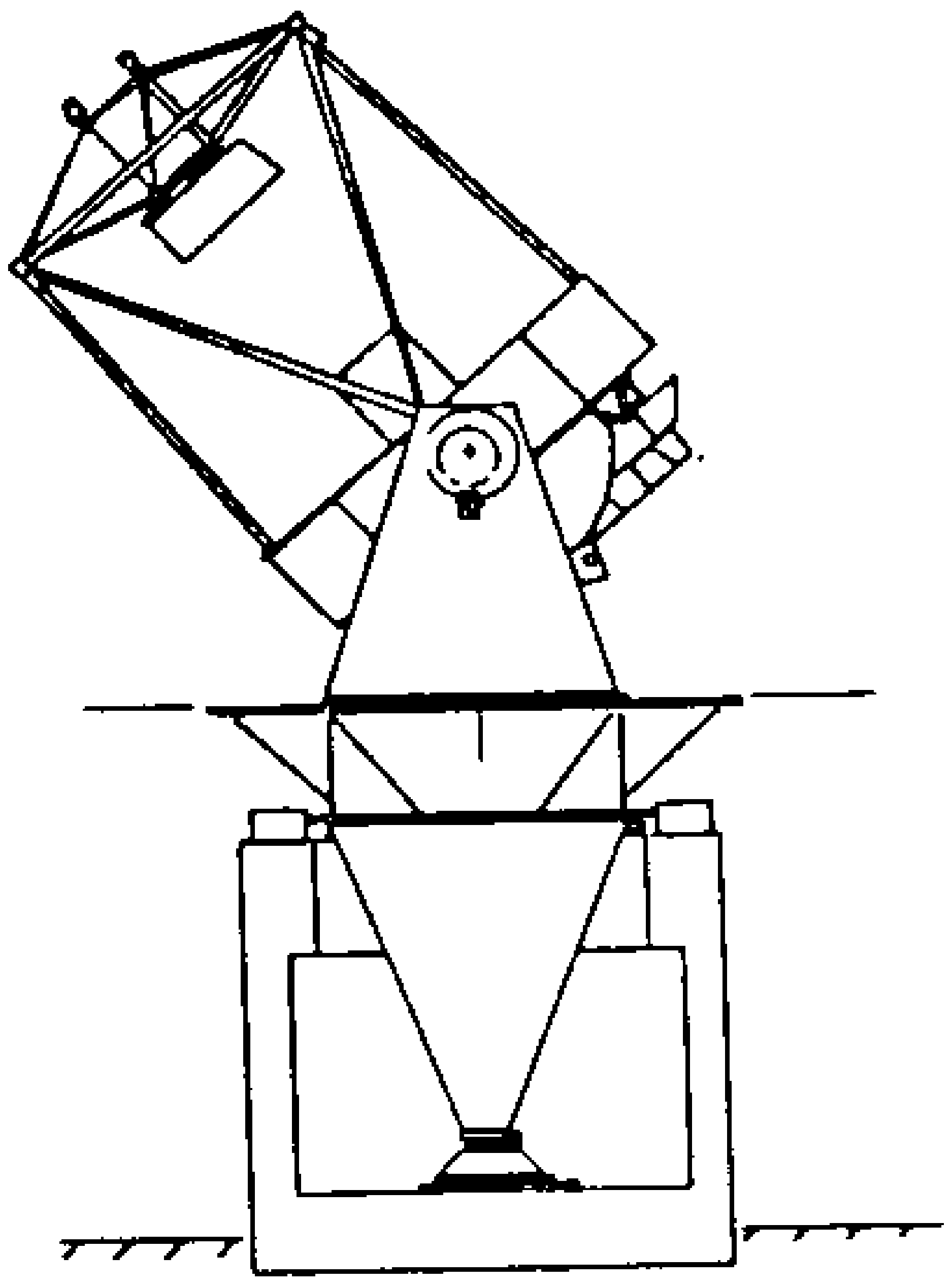

3.5 METER TELESCOPE

Tertiary Blank Selection Study for the WIYN 3.5 Meter Telescope

Technical Report

WODC 02-05-02

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1. Purpose and scope

This report addresses some of the issues basic to the selection of the tertiary mirror substrate. An implicit requirement is substrate rigidity; the material and configuration must allow polishing to a high quality flat, and it must hold its figure with a reasonably simple support scheme in the telescope environment. This limits the range of possible materials and configurations to only a few choices. The selection of one from this few is driven by considerations of thermal performance: the substrate susceptibility to thermal bowing, and the impact on seeing inside the telescope. A final important factor is the cost to the project.

2. Seeing contributions

It is widely accepted that image aberrations due to mirror seeing arise from air convection cells driven by a temperature difference between the mirror surface and the surrounding air. The typical size of these cells is on the order of a few centimeters. This high spacial frequency, stochastic contribution from each optical surface will scale (to the first order) with the effective distance of the surface from the instrument focal plane. For the tertiary and secondary mirrors the scaling distance is the physical distance from focus, but for the primary the scaling distance is effectively one system focal length - 22.05 m - from the focal plane.

Normalized to the effective focal length, the scaling factors for image aberrations due to mirror seeing for each mirror are:

Primary	**22.05/22.05 = 1.0**
Secondary	**6.925/22.05 = 0.314**
Tertiary	**3.175/22.05 = 0.144**

There are two ways in which the tertiary mirror contributes to seeing in the telescope; first directly by mirror seeing at the tertiary surface; second indirectly to the primary mirror seeing due to the mass and thermal inertia of the tertiary and its support package residing just above the primary. The first effect has a scaling factor 0.144, the second effect does not scale.

2a. Direct mirror seeing

In a now classic paper by Lowne (1979), mirror seeing was studied using a 254 mm diameter f/8 mirror in a variety of conditions and orientations. Lowne found that mirror seeing was strongly influenced by its angle of inclination or zenith distance z, with z = 0° being the worst case. Lowne also found that wind blown on the face of the mirror had a strong restorative effect on the image, and noted the dilatory effects of enclosing a mirror inside a tube since this prevents wind from reaching the glass surface.

More recent studies by Barr et al (1990) on a vertical pointing 1.8 meter borosilcate mirror found mirror seeing effects were evident when the glass-to-air temperature difference exceeded about 0.5° C. At temperature differences less than this their measurements did not show any images better than 0.07 FWHM.

A recent study by Racine et al (1991) was based on a statistical study of 562 CCD camera images taken at the prime focus of the CFHT 3.6 meter telescope. They found statistical evidence that mirror seeing effects are present for any temperature difference where the mirror is warmer than ambient air. For mirror seeing they propose a model based on Kolmorgorov turbulence:

$$\omega_m^{5/3} = a_m \Delta T_m^2 (1 - e^{\frac{-D \cot z}{h_0}})$$

where:

ω_m = Mirror seeing image degradation (FWHM)
a_m = A scaling coefficient
ΔT = Mirror to air temperature difference
D = Mirror diameter
h_0 = Scale height for decrease in air temperature difference

Racine used a value of $h_0 = 0.5$ meters, and found the scaling coefficient $a_m = 0.40$ as appropriate for the CFHT. Based on this model, the effect of inclination is strong for $D/h_0 < 1$ as in Lowne's study, but very weak for $D/h_0 >> 1$ as for the CFHT and WIYN primaries.

The tertiary mirror resides in the same thermal environment as the primary. It is reasonable to assume that h_0 will be the same as for the primary, but the optical diameter is only 0.5 meters. A reasonable estimate for the WIYN tertiary is $D/h_0 \approx 1$. Using Racine's model and the distance scaling factor 0.144, the tertiary's contribution at zenith pointing (tertiary inclination 45°) is about 0.03 FWHM at 1°C temperature difference and 0.07 FWHM at 2°C. At an average zenith distance of 30° the tertiary contribution is about 0.02 at 1° and 0.05 at 2° C temperature difference, indicating a modest dependence on zenith distance.

2b. Indirect mirror seeing

A potentially larger effect is the contribution to mirror seeing of the primary mirror. This effect is driven by the mass of the tertiary, support cell and baffle at some temperature difference from ambient creating convective cells in the in the primary beam. The effect should scale roughly as the ratio of the surface area of the tertiary and its cell to that of the primary (2/9.5).

A temperature difference in °C is roughly equal to the mirror's thermal time constant in hours times the ambient temperature slew rate in °C/hr. The thermal time constant, in turn, is determined from the effective thickness of the glass by:

$$\tau = t\rho c/h$$

where:

ρ is mass density
c is material specific heat

The effective thickness (**t**) is proportional to the lightweighting ratio. Heat transfer to or from the tertiary is dominated by convective coupling to the surrounding air. Cheng and Angel give an expression for convective heat transfer coefficient (**h**) as a function of wind velocity (**v**) and mirror diameter (**L**):

$$h = 3.4\ v^{0.8} / L^{0.2}$$

If the tertiary is enclosed in a traditional conical baffle, the heat transfer coefficient will be greatly reduced. However, if the tertiary baffle is designed to permit free air circulation then we can expect a reasonable coefficient of heat transfer from the tertiary and its cell. Assuming an "open" baffle, for a typical condition of 2 m/s breeze and a tertiary "diameter" of 1.2 m, **h** is about 6 $W/m^2/K$.

Using these equations we can calculate the thermal time constant, and estimate the temperature difference from ambient air. We can then use Racine's mirror seeing model to estimate the seeing contribution based on normal conditions (0.25° C/hr) and for adverse conditions of 0.5° C/hr or low wind. For substrates made of glass these are:

Indirect seeing contribution

lw ratio (%)	**weight (Kg)**	**τ (hrs)**	**FWHM .25°/hr**	**FWHM .5°/hr**
80	39	1.9	0.05	0.11
70	59	2.8	0.08	0.19
60	78	3.8	0.11	0.26
50	98	4.7	0.15	0.34
40	118	5.7	0.19	0.42
30	138	6.6	0.22	0.51
20	157	7.6	0.26	0.60
10	177	8.5	0.30	0.70
(solid)	197	9.5	0.34	0.79

Though this calculation is clearly approximate, the indicated trend is intuitively correct; a lighter weight mirror should more closely follow a dynamically changing ambient temperature.

Adding the direct and indirect seeing contributions in quadrature gives:

Direct and indirect seeing

lw ratio (%)	**Direct FWHM**	**Indirect FWHM**	**RMS Sum FWHM**
80	0.01	0.05	0.05
70	0.01	0.08	0.08
60	0.02	0.11	0.12
50	0.03	0.15	0.15
40	0.03	0.19	0.19
30	0.04	0.22	0.23
20	0.05	0.26	0.27
10	0.05	0.30	0.31
(solid)	0.06	0.34	0.35

Judging from this rough estimate, a glass tertiary blank should be about 70% to 80% light weight to meet the 0.06 FWHM goal under normal observing conditions.

Silicon Carbide (SiC) has considerably different material properties than glass. Because of its high rigidity, substrates up to 90% lightweight are possible. The material specific heat of SiC is nearly equal to that of glass, hence the thermal energy storage capacity of an SiC blank will be considerably less. At the same time, due to its high thermal conductivity, an SiC mirror will rapidly approach ambient air temperature.

Seeing studies to date have been done only for glass optics, so projecting seeing performance for such a different material as SiC is questionable. Never-the-less, evaluating the previous equations for a 90% light weight SiC tertiary blank (24 Kg total weight) indicates the seeing contribution will be about 0.03 FWHM.

3. Thermal bowing

Low cost makes borosilicate glass an attractive candidate for the WIYN tertiary, however its relatively high coefficient of thermal expansion makes it more susceptible to thermal bowing. For a borosilcate primary mirror bowing can be compensated by a refocus of the secondary mirror, but for an inclined folding flat the global effect of thermal bowing will introduce astigmatism to the wavefront.

3a. Wavefront aberrations due to thermal bowing

A flat mirror of thickness t meters subjected to a front-to-back temperature gradient of C (°C/meter) will take on a curvature, with the surface deformation described by:

$$S(r,\theta)=\frac{r^2\alpha C}{2}$$

where:

r,θ are polar coordinates centered on the optical axis
α = coefficient of thermal expansion

If the surface is inclined to the optical axis through an angle ϕ, the reflected wavefront aberration will be:

$$W(r,\theta)=\frac{r^2\alpha C}{\sin^2(\theta)\cos^2(\phi)+\cos^2(\theta)}$$

This expression can be divide into two components; a focus shift given by:

$$W_{020}=\frac{W(r,90^\circ)+W(r,0^\circ)}{2} \qquad (\textit{focus shift})$$

$$=\frac{r^2\alpha C}{2\cos^2(\phi)}+\frac{r^2\alpha C}{2}$$

and an astigmatism term given by:

$$W_{222}=W(r,90^\circ)-W(r,0^\circ) \qquad (\textit{astigmatism})$$

$$=\frac{r^2\alpha C}{\cos^2\phi}-r^2\alpha C$$

At an inclination angle of 45° these reduce to:

$$W_{020}=\frac{3}{2}r^2\alpha C$$

$$W_{222}=r^2\alpha C$$

In the WIYN telescope the focus will be constantly checked and corrected by focusing the secondary, so this term may be neglected.

To convert the wave aberration expression to real units, we must normalize by the radius of the wavefront at the tertiary. For an f/6.3 beam 3.175 meters from focus the wavefront is 0.5 meters in diameter (r=0.25 meters). The astigmatism due to thermal bowing is:

$$W_{222} = 0.063\alpha C \qquad (meters)$$

3b. Image aberration due to thermal bowing

Ray deviations at the focal plane due to astigmatism can be calculated by determining the slope of the wavefront (ie the derivative of W_{222}) and multiplying by the distance to the focal plane. At best focus the slope will be divided equally between the sagittal and tangential planes. Hence the wavefront slope will be:

$$slope = \pm\frac{1}{2}\,\frac{d\,W_{222}}{dr} = \pm r\alpha C$$

and the ray deviation ε at the focal plane is given by:

$$\varepsilon = \pm \mathbf{r\alpha C}(3.175) \qquad (meters)$$

Evaluating this expression at the edge of the wavefront will give the radius of 100% encircled energy (100% ee diameter = 2ε). The 50, 63, and 80% encircled energy can be estimated by evaluating ε at the radius which contains 50, 63, or 80% of the illuminated aperture area. For an evenly illuminated aperture with a 15% central obscuration, these are 0.76**r**, 0.83**r**, and 0.91**r** respectively.

For a WIYN tertiary 10 cm thick subjected to a front-to-back temperature difference of 1° C the encircled energy diameter is:

$$D_{ee} = 1.5 \times 10^5\, C_r\, \alpha \qquad (arcseconds)$$

where C_r = 1.00 for 100% ee
0.91 for 80% ee
0.83 for 63% ee
0.76 for 50% ee

The coefficients of expansion for various substrate materials are:

Material	α
Schott Tempax borosilicate (used by Hextek)	3.16 ppm/°C

Ohara E6 borosilicate (WIYN primary)	2.81
Schott Zerodur	0.12
Corning ULE	± 0.03
SiC	2.40

and the aberrated image size due to thermal bowing for these substrates materials will be:

Image size per °C front-to-back temperature difference

Material	**100% ee**	**80% ee**	**63% ee**	**50% ee**
Tempax	.48	.44	.40	.36
E6	.42	.38	.35	.32
Zerodur	.018	.016	.015	.014
ULE	.005	.005	.004	.004
SiC	.36	.33	.30	.27

For SiC this calculation is misleading; the thermal conductivity of SiC is over 100 times that of glass (k=200 W/M/°C), so a front-to-back temperature difference is much less likely to develop. In the telescope environment, SiC should surpass even ULE for resistance to thermal bowing. For the glasses temperature gradients are quite likely to develop.

Andrew Cheng modeled the thermal performance of a lightweight (borosilcate) glass secondary mirror in some detail under normal observing conditions. Assuming a factor 5 difference in heat transfer from the front and back surfaces, he found that front-to-back temperature differences as high as 0.6° C can develop.

If the tertiary experiences similar conditions then we can expect a Tempax tertiary to aberrate the image by 0.6 x 0.40 = 0.24 arcseconds 63% ee. This far exceeds the error budget goal of 0.06 FWHM for tertiary thermal contributions. Conversely, the 0.06 FWHM allowance implies the front-to-back temperature gradient would have to be held to no more than 0.15° C. This may be possible by diverting a small portion of the temperature controlled air from the primary cell into the tertiary cell, however this complicates the tertiary cell. For the WIYN project in particular, the requirement of folding the tertiary out of the way of the Cassegrain beam limits the available room behind the tertiary. This makes implementing an air circulating temperature control system extremely difficult.

4. Conclusions

The tertiary's inclination angle and proximity to the focal plane lessen its contribution due to direct mirror seeing. However, the indirect contribution to primary mirror seeing can be significant In order to limit its overall seeing contribution to be within the error budget allowance under normal observing conditions, the tertiary should be between 70% and 80% lightweight.

It is important to design the tertiary mount to allow free circulation of air around the mirror and cell. A traditional cone shaped baffle is not recommended.

Thermal bowing of an inclined mirror introduces a focus shift and an astigmatic aberration to the wavefront. For a borosilicate blank this will introduce image aberrations of about 0.40 arcseconds FWHM per degree C temperature difference front-to-back. In order to avoid this contribution a borosilicate blank would have to be temperature controlled to be isothermal to about 0.15° C. This may be possible using the temperature control technology developed for borosilicate primaries, but this greatly complicates the design of the tertiary mounting cell. Borosilicate seems a poor choice for the tertiary substrate material.

For Zerodur the effect of thermal bowing is tolerable, but not completely negligible, amounting to about 0.015 arcseconds FWHM per degree C front-to-back temperature difference. The materials of choice with regards to thermal bowing are SiC, ULE and Zerodur, in that order. Ironically, this is also in descending order of cost.

5. References

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