Disturbance rejection of the WIYN telescope position control servosystem

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ABSTRACT

The performance of a telescope position control servosystem depends on its ability to minimize changes in position due to wind and other disturbances. Modern, lightweight telescopes such as WIYN rely on feedback to achieve good disturbance rejection. While the simplicity of a direct, friction-drive system is attractive, the small drive ratio greatly diminishes the effectiveness of motor velocity control in reducing disturbance sensitivity. The WIYN servosystems use position feedback and motor torque control to achieve tracking accuracy and disturbance rejection. This type of control allows the use of larger position control bandwidth than is possible with motor velocity control.

The factors affecting disturbance rejection are discussed. Motor velocity controlled and motor torque controlled servosystems are described and compared. The WIYN telescope is analyzed to show the expected performance of each type of system, and measured performance is presented.

Keywords: telescope, position control, disturbance, telescope servosystems, control systems.

1. GENERAL CONSIDERATIONS

Pointing stability is an important factor in telescope performance. The telescope and its control system must minimize the effects of torque disturbances, particularly those due to wind, known as wind shake. The high quality of the WIYN telescope optics means that disturbances of just a few hundredths of an arcsecond may result in significant image degradation. The total error budget for wind shake is 0.12 arcseconds.

The WIYN telescope depends on its position control servos to manage disturbance response to a greater degree than many larger telescopes. The short focal length of the 3.5 Meter f/1.75 WIYN primary mirror means the telescope dimensions, total mass, and moments of inertia are small compared to older, larger designs. As a result, a simple and economical direct friction-drive system can drive the Azimuth and Elevation axes. However, as the overall dimensions of a telescope decrease, the moments of inertia decrease much faster than wind torque, making the inherent sensitivity to wind disturbance greater for a small telescope. Also, a lower drive ratio greatly decreases the effective inertia of the motors, further increasing the inherent sensitivity to disturbances. Effective motor inertia can be increased using a modest amount of motor velocity feedback, but this is effective only in systems with large drive ratios. Because the inherent disturbance sensitivity of a large telescope can be made quite small, a limited position control loop bandwidth has little effect on overall disturbance sensitivity. With the WIYN telescope, the moments of inertia are small and the effective motor inertia is negligible. Because of its small drive ratio, motor velocity feedback would not be effective unless the velocity loop bandwidth was impracticably large. Only the position control servo can reduce disturbance sensitivity to an acceptable value in the crucial 1 to 5 Hz frequency range, where wind disturbances may be significant.

This discussion considers only the effects of disturbance torque on the Azimuth and Elevation servos. Of course, other sources of image motion can result from disturbance torque, such as tilting of the fork and flexure of the telescope structure, which the Azimuth and Elevation servos do not affect.

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2. TELESCOPE MODELS

This analysis uses simple models of the telescope and mount which are sufficiently accurate and provide an intuitive understanding of the important factors involved in disturbance rejection. The results are based on model analysis, except for the test data shown in Figure 11. Additional test data was collected, showing the disturbance response to be consistent with the simple model.

3. WIYN TELESCOPE SERVO DESCRIPTION

The WIYN telescope, located at the Kitt Peak National Observatory, uses an Elevation-Azimuth mount. Each axis has two identical drive motors coupled to the telescope through steel friction rollers. The Elevation drive ratio is about 30:1, while the Azimuth drive ratio is about 35:1. The motors are current-driven, resulting in equal division of torque between the two motors on each axis and making the motor torque independent of speed. Position feedback is provided by friction-driven incremental encoders. A digital feedback compensator generates motor current commands in response to position errors. Motor velocity feedback is not used.

Many of the examples here use the numerical values typical of the WIYN Elevation axis. The principles apply equally to the Azimuth axis. Because the Elevation axis has the smaller moment of inertia and is subjected to greater wind torque, the performance of the Elevation servo with respect to disturbance rejection is more critical than that of the Azimuth axis.

4. OPEN LOOP DISTURBANCE RESPONSE

With no position feedback, the response to torque disturbances is defined here as the open loop disturbance response. Motor velocity feedback may or may not be present. Disturbance response for a single axis is defined as the ratio of position (angle) to applied disturbance torque: $D(s) = \frac{\theta_T(s)}{\tau_D(s)}$, where $\theta_T(s)$ is the telescope position, and $\tau_D(s)$ is the disturbance torque applied. Disturbance rejection is a qualitative description roughly comparable to the reciprocal of disturbance response. Hence, good disturbance rejection requires the disturbance response magnitude be small.

The simplest telescope model is that of a freely rotating rigid body with a moment of inertia J_T about the axis under consideration. For this model,

$$D(s) = \frac{1}{J_T s^2} \tag{1}$$

The frequency response is shown in Figure 1. Even without the effect of position feedback, the disturbance response is small at high frequencies if the moment of inertia is large. In practice, since wind torque is smaller at high frequencies, this open loop disturbance response is generally adequate at frequencies above about 10 Hz.

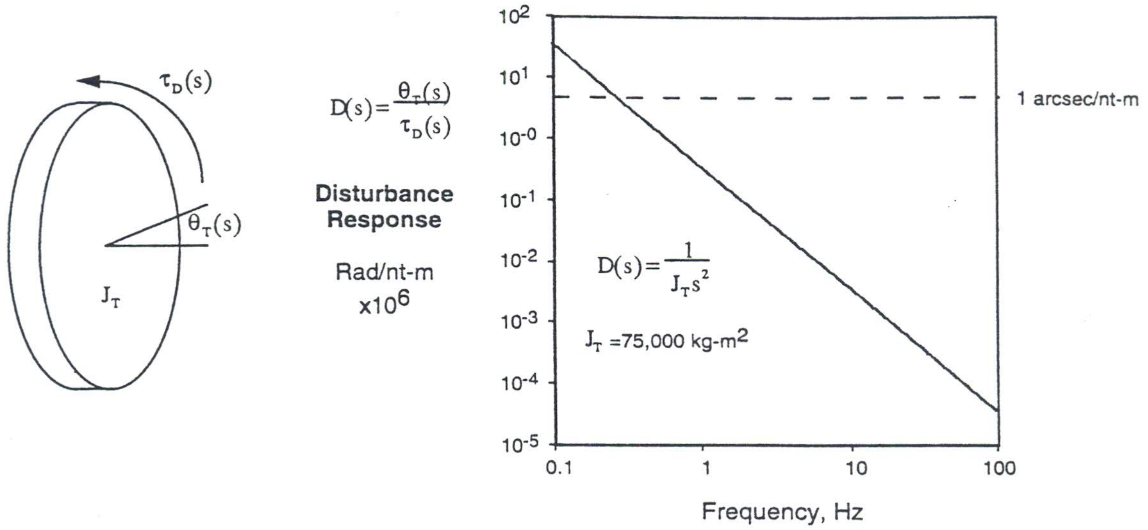

Figure 1. Disturbance Response for Freely Rotating Rigid Body

Next, consider a freely rotating system, but with the telescope coupled to the drive motor through a perfectly rigid drive mechanism and a reduction ratio of N:1, as illustrated in Figure 2. In this case,

$$D(s) = \frac{1}{s^2 (J_T + N^2 J_M)} \qquad (2)$$

where J_M is the motor's moment of inertia. Figure 2 shows the frequency response of this model. A drive ratio of 1400:1 is typical of other telescopes. The value of J_M, 0.34 kg-m^2, was arbitrarily chosen to make the disturbance response one-tenth of the Figure 1 value. This model behaves like the first, but with a larger moment of inertia. If $N^2 J_M$ is large compared to J_T, the disturbance response may be significantly smaller than that of the telescope alone. The fact that this effect depends on the square of the drive ratio is important. The motor inertia is always small compared to that of the telescope, so its effect is significant only if N^2 is very large. Motor velocity feedback may effectively increase J_M further, but the effect on disturbance response is significant only for large drive ratios.

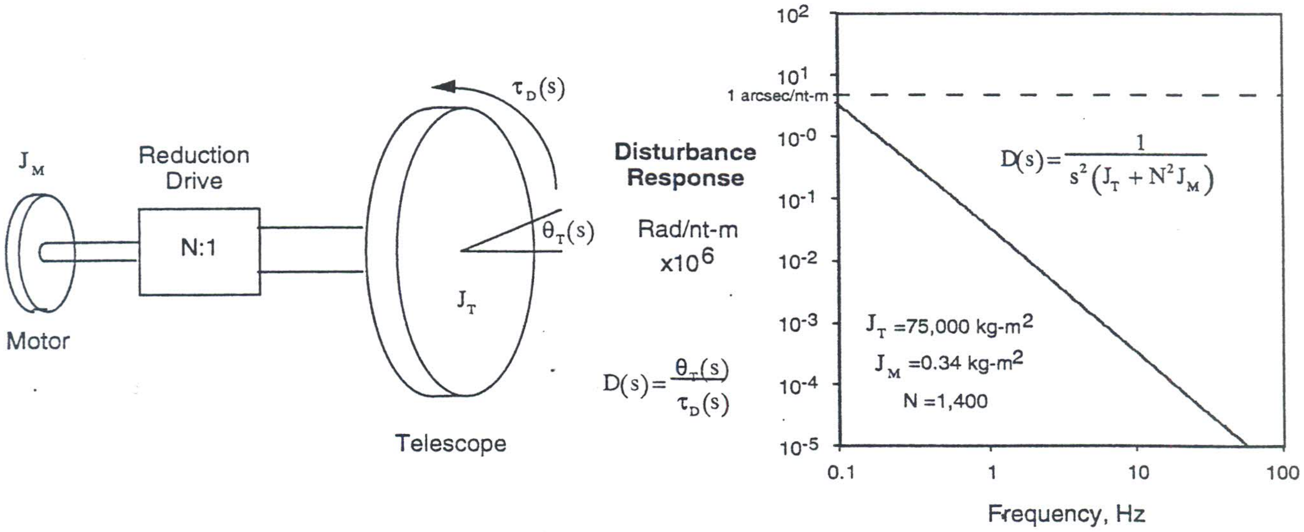

Figure 2. Telescope Rigidly Coupled to Motor

Finally, consider the above model, but with coupling of finite stiffness connecting the motor and telescope, as shown in Figure 3. This model includes the effects of the structural resonances which may have significant effects in the frequency range of interest, with damping neglected for simplicity. The response is:

$$D(s) = \frac{1}{s^2 J_T} \cdot \frac{s^2 + \dfrac{K_D}{N^2 J_M}}{s^2 + K_D \dfrac{(J_T + N^2 J_M)}{N^2 J_T J_M}} \qquad (3)$$

where K_D is the stiffness of the drive coupling. The response now includes an antiresonance at $f_A = \frac{1}{2\pi}\sqrt{\frac{K_D}{N^2 J_M}}$ and a resonance at $f_R = \frac{1}{2\pi}\sqrt{K_D \frac{(J_T + N^2 J_M)}{N^2 J_T J_M}}$. Figure 3 shows that at frequencies below f_A, the response is essentially that of the rigidly-coupled model (Equation 2), while at frequencies above f_R, the response is close to that of the freely rotating telescope alone (Equation 1). Thus, at low frequencies, the telescope may be considered rigidly coupled to the motor, while at high frequencies, it is effectively isolated from the motor. Things become more complicated near the antiresonance and resonance.

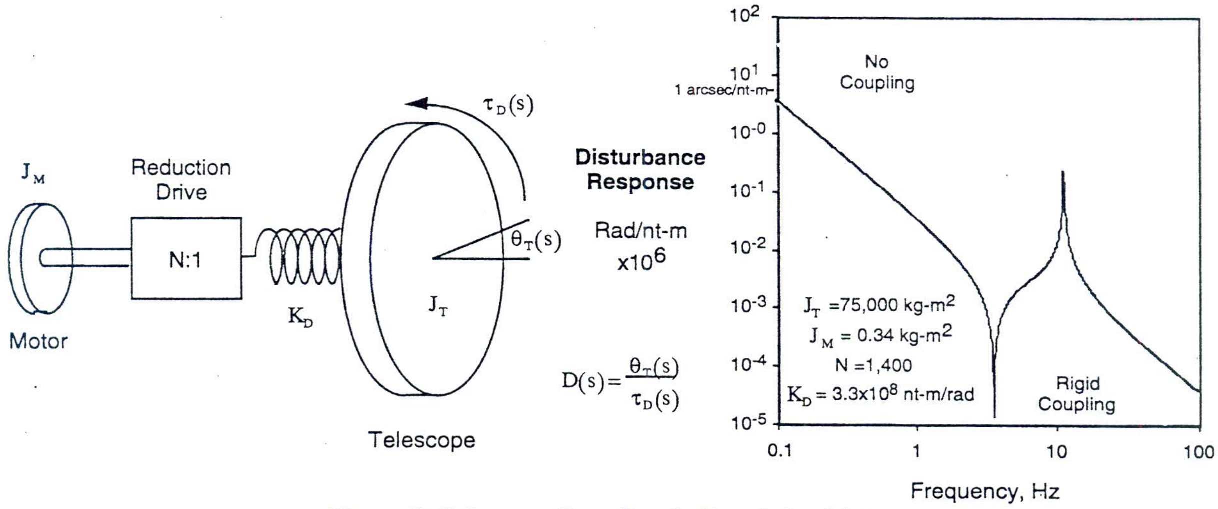

Figure 3. Telescope Compliantly Coupled to Motor

5. THE WIYN ELEVATION AXIS

The open loop disturbance response of the WIYN elevation axis can be calculated using the Equation 3 model. Typical values are $J_T = 7.5 \times 10^4$ kg-m^2, $J_M = 0.076$ kg-m^2, $K_D = 3.3 \times 10^8$ nt-m/rad, and $N = 30$. The resulting antiresonance and resonance are both near 350 Hz. Thus, at the lower frequencies of concern, the telescope and motors may be considered rigidly coupled together. However, the effective moment of inertia of the motors, $N^2 J_M$, is only 68 kg-m^2, smaller than that of the telescope alone by a factor of more than 1,000. Clearly, the motor inertia has no significant effect on open loop disturbance response. Only the moment of inertia of the telescope is important and the response up to 100 Hz is essentially identical to that shown in Figure 1.

To illustrate the effect of the drive ratio, imagine that the WIYN elevation axis used a drive ratio of 1400 instead of 30, with all other values the same. Now the antiresonance occurs at 7.5 Hz, and the resonance at 12.9 Hz. $N^2 J_M$ is now 1.5×10^5 kg-m^2, twice that of the telescope alone. At low frequencies, the motor inertia reduces the disturbance response by a factor of 3, compared to that obtained with the lower drive ratio. Figure 4 shows the open loop disturbance response with the 30:1 and hypothetical 1400:1 drive ratios.

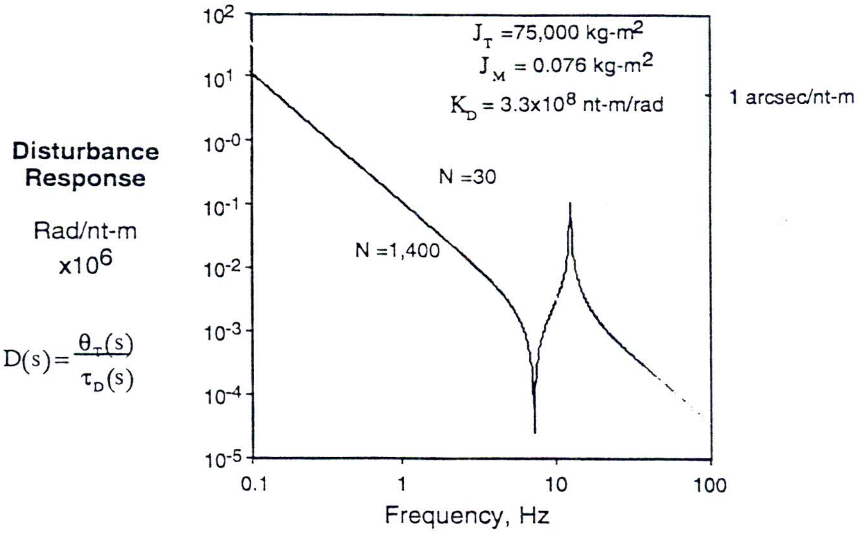

Figure 4. WIYN Elevation Axis, Open Loop Disturbance Response

6. MOTOR VELOCITY FEEDBACK

A simple, first-order motor velocity feedback loop is illustrated in Figure 5. For clarity, the velocity control input is not shown. Figure 5 is used to calculate the response of the motor to disturbance torque applied to it through the drive from the telescope. The motor disturbance response is defined as $D_M(s) = \frac{\theta_M(s)}{\tau_{DM}(s)}$. The value of $D_M(s)$ can be used to calculate the effective inertia of the motor, which, if large enough, may significantly affect the telescope's disturbance response. Without velocity feedback, $D_M(s) = \frac{1}{s^2 J_M}$, which is simply the inertial response of the motor. With feedback, analysis of Figure 5 shows:

$$D_M(s) = \frac{1}{s^2 J_M} \cdot \left(\frac{1}{1 + \frac{K_V}{s J_M}} \right) \tag{4}$$

Here it is useful to calculate the magnitude as a function of frequency:

$$|D_M(\omega)| = \frac{1}{\omega^2 J_M \sqrt{1 + \frac{K_V{}^2}{\omega^2 J_M{}^2}}} \tag{5}$$

which can also be expressed as: $|D_M(\omega)| = \frac{1}{\omega^2 J_{EQ}}$, where $J_{EQ} = J_M \sqrt{1 + \frac{K_V{}^2}{\omega^2 J_M{}^2}}$. J_{EQ} is then the equivalent moment of inertia of the motor, including the effect of velocity feedback (but not the effect of the drive ratio). For this first order velocity feedback loop, the open loop bandwidth, ω_{OL}, can be expressed as $\omega_{OL} = \frac{K_V}{J_M}$, so $J_{EQ} = J_M \sqrt{1 + \frac{\omega_{OL}{}^2}{\omega^2}}$. Thus, at frequencies that are low compared to the open loop bandwidth, the equivalent motor inertia may be much larger than J_M, while at high frequencies velocity feedback has no effect. Conversely, to have any effect on disturbance response, the open loop bandwidth must be considerably larger than the highest frequency of interest. For example, to increase the effective moment of inertia by a factor of 10 at a frequency of 5 Hz, the required velocity feedback open loop bandwidth is about 50 Hz.

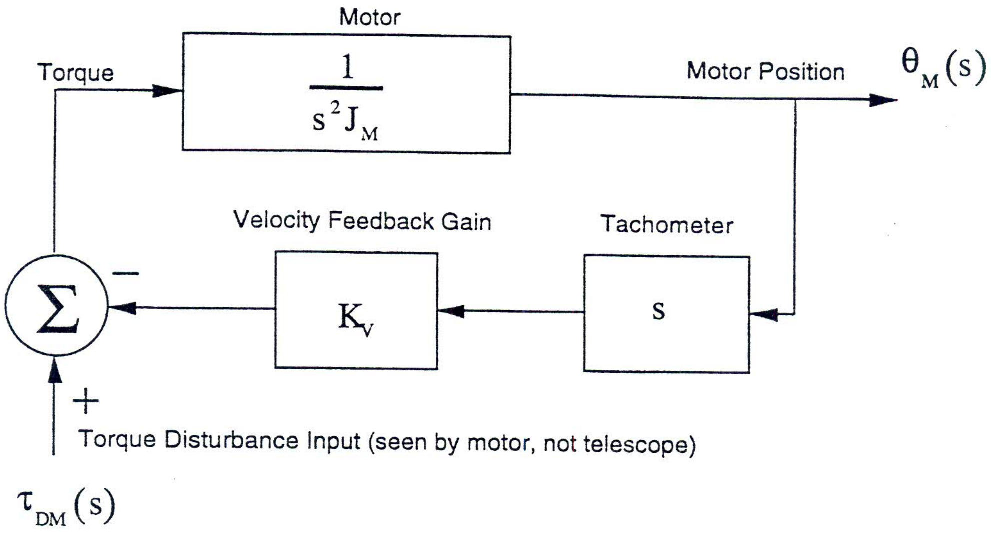

Figure 5. Motor Velocity Feedback Analysis

7. MOTOR VELOCITY FEEDBACK AND DRIVE RATIO

We can now see why motor velocity feedback effectively reduces disturbance sensitivity with a high drive ratio. Using the WIYN Elevation axis again as an example, but assuming a drive ratio of 1400:1, we have seen that even without motor velocity feedback, the effect of motor inertia reduces the disturbance sensitivity by a factor of three at low frequencies. Figure 6 compares the disturbance response for velocity feedback loop bandwidths of 10 and 50 Hz with that obtained without velocity feedback, for the hypothetical 1400:1 drive ratio. With velocity feedback, the response at low frequencies is significantly reduced. If we compare the Figure 6 response to that of Figure 1, it is clear that a considerable improvement in disturbance rejection is achieved at low frequencies even without including the effect of the position control loop. The position control loop may contribute to further improvement at low frequencies.

Figure 6 also shows that the resonance has moved down to about 11 Hz. This resonance limits the position control loop bandwidth, as discussed in Reference 1.

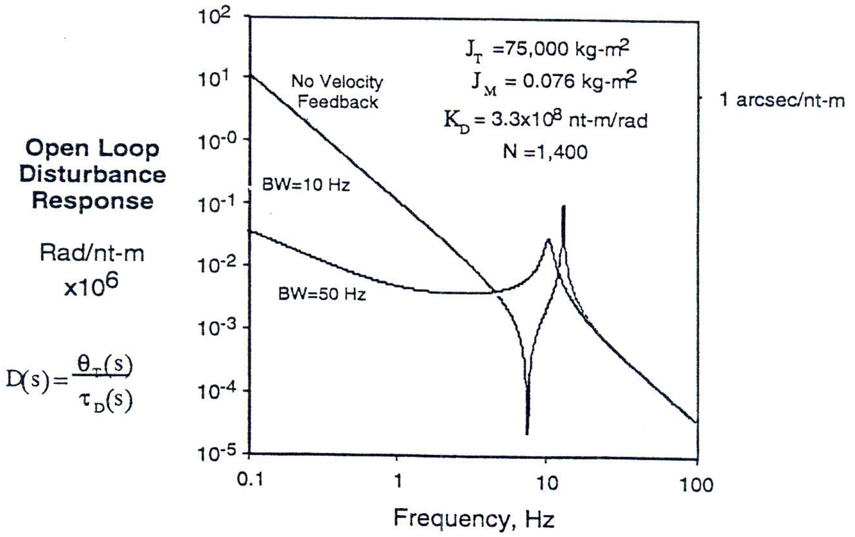

Figure 6. WIYN Elevation Axis, 1400:1 Drive Ratio, with Motor Velocity Feedback

The situation is quite different, however, with the real WIYN Elevation drive ratio of 30:1. As shown in Figure 7, motor velocity feedback has virtually no effect above 1 Hz unless the velocity loop bandwidth is at least 5 kHz. Without motor velocity feedback, the effective motor inertia is smaller than that of the telescope by a factor of over 1,000. To make the effective motor inertia equal to 10 times that of the telescope at 5 Hz, the velocity feedback open loop bandwidth must be over 50 kHz. This improves disturbance rejection at 5 Hz by a factor of about 10, as Figure 7 indicates. But the velocity feedback loop, which necessarily includes the mechanical coupling between the motor and tachometer, certainly has mechanical resonances below 50 kHz, and even below 5 kHz, resulting in instability with such high open loop bandwidth. Even with a higher order velocity feedback loop, it is unlikely that enough open loop gain can be achieved at low frequencies without resulting in instability. With the 1400:1 drive ratio example, on the other hand, the required 50 Hz bandwidth is easily attained in practice. Clearly, motor velocity feedback is of little value with the low drive ratios used on the WIYN telescope.

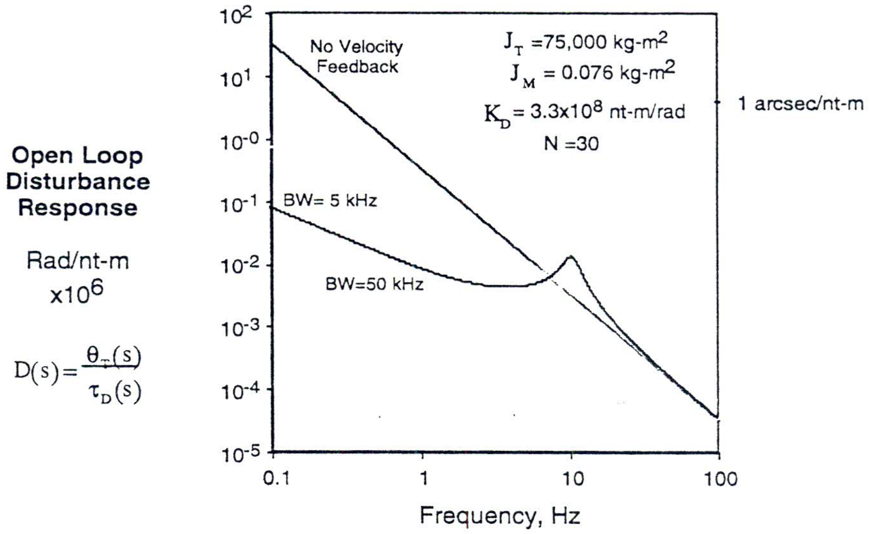

Figure 7. WIYN Elevation Axis, 30:1 Drive Ratio, with Motor Velocity Feedback

8. POSITION FEEDBACK

The effect of position feedback on disturbance response can be calculated using Figure 8. $D(s)$ is the open loop disturbance response, which may include the effects of motor inertia and motor velocity feedback. $H(s)$ is the response of the position control feedback compensator, the one part of the feedback loop over which the designer has complete control. $B(s)$ is the drive response to the compensator output; it is included because the motor torque is not applied at the same point in the system as disturbance torque. The closed loop disturbance response of the entire system, $D_{CL}(s)$, is defined as $\frac{\theta_T(s)}{\tau_D(s)}$ as above, but now position feedback is present. Analyzing Figure 8 shows that $D_{CL}(s) = \frac{D(s)}{1+H(s)B(s)D(s)}$. If $H(s)B(s)D(s)$ is defined as $A_{OL}(s)$, the closed loop disturbance response is:

$$D_{CL}(s) = \frac{D(s)}{1 + A_{OL}(s)} \tag{6}$$

The magnitude of $D_{CL}(s)$ as a function of frequency is then:

$$|D_{CL}(\omega)| = \frac{|D(j\omega)|}{|1 + A_{OL}(j\omega)|} \tag{7}$$

where $j = \sqrt{-1}$. Equation 7 shows that the closed loop disturbance response is significantly smaller than the open loop disturbance response only if $|A_{OL}(j\omega)|$ is much greater than 1. To minimize disturbance response, $|A_{OL}(j\omega)|$ should be as large as possible over the frequency range of concern, which generally extends up to the frequency at which the open loop disturbance rejection is sufficient.

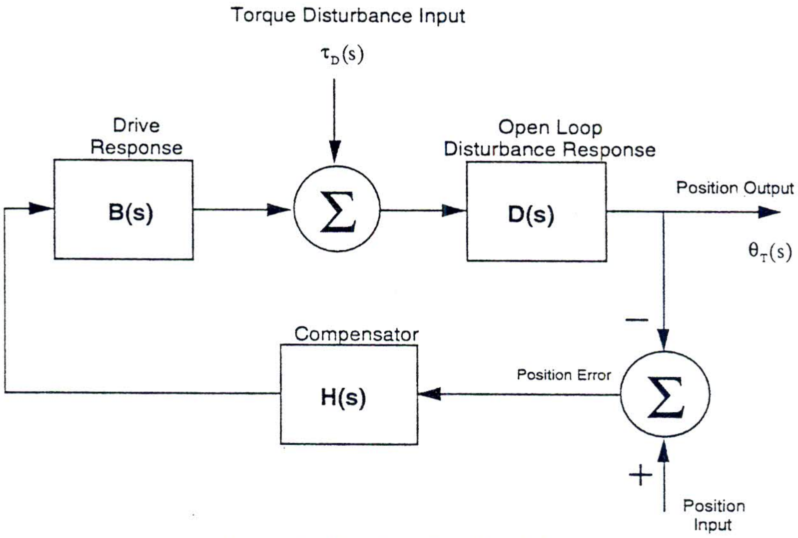

Figure 8. Position Feedback Loop

The disturbance response could be made arbitrarily small simply by selecting the appropriate compensator response, $H(s)$, except for feedback loop stability requirements. However, $|A_{OL}(j\omega)|$ must be small at high frequencies where mechanical resonances, time delays, etc., may result in instability. There is also a limit on how fast $|A_{OL}(j\omega)|$ may decrease as frequency increases. In general, the best that one can do is make $|A_{OL}(j\omega)|$ large at low frequencies, and drop below unity at the highest frequency that does not result in instability (the frequency at which $|A_{OL}(j\omega)| = 1$ is defined as the crossover frequency). As a result, position feedback may improve disturbance rejection at low frequencies, and has no effect on the disturbance response at high frequencies. Near the crossover frequency, the closed loop disturbance response may even be larger than the open loop response, depending on the compensator transfer function and other design tradeoffs. The key point is that the position feedback cannot improve disturbance rejection at any frequency above its crossover frequency. For example, improving disturbance rejection in the 4 Hz range requires a crossover frequency of greater than about 7 Hz.

9. CLOSED LOOP DISTURBANCE RESPONSE

The position control feedback loop affects the overall disturbance response, since its purpose is to minimize all position errors. The practical bandwidth of the position control loop is limited by mechanical resonances in the system. The bandwidth limit for the WIYN Elevation servo is in the range of 4 to 6 Hz. Figure 9 shows the predicted disturbance response for a position loop bandwidth of 4 Hz and various values of motor velocity feedback loop bandwidth. Even with a velocity feedback loop bandwidth of 500 Hz, the effect is small; a bandwidth of about 5 kHz is required to significantly reduce disturbance response in the 2 to 5 Hz range. Given that a tachometer capable of producing a detectable output at low speeds is physically quite large, there is little hope of stabilizing a velocity feedback loop with a 5 kHz bandwidth.

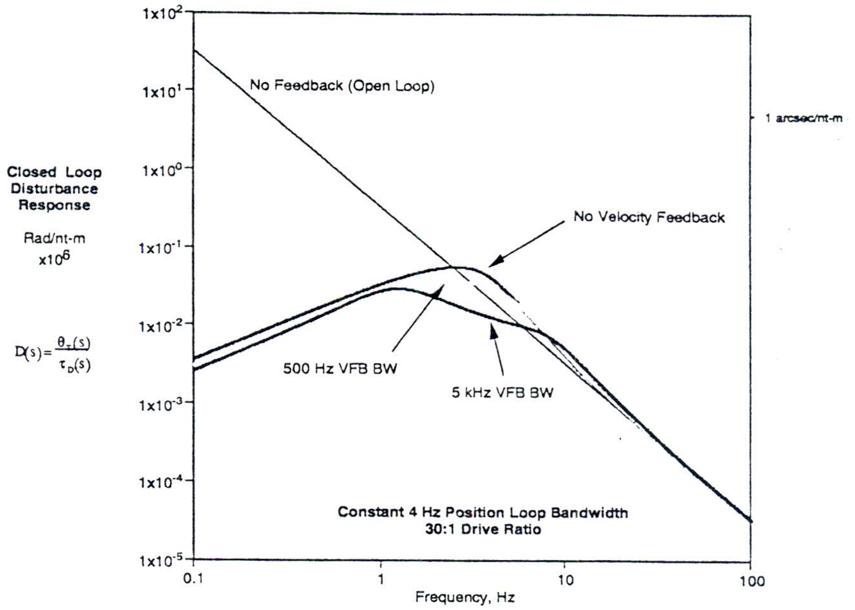

Figure 9. WIYN Elevation Axis with Position and Motor Velocity Feedback

The bandwidth of the position control loop, however, has a significant effect on closed loop disturbance response, as shown in Figure 10. Each doubling of the bandwidth reduces the disturbance response at low frequencies by a factor of 8, and the maximum disturbance response value by about a factor of 4. Clearly, it is desirable to maximize the position loop bandwidth. Without motor velocity feedback, the position loop bandwidth is limited by the effects of mechanical resonances. Because the resonance resulting from the motor inertia and drive compliance is at a high frequency (350 Hz for the WIYN Elevation axis), it is likely that other mechanical resonances at lower frequencies will limit the usable bandwidth.

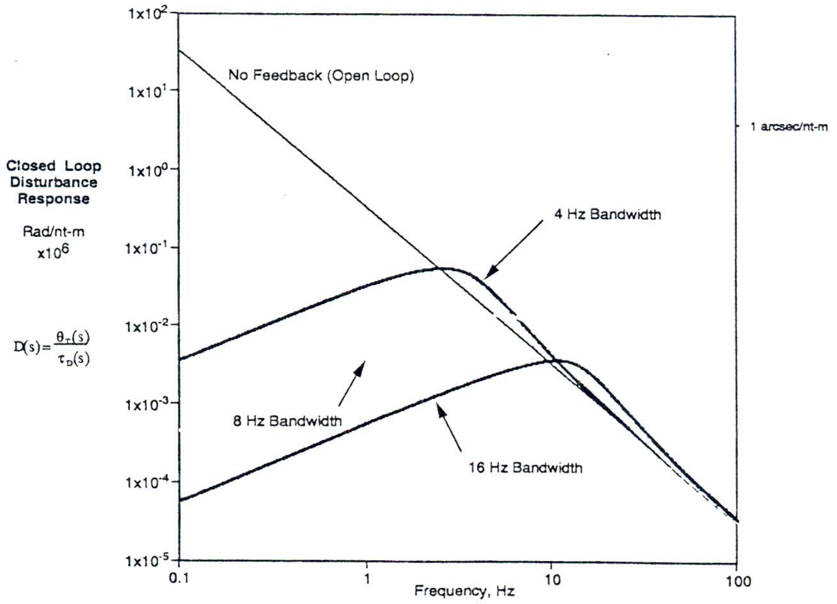

Figure 10. WIYN Elevation Axis with Position Feedback Only

10. WIYN TEST DATA

The disturbance response of the WIYN servos was measured by applying sinusoidal currents to the drive motors with the position feedback loop closed. The effect of the significant Coulomb (speed invariant) friction was eliminated by slowly slewing the telescope at constant speed during the measurement. As Figure 11 shows, the test results for the Elevation axis match the model response quite well. The drop in measured response at about 7 Hz is caused by a well-understood antiresonance due to lateral translation of the fork, and has no significant effect on disturbance response or stability. At higher frequencies, the test data show the effects of mechanical resonances not included in the model used here.

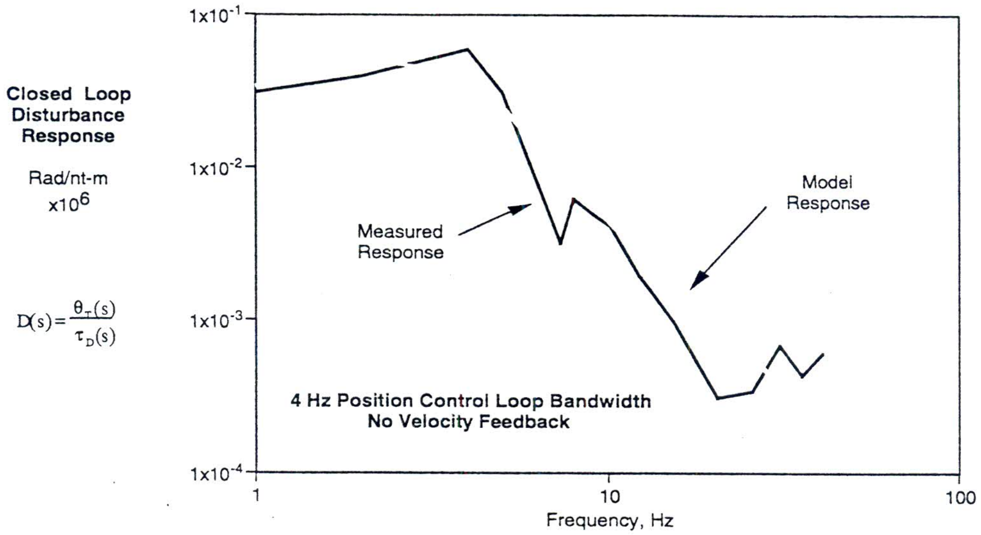

Figure 11. WIYN Elevation Axis, Measured Disturbance Response

11. CONCLUSIONS AND RECOMMENDATIONS

The choice of low drive ratios for the WIYN Elevation and Azimuth drives was primarily based on simplicity and cost. Because of the telescope's low inertia, moderate-sized motors easily provide enough torque to accelerate the telescope with a low drive ratio. It is now clear, however, that the approach to achieving adequate disturbance rejection must be different from that successfully used with larger telescopes with high drive ratios. The WIYN telescope is inherently more susceptible to wind disturbances, and motor velocity feedback is of little value.

Two methods of improving the disturbance response of the WIYN telescope servos suggest themselves. The magnitude of the disturbance torque may be minimized, and the position control loop bandwidth may be maximized. The former is a matter of mitigating the effect of wind on the telescope, perhaps by using baffles and limiting ventilation to the minimum necessary level during periods of high winds. Careful optimization of the position servo compensator allows the use of maximum bandwidth in the presence of mechanical resonances. The largest usable bandwidth for the Elevation axis is currently about 6 Hz. Increasing the position control bandwidth to 8 or 10 Hz will significantly improve disturbance rejection and reduce wind shake. Identifying the causes of the limiting mechanical resonances, and making mechanical modifications to reduce their effects, may allow the desired increase in bandwidth.

The mechanical design in future telescopes similar to WIYN must allow the largest possible position control loop bandwidth, with particular attention given to mechanical resonances in the 20 to 100 Hz range. If resonances in this range are well damped or decoupled from the servos, position control loop bandwidths of 10 Hz or greater should be possible, resulting in excellent disturbance rejection.

12. ACKNOWLEDGMENTS

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13. REFERENCES

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